Abstract

In a previous paper by Thomassen and Van Wouwe [5] the notion of an n-fold compound option was introduced as a generalization of Geske’s compound option [3]. To compute such an n-fold numerically remains possible but tedious because most algorithms are not capable to compute multivariate normal CDF’s for higher dimensions \( n \geq 3 \).

For that reason a proof to regard an n-fold as an (n-k)-fold on a k-fold is given and allows for example to decompose a 6-fold compound option into a 2-fold and a 4-fold.

keywords: financial, n-fold compound options.

1 Introduction

Definition of an n-fold compound option:
The compound call option as introduced by Geske [3] (what we call a 2-fold compound call option) is a call option with a call option as underlying. The compound call of order n (or the n-fold compound call option, with exercise date and price given by \( t_1 \) and \( K_1 \)) is a generalization where the underlying asset is a compound call of order n-1 (with exercise date and price given by \( t_2 \) and \( K_2 \)) which is itself a call on a compound call of order n-2 ... until the final underlying asset being a European call (with exercise date and price given by \( t_n \) and \( K_n \)). We are interested to know how to decompose theoretically an n-fold compound option.

We will use the following notations:
$V$ : current market value of the firm,
$t_i$ : maturity date of investment for the compound call option $C_i$ of order $n+1 - i$,
$K_i$ : exercise price for the compound call option $C_i$,
$C_i$ : current value of the compound call option with the option $C_{i+1}$ as underlying,
$r$ : risk-free rate of interest,
$\sigma^2_V$ : instantaneous variance of the return on the assets of the firm,
$N_n(a_1, a_2, \ldots, a_n; F)$ : n-variate cumulative normal distribution function with $a_i$ as upper limits and $F$ as the correlation matrix.

As we are working in a Black Scholes model [1], we use the following well-known economic assumptions [4] to build our mathematical model:

- there is no credit risk, only market risk,
- the market is maximally efficient, i.e. it is infinitely liquid and does not exhibit any friction,
- continuous trading is possible,
- the time evolution of the asset price is stochastic and exhibits geometric Brownian motion,
- the risk-free interest rate $r$ and the volatility $\sigma_V$ are constant,
- the underlying pays no dividends,
- the underlying is arbitrage-free.

2 How to decompose the n-fold compound option

In this section we will prove how the price for an n-fold compound option can be determined in two different ways. Both methods will result in the creation of a self-financing portfolio constructed by buying the n-fold call option and selling a replicating portfolio.

In the two models a different replicating portfolio is created. Since both models end up with the pricing of an n-fold compound call option, the equivalence between the resulting PDE’s will have to be demonstrated.
2.1 Deriving the PDE with the value of the firm as the underlying

In a first situation, a riskless hedge is created and maintained with two securities namely the value of the firm and the n-fold compound call. In a recent paper Thomassen and Van Wouwe [5] have developed this idea and have proven that the following expression can be found for an n-fold compound call option.

**Theorem 2.1** Suppose for \( s = n, n-1, \ldots, 2 \) the calls \( C_s \) are known and given by:

\[
C_s = V N_{n+1-s}(a_s, a_{s+1}, \ldots, a_n : A_s^{n+1-s})
- \sum_{m=s}^{n} K_m e^{-r(t_m-t)} N_{m+1-s}(b_s, b_{s+1}, \ldots, b_m : A_s^{m+1-s})
\]

where we use the notations

\[
a_\ell = b_\ell + \sigma \sqrt{t_\ell - t}
\]

\[
b_\ell = \frac{\ln V - (r - \frac{\sigma^2}{2})(t_\ell - t)}{\sigma \sqrt{t_\ell - t}}
\]

\[
\overline{V}_\ell = \text{the solution of}
\]

\[
C_{\ell+1}(V, t_\ell) = K_\ell
\]

\[
\overline{V}_n = K_n
\]

\[
\rho_{ij} = \rho_{ji} = \sqrt{\frac{t_i - t}{t_j - t}} \quad i < j
\]

\[
A_s^\ell = (a_{ij}^\ell)_{i,j=1,\ldots,\ell} \quad \text{with} \quad \left\{ \begin{array}{l}
a_{ii} = 1 \\ a_{ij} = a_{ji} = \rho_{i+s-1,j+s-1}
\end{array} \right.
\]

Then, the n-fold compound option is priced as follows:

\[
C_1 = V N_n(a_1, a_2, \ldots, a_n : A_1^n)
- \sum_{m=1}^{n} K_m e^{-r(t_m-t)} N_m(b_1, b_2, \ldots, b_m : A_1^m)
\]

This value is derived by solving the PDE:

\[
\frac{\partial C_1}{\partial t} = r C_1 - r V \frac{\partial C_1}{\partial V} - \frac{1}{2} \sigma V^2 \frac{\partial^2 C_1}{\partial V^2}
\]

with boundary condition

\[
C_1(V, t_1) = \max (0, C_2(V, t_1) - K_1)
\]
2.2 Deriving the PDE for an n-fold as an (n-k)-fold on a k-fold

In this second situation a different approach is used to determine the PDE for an n-fold compound call option. Looking at the definition of an n-fold compound call option, one could interpret the n-fold compound option as an option on an option and so on. More generally the n-fold compound option could be recognized as an (n-k)-fold option on a k-fold option.

In this section this is precisely what will be demonstrated. It will be shown that a PDE for the n-fold compound option can be obtained by developing a portfolio based on the strategy of hedging one unit of a k-fold compound option and selling an appropriate number of (n-k)-fold options (which is finally the n-fold compound call) with an adjusted volatility.

So suppose now that one wants to decompose the n-fold compound option into 2 parts: a k-fold and an (n-k)-fold. We then have the following:

\[ C_{n-k+1} = C_{n-k+1}(V, t) \text{ is a k-fold,} \]
\[ C_1 = C_1(C_{n-k+1}, t) \text{ is an (n-k)-fold.} \] (11) (12)

Concerning the k-fold \( C_{n-k+1}(V, t) \) we can use the previous section to derive its PDE as a function of \( V \):

\[ \frac{\partial C_{n-k+1}}{\partial t} = r C_{n-k+1} - r V \frac{\partial C_{n-k+1}}{\partial V} - \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 C_{n-k+1}}{\partial V^2} \] (13)

To derive the PDE for the (n-k)-fold \( C_1 \) with \( C_{n-k+1} \) as the underlying asset, one uses the following strategy (see Black and Scholes [1] for an analogue). Buy 1 asset of the k-fold \( C_{n-k+1} \) and sell \( \frac{1}{\sigma_{n-k+1}} \) parts of the (n-k)-fold \( C_1 \).

The value of our portfolio becomes:

\[ P = C_{n-k+1} - \frac{1}{\partial C_1} C_1 \] (14)

Because the value of the portfolio is independent of changes in the k-fold, the return in this hedged position is certain and equals:

\[ \Delta P = \left( C_{n-k+1} - \frac{1}{\partial C_1} C_1 \right) r \Delta t \] (15)
On the other hand one can determine \( \Delta P \) by Taylor expansion for which one needs \( \Delta C_{n-k+1} \).

Because

\[
\Delta C_{n-k+1} = C_{n-k+1}(V + \Delta V, t + \Delta t) - C_{n-k+1}(V, t)
\]  

(16)

and given the geometric Brownian motion of the underlying value \( V \) (where \( \Delta Z = \epsilon \sqrt{\Delta t} \) and \( \epsilon \sim N(0, 1) \))

\[
\Delta V = \mu V \Delta t + \sigma V \Delta Z
\]  

(17)

one finds for this change

\[
\Delta C_{n-k+1} = \mu_{n-k+1} C_{n-k+1} \Delta t + \sigma_{n-k+1} C_{n-k+1} \Delta Z
\]  

(18)

with

\[
\mu_{n-k+1} = \frac{1}{C_{n-k+1}} \left( \frac{\partial C_{n-k+1}}{\partial t} + \mu V \frac{\partial C_{n-k+1}}{\partial V} + \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 C_{n-k+1}}{\partial V^2} \right)
\]  

(19)

\[
\sigma_{n-k+1} = \sigma V \frac{\partial C_{n-k+1}}{\partial V}
\]  

(20)

In an analogous way one can determine the changes in the \((n-k)\)-fold \( C_1 \):

\[
\Delta C_1 = \mu_1 C_1 \Delta t + \sigma_1 C_1 \Delta Z
\]  

(21)

where

\[
\mu_1 = \frac{1}{C_1} \left( \frac{\partial C_1}{\partial t} + \mu_{n-k+1} C_{n-k+1} \frac{\partial C_1}{\partial C_{n-k+1}} + \frac{1}{2} \sigma_{n-k+1}^2 C_{n-k+1}^2 \frac{\partial^2 C_1}{\partial C_{n-k+1}^2} \right)
\]  

(22)

\[
\sigma_1 = \sigma_1 \frac{C_{n-k+1}}{C_1} \frac{\partial C_1}{\partial C_{n-k+1}}
\]  

(23)

So \( \Delta P \) also equals

\[
\Delta P = \Delta C_{n-k+1} - \frac{1}{\frac{\partial C_1}{\partial C_{n-k+1}}} \Delta C_1
\]

\[
= -\frac{1}{\frac{\partial C_1}{\partial C_{n-k+1}}} \left( \frac{\partial C_1}{\partial t} + \frac{1}{2} \sigma_{n-k+1}^2 C_{n-k+1}^2 \frac{\partial^2 C_1}{\partial C_{n-k+1}^2} \right) \Delta t
\]  

(24)
Putting equation (15) and equation (24) together, one obtains the PDE:

\[ \frac{\partial C_1}{\partial t} = r C_1 - r C_{n-k+1} \frac{\partial C_1}{\partial C_{n-k+1}} - \frac{1}{2} \sigma_{n-k+1}^2 C_{n-k+1} \frac{\partial^2 C_1}{\partial C_{n-k+1}^2} \]  

(25)

The derivation also reveals that if the (n-k)-fold $C_1$ is computed with as underlying asset the k-fold $C_{n-k+1}$, a new volatility $\sigma_{n-k+1}^2$ should be used as presented in formula (20).

### 2.3 Equivalence of the two models

We now are confronted with two PDE’s leading to the n-fold compound option. In the following theorem it will be shown that these two PDE’s resulting from two different models are equivalent and therefore will result in the same solution for the n-fold compound option as has to be expected.

This equivalence is not only of theoretical importance but is perhaps even more important towards the possible numerical valuation of n-fold compound options. The direct calculation of an n-fold option with $n > 4$ becomes very tedious because there is no software available to calculate the multivariate normal distribution in an adequate way.

By the equivalence of the two approaches for an n-fold compound option however it becomes possible to decompose an n-fold in a number of k-folds so that the numerical calculation remains appropriate.

The following theorem will show that the PDE constructed by assuming the n-fold as an (n-k)-fold on a k-fold, can be transformed into the other PDE.

**Theorem 2.2 First PDE:** $C_1$ is the solution of

\[ \frac{\partial C_1}{\partial t} = r C_1 - r V \frac{\partial C_1}{\partial V} - \frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 C_1}{\partial V^2} \]  

(26)

**Second PDE:** $C_1$ is the solution of

\[ \frac{\partial C_1}{\partial t} = r C_1 - r C_{n-k+1} \frac{\partial C_1}{\partial C_{n-k+1}} - \frac{1}{2} \sigma_{n-k+1}^2 C_{n-k+1} \frac{\partial^2 C_1}{\partial C_{n-k+1}^2} \]  

(27)

with $C_{n-k+1}$ the underlying, satisfying itself a similar PDE

\[ \frac{\partial C_{n-k+1}}{\partial t} = r C_{n-k+1} - r V \frac{\partial C_{n-k+1}}{\partial V} - \frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 C_{n-k+1}}{\partial V^2} \]  

(28)

PDE (27) can be rewritten into PDE (26).
Proof

We start with PDE (27). Given $C_1$ as a function of $C_{n-k+1}$ and of time $t$, it can be written as follows:

$$C_1 = C_1(C_{n-k+1}, t).$$  \hspace{1cm} (29)

To avoid confusion we will use the following notations:

- $D_1$ means the partial derivative towards the first variable,
- $D_2$ means the partial derivative towards the second variable,
- $C_1^*$ means that we work with $C_{n-k+1}$ as underlying asset,
- $C_1$ means that we work with $V$ as underlying asset.

(Of course $C_1^*(C_{n-k+1}, t) \equiv C_1(V, t)$)

So PDE (27) can be rewritten as:

$$D_2 C_1^* = r C_1^* - r C_{n-k+1} D_1 C_1^* - \frac{1}{2} \sigma_{n-k+1}^2 C_{n-k+1}^2 D_{11} C_1^*$$ \hspace{1cm} (30)

According to the partial differential theory, we know:

$$D_1 C_1 = D_1 C_1^* \cdot \frac{\partial C_{n-k+1}}{\partial V}$$ \hspace{1cm} (31)

$$D_2 C_1 = D_1 C_1^* \cdot \frac{\partial C_{n-k+1}}{\partial t} + D_2 C_1^*$$ \hspace{1cm} (32)

Furthermore we can derive the following:

$$D_{11} C_1 = \frac{\partial}{\partial V} \left[ D_1 C_1^* \cdot \frac{\partial C_{n-k+1}}{\partial V} \right]$$
$$= \frac{\partial}{\partial V} [D_1 C_1^*] \cdot \frac{\partial C_{n-k+1}}{\partial V} + D_1 C_1^* \cdot \frac{\partial}{\partial V} \frac{\partial C_{n-k+1}}{\partial V}$$
$$= D_1 \left[ \frac{\partial C_1}{\partial V} \right] \cdot \frac{\partial C_{n-k+1}}{\partial V} + D_1 C_1^* \cdot \frac{\partial^2 C_{n-k+1}}{\partial V^2}$$
$$= D_1 \left[ D_1 C_1^* \cdot \frac{\partial C_{n-k+1}}{\partial V} \right] \cdot \frac{\partial C_{n-k+1}}{\partial V} + D_1 C_1^* \cdot \frac{\partial^2 C_{n-k+1}}{\partial V^2}$$
$$= D_{11} C_1^* \left( \frac{\partial C_{n-k+1}}{\partial V} \right)^2 + D_1 C_1^* \cdot D_1 \left[ \frac{\partial C_{n-k+1}}{\partial V} \right] \cdot \frac{\partial C_{n-k+1}}{\partial V}$$
\[ \begin{align*}
+ D_1 C_1^* \cdot & \frac{\partial^2 C_{n-k+1}}{\partial V^2} \\
= \quad & D_{11} C_1^* \left( \frac{\partial C_{n-k+1}}{\partial V} \right)^2 + D_1 C_1^* \cdot \frac{\partial^2 C_{n-k+1}}{\partial V^2}
\end{align*} \]

These equations make it possible to rewrite PDE (27) into PDE (26):

\[ D_2 C_1^* = r C_1^* - r C_{n-k+1} D_1 C_1^* - \frac{1}{2} \sigma^2_{n-k+1} C_{n-k+1}^2 D_{11} C_1^* \]

\[ \downarrow \text{ use equations (32) and (20)} \]

\[ D_2 C_1 - D_1 C_1^* \cdot \frac{\partial C_{n-k+1}}{\partial t} = r C_1^* - r C_{n-k+1} D_1 C_1^* - \frac{1}{2} \sigma^2_{n-k+1} C_{n-k+1}^2 \left( \frac{\partial C_{n-k+1}}{\partial V} \right)^2 D_{11} C_1^* \]

\[ \downarrow \text{ use equation (33)} \]

\[ D_2 C_1 = r C_1^* + D_1 C_1^* \cdot \left( \frac{\partial C_{n-k+1}}{\partial t} - r C_{n-k+1} \right) - \frac{1}{2} \sigma^2_{n-k+1} \frac{\partial C_{n-k+1}}{\partial V} \left( D_{11} C_1 - D_1 C_1^* \frac{\partial C_{n-k+1}}{\partial V^2} \right) \]

\[ \downarrow \]

\[ D_2 C_1 = r C_1^* - \frac{1}{2} \sigma^2_{n-k+1} V^2 D_{11} C_1 + D_1 C_1^* \cdot \left( \frac{\partial C_{n-k+1}}{\partial t} - r C_{n-k+1} \right) + \frac{1}{2} \sigma^2_{n-k+1} \frac{\partial^2 C_{n-k+1}}{\partial V^2} \]

\[ \downarrow \text{ use equation (28)} \]

\[ D_2 C_1 = r C_1^* + D_1 C_1^* \cdot \left( -r V \frac{\partial C_{n-k+1}}{\partial V} \right) - \frac{1}{2} \sigma^2_{n-k+1} V^2 D_{11} C_1 \]

\[ \downarrow \]

\[ \frac{\partial C_1}{\partial t} = r C_1 - r V \frac{\partial C_1}{\partial C_{n-k+1}} \frac{\partial C_{n-k+1}}{\partial V} - \frac{1}{2} \sigma^2_{n-k+1} V^2 \frac{\partial^2 C_1}{\partial V^2} \]

which equals exactly equation (26).
3 Numerical examples

This principle can be of importance if one is interested in the valuation of a new drug application. As was pointed out in the article by Cassimon et al. [2] a new drug application can be seen as an n-fold compound option where, depending on the value of a stage, a new stage will be entered or the research will be stopped due to unsatisfying results.

In very practical situations, the value of such a new drug application can be determined as a 6-fold compound option with the following values for the variables:

- risk free rate of interest \( r = \ln 1.053 \),
- volatility \( \sigma = 1.02 \),
- exercise prices in thousand USD:
  \( k_0 = 2200, k_1 = 13800, k_2 = 2800, k_3 = 6400, k_4 = 18100, k_5 = 3300, k_6 = 31200 \)
  where \( k_6 \) represents the investment cost at time \( t_6 \)
- maturity dates:
  \( t_1 = 2, t_2 = 6, t_3 = 7, t_4 = 9, t_5 = 12, t_6 = 14, \)
- present value at time \( t_6 \): \( V = 85,000 \) thousand USD

Due to some computational problems, it is not possible to compute straight-forward a 6-fold compound option. However theorem 2.2 allows us to decompose the 6-fold into compound options of a smaller order (for instance as a 2-fold on a 4-fold, as a 4-fold on a 2-fold or as a 3-fold on a 3-fold), leading to the value of the 6-fold, for different choices of volatility \( \sigma \) and underlying value \( V \) at time \( t_6 \). It can immediately be seen that the 6-fold increases if either one of these values increases, as was shown by Thomassen and Van Wouwe [5].

The economic decision making will depend on the comparison of the value of the 6-fold compound option with the initial cost \( k_0 \).

The 6-fold computed as a 2-fold on a 4-fold
<table>
<thead>
<tr>
<th>$\sigma \setminus V$</th>
<th>75 000</th>
<th>80 000</th>
<th>85 000</th>
<th>90 000</th>
<th>95 000</th>
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The 6-fold computed as a 4-fold on a 2-fold

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The 6-fold computed as a 3-fold on a 3-fold

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<td>19 788.0</td>
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<tr>
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<td>0.94</td>
<td>21 621.8</td>
<td>23 623.2</td>
<td>25 650.4</td>
<td>27 701.1</td>
<td>29 772.8</td>
</tr>
<tr>
<td>0.98</td>
<td>22 435.3</td>
<td>24 464.1</td>
<td>26 516.9</td>
<td>28 591.3</td>
<td>30 685.2</td>
</tr>
<tr>
<td>1.02</td>
<td>23 205.7</td>
<td>25 260.0</td>
<td>27 336.5</td>
<td>29 432.8</td>
<td>31 547.2</td>
</tr>
</tbody>
</table>

Since a wide variety of decompositions for the 6-fold are available, theorem (2.2) can be applied for each of these situations. The resulting numerical values therefore should be close to each other as is demonstrated by the joined tables.
4 Conclusions

We proved that it is possible to decompose an n-fold into a k-fold that can be computed using compound option theory (see Thomassen and Van Wouwe [5]), and an (n-k)-fold that can be computed using the k-fold as underlying asset and considering the appropriate value for the volatility.

Considering an n-fold as a composition of several compound options of lower order, is a generalization of the previous result and can be carried out analogously when we adjust the volatility of the underlying assets correctly.

References


