EVALUATION OF THE THROUGHPUT OF TCP IN A NETWORK WITH WIRELESS ACCESS*

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ABSTRACT

The throughput of a TCP connection in the presence of segment losses, due to the errors in a wireless link, is investigated by means of an analytical model. We show how a TCP connection that does not experience random losses which shares a bottleneck link with a TCP connection that does experience random losses benefits from the throughput degradation of the latter. Simulation results validate the analytical model.
The correlation structure of the losses due to the wireless medium is an essential characteristic that cannot be neglected. Therefore this correlation is incorporated in the model and its influence on the TCP throughput is studied.
Since TCP always responds to segment losses by congestion control (i. e. by shrinking the window), the error control is often pushed to the data link layer such that TCP only experiences segment losses due to congestion. The additional time to recover errors on the data link layer leads to a longer round trip time. We investigate trade-offs between throughput decrease due to random losses and throughput decrease due to longer round trip times.

1 Introduction

TCP provides a reliable connection-oriented byte stream service between two end systems. In addition, TCP implements a flow control mechanism [11] that allows the reduction of transmission rates when network congestion occurs. The different versions [6] of TCP (Tahoe, Reno, Vegas, ...) all use a window mechanism, in which the size of the window is dynamically adjusted based on acknowledgements that are sent back by the receiving station. When a segment loss is detected, the window size is reduced and the lost segment(s) is (are) retransmitted.

When the transmission medium is unreliable (e. g. in a wireless environment) additional segment losses may occur due to the high bit error rate (BER) of the medium. TCP will react on these segment losses by reducing (unnecessarily) the transmission rate.

In this paper we investigate the influence of this phenomenon on the TCP throughput and fairness. In order to investigate the fairness issue in the presence of random losses, we consider a network, where a TCP connection that experiences random losses shares a congested link with a TCP connection that does not experience such losses. Section 2 gives a detailed description of the used network configuration and its important parameters, while Sections 3 and 4 describe the modelling of the wireless access channel and the TCP sources. The analytical derivation of the throughput of both flows can be found in Section 5. Finally, Section 6 illustrates how the throughput of the wired connection benefits from the random losses of the other connection.

Segment losses in the wireless access network due to the unreliability of the medium may be solved on the data link layer, before offering the segments to the transport layer. Error recovery on the data link layer introduces some extra delay, and hence the round trip time (RTT) of the TCP connection is increased, leading to a decrease in throughput. We determine a trade-off between the throughput decrease due to an increase in round trip time (with error recovery at data link layer) and the decrease in throughput due to segment loss (without error recovery at data link layer). This analysis and the corresponding results can be found in Section 7.

An important characteristic of the losses due to the wireless access medium is the correlation structure. The analytical model also comprises this. We conclude this paper (in Section 8) with a study of the influence of this correlation on the TCP throughput.

Performance analysis of TCP in the presence of random losses has been investigated in [4]. In this paper, two analytical models are developed which allow to investigate the effect of TCP parameters (e. g. the Fast Retransmit Threshold) on the throughput. Other models for TCP flow control can be found in [2], [3], etc. TCP throughput improvement by pushing the error control to lower layers have been proposed and analysed in e. g. [10], [5] and [1].

2 Network Model

Consider the network model depicted in Figure 1. SES1 is a source with a wired connection to the network, while SES2 is connected to the network via a wireless access network. Assume that both sources have an active TCP connection routed over two intermediate routers that are connected with a bottleneck link. The connections have destination DES1, resp. DES2. Suppose that information is transmitted in TCP segments of constant length (m bytes). Assume the following parameters for this network: the bandwidth of the shared link is b segments/time unit, the buffer capacity of the first router is B segments and the propagation delay between SESi and DESi is τ_i time units.

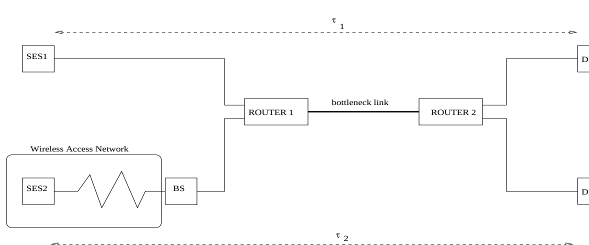

Figure 1: Network Model

We define W_{max} as the maximum number of segments the network can accommodate before a loss occurs. Then $W_{max} = B + BW$, where BW is the number of segments that are on the link when it is fully utilised. BW satisfies the following relations:

$$b = \frac{cwnd_1(t)}{2\tau_1} + \frac{cwnd_2(t)}{2\tau_2}$$

$$BW = cwnd_1(t) + cwnd_2(t)$$

where $cwnd_i(t)$ denotes the transmission window of TCP connection i at time t. In case of equal round trip times (i. e. $\tau_1 = \tau_2 = \tau$), $W_{max} = B + 2\,\tau\, b$.

3 The Wireless Medium

The wireless access part introduces additional errors for connection 2. To model the correlated character of these errors, the wireless channel is represented by a variant of the Gilbert-Elliott (GE) model, which is often used in performance studies of wireless links ([5],[9]). The GE-model is a two-state Markov chain where the two states are named *Good* and *Bad*. In the *Good* state segment errors occur with probability p^g and in the *Bad* state with probability p^b, $p^g \ll p^b$. The duration of the *Good*, resp. *Bad* state, is geometrically distributed with mean $\frac{1}{\delta}$, resp. $\frac{1}{\gamma}$ RTTs. We denote the underlying transition matrix of the Markov chain by M,

$$M = \begin{pmatrix} (1-\delta) & \delta \\ \gamma & (1-\gamma) \end{pmatrix}.$$

The fraction of time losses may occur can be computed as follows:

$$\mathrm{LP}_{\mathrm{wir}} = \frac{p^g\,\frac{1}{\delta} + p^b\,\frac{1}{\gamma}}{\frac{1}{\delta} + \frac{1}{\gamma}}.$$

Note that the segment error probabilities of the *Good* and *Bad* state (p^g and p^b) can be derived from the respective bit error rates BER^g and BER^b. If we denote the number of payload bits in a segment by K_{payload}, then $p^g = 1 - (1-\mathrm{BER}^g)^{K_{\mathrm{payload}}}$ and $p^b = 1 - (1-\mathrm{BER}^b)^{K_{\mathrm{payload}}}$.

We denote by p_i^g the probability that due to the wireless transmission medium at least one segment is lost when i segments are sent and when the wireless channel is in the *Good* state, i. e. $p_i^g = 1-(1-p^g)^i$. Analogous, we define $p_i^b = 1-(1-p^b)^i$ for the *Bad* state.

4 TCP Dynamics

The TCP congestion control mechanism is supposed to be a simplified version of TCP Reno, the dynamics of which are described in what follows. Assume that segments have constant length of m bytes. A TCP connection can be in two phases [8], namely in a *Slow Start* phase and a *Congestion Avoidance* phase. Denoting the current window size at time t by $cwnd(t)$ and the threshold value at which TCP changes phase by TH, then upon arrival of an acknowledgement after a time t_{ACK}:

$$cwnd(t + t_{\mathrm{ACK}}) = \begin{cases} cwnd(t) + m & cwnd(t) < TH \\ cwnd(t) + \frac{m^2}{cwnd(t)} & cwnd(t) \geq TH \end{cases}.$$

In TCP Tahoe, when a segment loss occurs at time t, then $TH = max(2m, cwnd(t)/2)$ and $cwnd(t^+) = m$. TCP Reno is similar to TCP Tahoe, except upon segment loss instants. When a segment loss is detected, it enters the *Fast Retransmit/Recovery* Phase, in which the window size is increased by one segment when a duplicate acknowledgement is received and $cwnd(t)$ is reset to TH when the non-duplicate acknowledgement corresponding to the retransmitted segment is received. This means that at segment loss, the window size is temporarily increased (to react fast on segment loss) but is soon restored to the new threshold. In this paper we ignore the Slow Start and Fast Retransmit/Recovery Phase, as the duration of it should be limited. Hence the window size always grows linearly when no loss occurs and is reduced to half of the current size upon segment loss, as will be explained in more detail in Section 5.1. For reasons of computational complexity, we let the window size be expressed in number of segments instead of in bytes.

5 TCP Throughput in a Wireless Access Environment

5.1 Window Size Evolution

Let us consider the network model of Figure 1 with one of the SES connected to the network via a wireless link with $\tau_1 = \tau_2 = \tau$. Thus apart from the segment losses on the bottleneck link, connection 2 is faced with segment losses due to the unreliability of the wireless medium.

Similar to the model used in [7], we assume that in case of congestion, i. e. when the sum of the window sizes $cwnd_1$ and $cwnd_2$ reaches the value W_{max}, *both connections* lose a segment and consequently their window size is divided by 2 (but minimum two segments). Define a *cycle* to be the time between two consecutive losses due to congestion. Note that in view of the above remark, the notion of cycle is independent from the connection. Let M_i be the state of the wireless channel at the end of the i^{th} cycle. Denote by W_i^1, resp. W_i^2 , the window size of connection 1, resp. connection 2, at the end of the i^{th} cycle. A typical evolution of the window sizes of the two connections is depicted in Figure 2, where X denotes a segment loss.

Clearly $W_i^1 + W_i^2 = W_{max}$. Therefore the state of the system at the end of the i^{th} cycle can be completely described by the pair $s_i = (M_i, W_i^2)$. We number the states from 1 to $2W_{max}$. For a state $s_i \in [1, W_{max}]$, the state of the wireless medium (M_i) is *Good* and the window size of flow 2 (W_i^2) is equal to s_i. For a state $s_i \in [W_{max}+1, 2W_{max}]$: $M_i =$ *Bad* and $W_i^2 = s_i - W_{max}$.

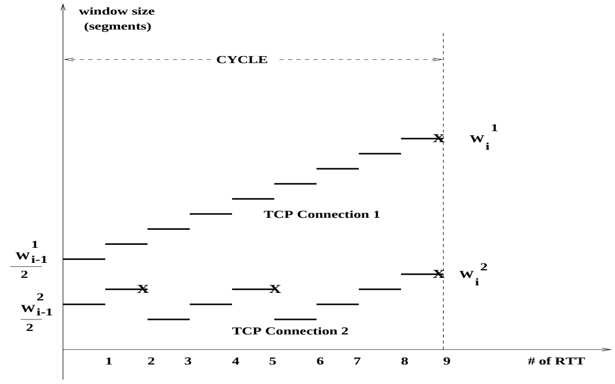

Figure 2: Window Evolution

5.2 Throughput during a Cycle

Now we compute the throughput of connection 2 during a typical cycle. Assume that connection 2 starts at system state $s_0 = (m_0, w_0^2)$ with medium state m_0 and a window size w_0^2; then connection 1 must start with window size w_0^1, with $2(w_0^1 + w_0^2) = W_{max}$. Define the $2W_{max} \times 2W_{max}$ matrix Q, describing the joint evolution per round trip time of the state of the wireless medium and the window size of connection 2 without the presence of connection 1, i. e. when connection 2 can take the whole system capacity W_{max}. Then

$$Q = \begin{pmatrix} (1-\delta)A & \delta A \\ \gamma B & (1-\gamma)B \end{pmatrix}$$

with

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & p_2^g & 1-p_2^g & 0 & \dots & 0 \\ 0 & p_3^g & 0 & 1-p_3^g & \dots & 0 \\ 0 & p_4^g & 0 & 0 & \dots & 0 \\ 0 & p_5^g & 0 & 0 & \dots & 0 \\ 0 & 0 & p_6^g & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1-p_{(W_{max}-1)}^g \\ 0 & 0 & 0 & 0 & \dots & 1-p_{W_{max}}^g \end{pmatrix}$$

and

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & p_2^b & 1-p_2^b & 0 & \dots & 0 \\ 0 & p_3^b & 0 & 1-p_3^b & \dots & 0 \\ 0 & p_4^b & 0 & 0 & \dots & 0 \\ 0 & p_5^b & 0 & 0 & \dots & 0 \\ 0 & 0 & p_6^b & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1-p_{(W_{max}-1)}^b \\ 0 & 0 & 0 & 0 & \dots & 1-p_{W_{max}}^b \end{pmatrix}.$$

Submatrix A is a $W_{max} \times W_{max}$ matrix which describes the evolution per round trip time of the window size of connection 2 (without the presence of connection 1), when the state of the wireless medium at the beginning of the round trip time is *Good*. Matrix B is analogous defined but for the case where the wireless medium is in the *Bad* state at the beginning of the round trip time. (Remark that when a loss occurs the window size is set to $\min(2, \lfloor \frac{cwnd}{2} \rfloor)$. This explains why the first column of A and B contains zeros.)

Now we take into account the presence of connection 1. Due to the constraint that $cwnd_1 + cwnd_2 < W_{max}$, the evolution of connection 2 can be described as follows. Define the starting position of the system state by the probability vector $\overline{v}_0(s_0)$, with

$$\overline{v}_0(s_0) = (0, 0, \ldots, 0, 1, 0, \ldots, 0)$$

where the 1 occurs at the s_0^{th} position.

At the beginning of the round trip time, the windows of connection 1 and 2 are w_0^1 and w_0^2. The cycle will end when a segment is lost. Segment loss occurs when $w_0^1 + w_0^2 \geq W_{max}$. Hence, by multiplying vector $\overline{v}_0(s_0)$ with the matrix Q in which the rows corresponding to a system state with a window size of flow 2 larger then $W_{max} - w_0^1 - 1$ are replaced by rows with zeros, we obtain the possible paths that the window size of connection 2 may follow in a complete round trip time, starting from state s_0. Thus,

$$\overline{v}_1(s_0) = \overline{v}_0(s_0) \times \begin{pmatrix} Q(1 : W_{max} - w_0^1 - 1, 1 : 2\,W_{max}) \\ O(1 : w_0^1 + 1, 1 : 2\,W_{max}) \\ Q(W_{max} + 1 : 2\,W_{max} - w_0^1 - 1, 1 : 2\,W_{max}) \\ O(1 : w_0^1 + 1, 1 : 2\,W_{max}) \end{pmatrix}.$$

(We have used notation $A(a : b, c : d)$ to indicate a $(b - a + 1) \times (d - c + 1)$ matrix consisting of the rows a to b and columns c to d of the matrix A; O is a matrix with all entries 0). To obtain the various paths for higher number of round trip times, we use the following analogous recursive formula:

$$\overline{v}_i(s_0) = \overline{v}_{i-1}(s_0) \times \begin{pmatrix} Q(1 : W_{max} - w_0^1 - i, 1 : 2\,W_{max}) \\ O(1 : w_0^1 + i, 1 : 2\,W_{max}) \\ Q(W_{max} + 1 : 2\,W_{max} - w_0^1 - i, 1 : 2\,W_{max}) \\ O(1 : w_0^1 + i, 1 : 2\,W_{max}) \end{pmatrix}.$$

Clearly $\overline{v}_{W_{max} - w_0^1 - 1}(s_0) = (0, 0, \ldots, 0)$.

Due to the cyclical evolution of the system, the long run throughput of both connections can be computed as the number of segments transmitted in a cycle, divided by the cycle duration. The cycle duration is given by the length of a RTT multiplied with the number of RTTS in the cycle. To determine the number of segments transmitted in a cycle, we need to add the number of segments transmitted during each of the RTTs in the cycle.

The throughput of connection 2, given that the starting system state is s_0, is then given by

$$T_2(s_0) = \frac{1}{(2\tau + WT) \sum_{(i)} \overline{v}_i(s_0)\overline{e}} \sum_{i=0}^{W_{max} - w_0^1 - 2} \Bigg[\Big(\overline{v}_i(s_0)(1 : W_{max} - w_0^1 - i - 1) + \overline{v}_i(s_0)(W_{max} + 1 : 2\,W_{max} - w_0^1 - i - 1) \Big) \times \begin{pmatrix} 1 \\ 2 \\ \vdots \\ W_{max} - w_0^1 - i - 1 \end{pmatrix} + \theta_i \Bigg]$$

with:

- *WT* the mean time a segments spends in the switch. We approximate this time by half the maximal switch response time, that is $WT = \frac{1}{2}\frac{B}{b}$ time units.
- θ_i represents the number of segments of the last window in RTT $(i+1)$ that are not subjected to loss,

$$\theta_i = \sum_{w_i^2 = 2}^{W_{max} - w_0^1 - i - 1} w_i^2 \Bigg[1_{\{w_i^2 + 1 + w_0^1 + i + 1 \geq W_{max}\}} \big(\overline{v}_{i+1}(s_0)(w_i^2 + 1) + \overline{v}_{i+1}(s_0)(W_{max} + w_i^2 + 1) \big) + 1_{\{\lfloor \frac{w_i^2}{2} \rfloor + w_0^1 + i + 1 \geq W_{max}\}} \Big(\overline{v}_{i+1}(s_0)(\lfloor \frac{w_i^2}{2} \rfloor) + \overline{v}_{i+1}(s_0)(W_{max} + \lfloor \frac{w_i^2}{2} \rfloor) \Big) \Bigg].$$

5.3 Computation of the Throughput

In the above computation of the throughput, it is assumed that TCP connection 2 starts with system state $s_0 = (m_0, w_0^2)$. In order to derive a general expression for the throughput, the steady state of the system at the end of a cycle is needed, i. e. we need the transition probabilities $\phi_{i,j} = \mathrm{P}\{$system state at the end of n^{th} cycle $= j \mid$ system state at the end of $(n-1)^{\text{th}}$ cycle $= i\}$, and compute from these probabilities the steady state vector for the system state at the end of a cycle.

When the system state at the end of cycle $(n-1)$ is $i = (m, w^2)$, cycle n starts with $w_0^2 = \max(2, \lfloor \frac{w^2}{2} \rfloor)$ and $w_0^1 = \frac{W_{max}}{2} - w_0^2$. The state of the medium at the beginning of the n^{th} cycle is m_0 ($m_0 =$ *Good* or *Bad*) with probability $M(m, m_0)$. Denote the system state (Good, w_0^2), res. (Bad, w_0^2), with s_g, res. s_b.

Assume cycle n ends in state $j = (\tilde{m}, \tilde{w}^2)$. A cycle always ends with loss(es) due to congestion, i. e. when the sum of both windows reaches W_{max}. If cycle n is k RTTs longs, then $w_k^1 + w_k^2 \geq W_{max}$. After k round trip times, the window size of connection 1 is $w_0^1 + k$. Thus only $k \geq W_{max} - w_0^1 - \tilde{w}^2$ give rise to segment loss. Hence,

$$\phi_{i,j} = \sum_{k=W_{max}-w_0^1-\tilde{w}^2}^{W_{max}-w_0^1-2} (\overline{v}_k(s_g)_{(\tilde{m},\tilde{w}^2)} M(m, Good) + \sum_{k=W_{max}-w_0^1-\tilde{w}^2}^{W_{max}-w_0^1-2} (\overline{v}_k(s_b)_{(\tilde{m},\tilde{w}^2)} M(m, Bad).$$

The matrix $\Phi = (\phi_{i,j})$ is stochastic. Let $\overline{\phi} = (\phi_1, \phi_2, \dots)$ be the steady state vector of the system state at the end of a cycle, i.e. $\overline{\phi} = \overline{\phi}\Phi$ and $\overline{\phi e} = 1$. The throughput of connection 2 is then given by

$$T_2 = \sum_{i=(m,w^2)} \big(T_2(s_g)\, M(m, Good)\, \phi_i\, \frac{E_{s_g}}{E}\big) + \sum_{i=(m,w^2)} \big(T_2(s_b)\, M(m, Bad)\, \phi_i\, \frac{E_{s_b}}{E}\big).$$

with:

- $E_{s_g} = E$[length of cycle starting with s_g]
- $E_{s_b} = E$[length of cycle starting with s_b]
- $E = E$[length of cycle].

The mean lengths E_{s_g}, E_{s_b} and E can be easily derived from the vectors $\overline{v}_i$.

The throughput T_1 of flow 1 can be derived on a similar manner.

5.4 Special Case

In the model presented above, we introduced correlation between the wireless losses by using the GE-model for the wireless medium. A more simplified case is to model the wireless medium with one state: the channel is always in the *Bad* state, and thus has a fixed segment loss probability $p_{loss-w} = p^b$. Losses due to the wireless medium are now not correlated since the loss probability remains fixed and independent of previous RTTs.

The analysis to derive the throughput can be easily adjusted to fit this case. The system state at the end of the i^{th} cycle is now completely described by the window size of connection 2, w_i^2. As before, $2(w_i^1 + w_i^2) = W_{max}$.

We repeat some of the most important expressions of the throughput derivation below.

$$\overline{v}_i(w_0^2) = \overline{v}_{i-1}(w_0^2) \times \begin{pmatrix} B(1 : W_{max} - w_0^1 - i, 1 : W_{max}) \\ O(1 : w_0^1 + i, 1 : W_{max}) \end{pmatrix}$$

$$T_2(s_0) = \frac{X}{(2\tau + WT)\sum_{(i)} \overline{v}_i(s_0)\overline{e}}$$

with

$$X = \sum_{i=0}^{W_{max}-w_0^1-2} \left[\overline{v}_i(w_0^2)(1 : W_{max} - w_0^1 - i - 1) \times \begin{pmatrix} \frac{1}{2} \\ \vdots \\ W_{max}-w_0^1-i-1 \end{pmatrix} + \theta_i \right]$$

6 Throughput in a Mixed Wired and Wireless Environment

Assume that two TCP connections as depicted in Figure 1 share a link. We show for different system configurations the throughput T of both connection 1 (wired access) and connection 2 (wireless access). The goal is to investigate the impact of the losses in the wireless access network on the throughput of both connections. As a first step we consider wireless access channels that can only be in one state, i.e. the wireless channel is always in the *Bad* state with segment loss probability p_{loss-w}. Later on (in Section 8) we will also consider configurations where the wireless channel is modelled by a two-state GE-model.

Different network configurations have been used. Table 1 summarises the network parameters for the different scenarios.

Figure	b	B	τ	W_{max}
3	2	10	5	30
4	8	8	2	40
5	4	10	5	50
6	2	60	10	100

Table 1: Network parameters of the different scenarios

For these configurations, the throughput T_1 and T_2 of both connections is shown in Figures 3-6. (Remark that due to the approximations made in the model, the sum of the individual throughputs is higher than the bottleneck bandwidth on some figures.) In all cases, the connection that does not experience random losses benefits in throughput from the window reduction imposed by TCP on the other connection as a result of the random segment losses.

When there is a wireless link in the end-to-end path of a TCP flow, TCP performance suffers significantly. This occurs because TCP shrinks its transmission window each time it detects a loss. This is acceptable when the loss is a notification of congestion somewhere along the transmission path, but unnecessary when the loss is caused by the wireless medium.

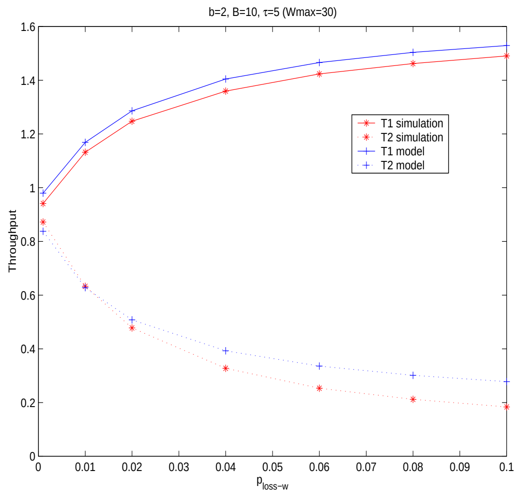

Figure 3: T_1 and T_2 for $W_{max} = 30$

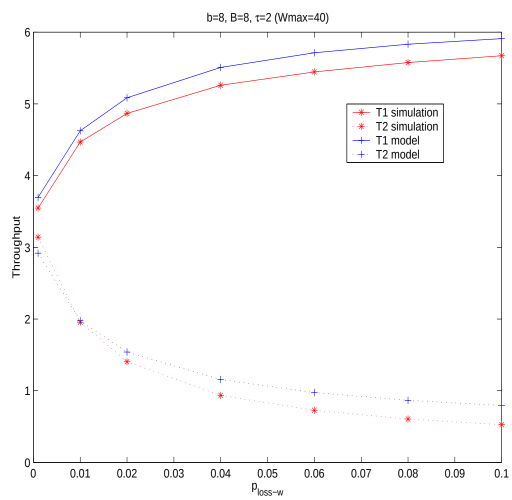

Figure 4: T_1 and T_2 for $W_{max} = 40$

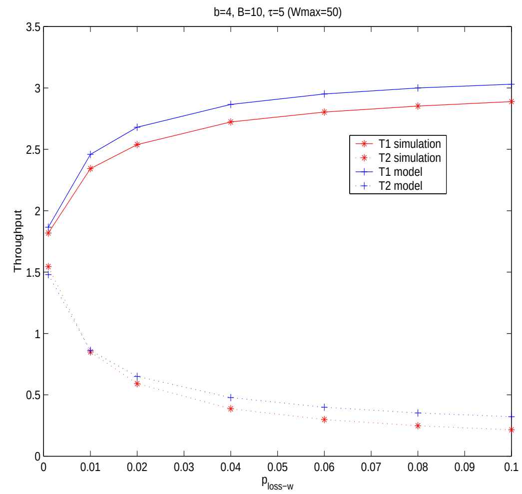

Figure 5: T_1 and T_2 for $W_{max} = 50$

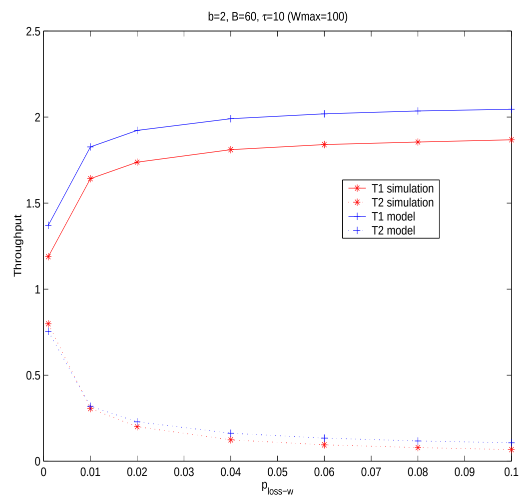

Figure 6: T_1 and T_2 for $W_{max} = 100$

The figures clearly show that throughput of the second flow deteriorates when the wireless losses increase. The other connection benefits from this because it can use the leftover capacity.

Figures 3-6 also contain the corresponding simulation results. This simulation implements the complete TCP protocol, thus *Slow Start* and *Fast Retransmit/Recovery* are included. The figures show that the results from the analytical model match the simulation results very well.

7 Throughput in a Wired Access Environment with Link Level Error Control

Segment losses in the wireless access network due to the unreliability of the medium may be solved on the data link layer, before offering the segment to the transport layer. The use of e. g. an automatic repeat request-based link layer recovery scheme, prevents the TCP connection from a considerable throughput deterioration due to the congestion control mechanism, but involves a supplementary delay due to the error recovery on the data link layer, which also decreases the throughput. We determine a trade-off between the throughput decrease due to an increase in round trip time (with error recovery at data link layer) and the decrease in throughput due to segment loss (without error recovery at data link layer).

We consider the following two systems. First, consider a system with wireless access, without error recovery scheme. In this case, the model developed in this paper applies (with $\tau_1 = \tau_2 = \tau$). The throughput of connection 2 (i. e. the one with the wireless access) can be computed using the method described in Section 5. Next, consider a system with wireless access and with error recovery scheme. This system can be modelled as if there is no random loss (i.e. $p_{loss-w} = 0$), but there is an additional delay to be taken into account due to the operation of the error recovery scheme in the wireless access network. This leads to a round trip delay for connection 2 equal to $\tau_2 > \tau_1$. The throughput T_2^e of this system is derived in Section 7.1. Finally we can compute the maximum value of τ_2 that is allowed to obtain the same throughput as in a system where no error recovery scheme on the data link layer is used.

7.1 TCP Throughput in a Wireless Environment with Error Recovery

In this section we evaluate the throughput of a TCP connection over a wireless link, where the loss of segments is hidden from the TCP layer by the data link layer. Consider the network model depicted in Figure 1, without additional segment loss in the wireless access network, but with different round trip delays, i.e. $\tau_2 > \tau_1$. Denote by W_i^j the window size of connection j at the end of the i^{th} cycle (i.e. when the i^{th} segment loss occurs). Then the following recursive relation is valid (see [7])

$$W_i^j = \frac{W_{i-1}^j}{2} + \frac{\tau_1\,\tau_2}{2\tau_j(\tau_1+\tau_2)} W_{max}$$

leading to

$$W_i^j = \frac{\tau_1\,\tau_2}{\tau_j(\tau_1+\tau_2)} W_{max} - \left(\frac{1}{2}\right)^{i-1}\left(\frac{\tau_1\,\tau_2}{\tau_j(\tau_1+\tau_2)} W_{max} - W_0^j\right).$$

From this result it follows that the window size of both connections converge exponentially to a value which is proportional to the inverse of the propagation delay. W_{max} is proportionally distributed between both connections according to the inverse of their propagation delays. Moreover, it can be shown from this result that the throughput is proportional to the inverse of the square of the propagation delay. In particular, the throughput of connection j, $j = 1, 2$, during the i^{th} cycle is given by

$$T_i^j = \frac{1}{4\tau_j}\left(2W_i^j - \frac{1}{2\tau_j}\frac{\tau_1\,\tau_2}{(\tau_1+\tau_2)} W_{max}\right).$$

Taking the limit for $i \to \infty$ leads to the throughput of connection j, $j = 1, 2$,

$$T_j^e = \frac{3}{8\tau_j^2}\frac{\tau_1\,\tau_2}{(\tau_1+\tau_2)} W_{max}.$$

7.2 Throughput comparison of system with and without error-recovery

In what follows we compute the maximum value of τ_2 that is allowed to obtain the same throughput as in a system where no error recovery scheme on the data link layer is used. From 7.1, the throughput for connection 2 in the second system (i.e. the system with error recovery) is given by

$$T_2^e = \frac{3}{8\tau_2^2}\frac{\tau_1\,\tau_2}{(\tau_1+\tau_2)} W_{max}.$$

The throughput of connection 2 in the first system (i.e. the system without error recovery) can be computed according to the method explained in Section 5. Denote the result by T_2. We determine τ_2 as the maximal value for which

$$T_2 \le \frac{3}{8\tau_2^2}\frac{\tau_1\,\tau_2}{\tau_1+\tau_2} W_{max}.$$

7.3 Numerical Results

Consider a network with the following parameters: $b = 4$ segments/time unit, $B = 6$ segments, $\tau = 3$ time units and hence $W_{max} = 30$ segments. In Figure 7 we show the maximal allowed round trip time τ_2 that leads to a throughput equal to the one in a system without error recovery on data link layer when p_{loss-w} ranges from 0.004 to 0.08. Figure 8 shows similar results for a network with parameters $b = 4$ segments/time unit, $B = 10$ segments, $\tau = 5$ time units ($W_{max} = 50$ segments).

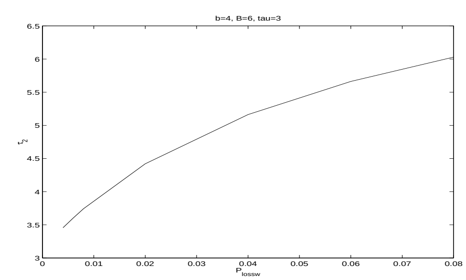

Figure 7: τ_2 versus p_{loss-w} for W_{max}=30

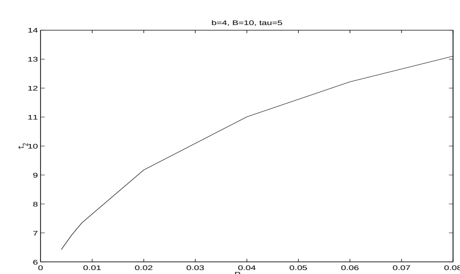

Figure 8: τ_2 versus p_{loss-w} for W_{max}=50

In both cases we see that the allowed round trip time increases with the loss probability. Of course, when the segment loss probability of the wireless medium increases, the throughput deterioration becomes worse. This gives us more room to solve the problem on the data link layer: the aimed throughput T_2 is lower, thus the additional delay, introduced by the error-recovery scheme, may be higher.

8 Influence of Correlation of Wireless Losses on Throughput

The correlation structure of the wireless losses is an essential characteristic that cannot be neglected. The wireless medium model presented in Section 3 is used to incorporate this correlation. The goal of this section is to investigate the influence of the correlation between the losses caused by the wireless access channel on the throughput of the wired and wireless connection.

The network configurations of Table 1 have been used. In all four scenarios, we used $p^b = 1$ and $p^g = 0.01$. The mean durations of the *Good* and the *Bad* state ($\frac{1}{\delta}$ and $\frac{1}{\gamma}$) are varied, such that the obtained $\mathrm{LP}_{\mathrm{wir}}$ stays fixed to 0.1. All the figures show two curves. The upper curve shows the throughput of flow 1 versus the mean time in the *Good* state, the lower gives the same but for flow 2.

Impact of wireless access: In Figures 9, 10, 11 and 12, we clearly notice that when the throughput of flow 2 decreases, flow 1 can benefit from this and its throughput increases. As already concluded in Section 6, the connection that does not experience random losses benefits in throughput from the window reduction imposed by TCP on the other connection as a result of the random segment losses.

Influence of correlation between wireless losses: We can also conclude from the figures that when the mean length of the *Good* state increases, the throughput of flow 2 increases. Hence,

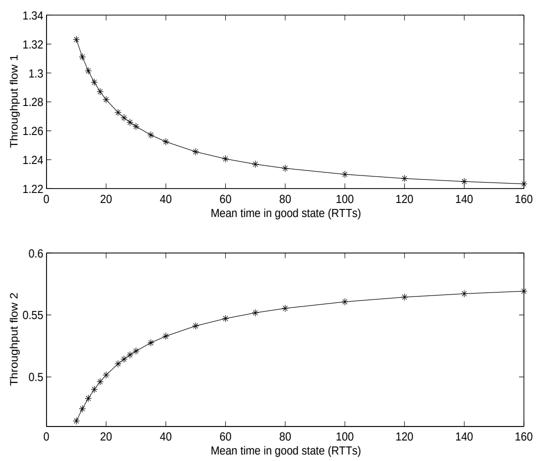

Figure 9: T_1 and T_2 for $W_{max} = 30$

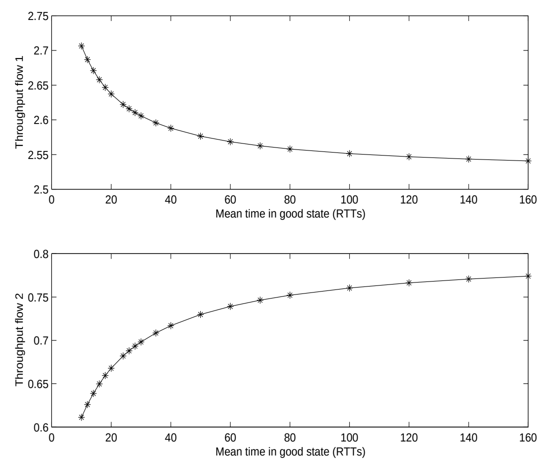

Figure 11: T_1 and T_2 for $W_{max} = 50$

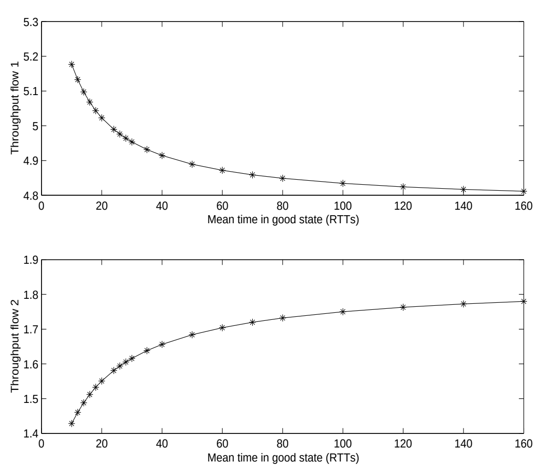

Figure 10: T_1 and T_2 for $W_{max} = 40$

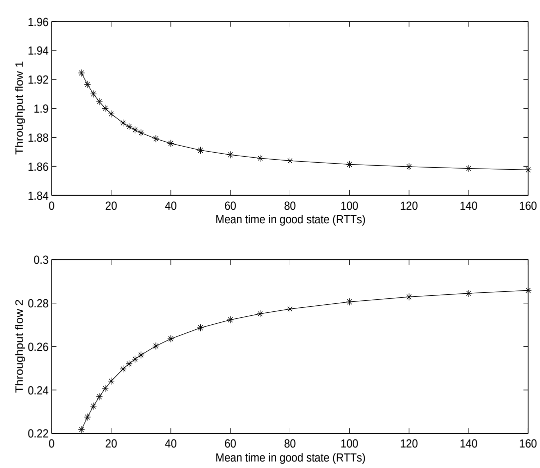

Figure 12: T_1 and T_2 for $W_{max} = 100$

the throughput of flow 1 decreases. When the mean time in the *Good* state increases, the mean time in the *Bad* state also increases for we keep $\mathrm{LP_{wir}}$ fixed. This means that there are longer periods with few losses and longer periods with many losses. Losses are more correlated. This has a positive influence on the TCP behaviour of the wireless flow. When the wireless losses are more grouped, TCP can solve several losses by one retransmission action. Thus, the window size of flow 2 is not reduced to half for each loss. When the losses are grouped, flow 2 will have to half its window less often which explains the positive effect on the throughput of connection 2.

Influence of system dimension: When the system becomes larger (W_{max} increases), the influence of the wireless link becomes larger. Thus, the impact of an improvement (more correlated losses) also becomes larger. This phenomenon is illustrated by computation of the percentage of gain in throughput for flow 2. We compare the throughput when there is no correlation with the throughput corresponding to the highest correlation. The throughput corresponding to no correlation is derived from the model of a system where the wireless link can only be in one state (as in Section 5.4), i. e. the system is always in the *Bad* state. Table 2 summarises these results. It is clearly seen that when system dimension grows, the gain also increases.

W_{max}	T_2 no correlation	T_2 highest correlation	Perc. of gain
30	0.2779	0.5692	51.18%
40	0.7938	1.7799	55.40%
50	0.3227	0.7741	58.31%
100	0.1069	0.2858	62.60%

Table 2: Percentages of throughput gain for Figures 9-12

Influence of Wireless Loss Rate: When the wireless losses decrease, the impact of the correlation on the throughput is less significant. Figure 13 below shows the system with $W_{max} = 30$, but this time $p^g = 0.005$, $p^b = 0.5$ and $\mathrm{LP_{wir}} = 0.01$. Comparison with Figure 9, leads to the conclusion that when we increase the mean time in the *Good* state, the throughput increases much faster in the scenario with $\mathrm{LP_{wir}} = 0.1$ then in the scenario with $\mathrm{LP_{wir}} = 0.01$.

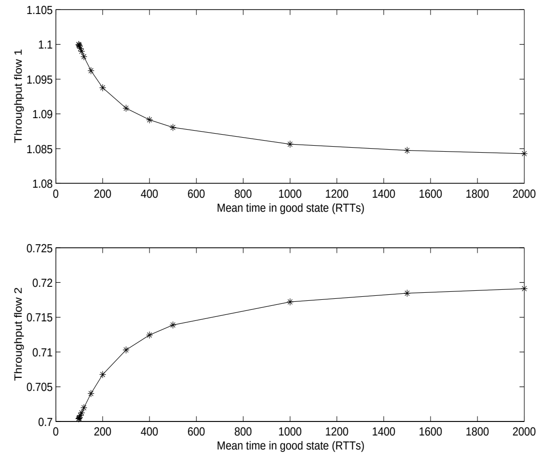

Figure 13: T_1 and T_2 for $W_{max} = 30$ and $\mathrm{LP_{wir}} = 0.01$

9 Conclusions

We have derived a simple analytical model to evaluate the performance of a TCP connection that experiences random segment losses due to a wireless access medium. We have illustrated how a TCP connection that does not experience random losses which shares a bottleneck link with a TCP connection that does experience random losses benefits from the throughput degradation of the latter. A possible solution to this throughput degradation, could be to solve the wireless errors on a lower layer instead of letting TCP solve these losses. We have evaluated the additional delay a TCP connection can support to solve the error recovery problem on the data link layer in order to gain throughput compared to the case where the error recovery is solved by the TCP protocol itself. Finally we looked at the influence of the correlation of the losses caused by the wireless medium on the throughput of both flows. We can conclude that when correlation increases, the throughput of the wireless connection improves. Thus the throughput of the wired connection decreases. The strength of the improvement depends on the system dimension and the amount of wireless losses.

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References

[1] I. Akyildiz and I. Joe, "*TCP performance improvement over wireless ATM networks through a new AAL protocol*", Proc. of IEE GLOBE-COM'98, pp. 508-512

[2] E. Altman, F. Boccara, J. Bolot, P. Nain, P. Brown, D. Collange and C. Fenzy, "*Analysis of TCP/IP flow control in high-speed wide area networks*", Proc. of 34th IEEE Conference on Decisions and Control, pp. 368-373, 1995

[3] G. Bao, "*Performance Evaluation of TCP/RLP protocol stack over CDMA wireless link*", Wireless Networks 2(3), pp. 229-237, 1996

[4] A. Chan, D. Tsang and S. Gupta, "*Performance analysis of TCP in the presence of random losses/errors*", Proceedings GLOBECOM '98, pp. 513-518

[5] H. Chaskar, T. Lakshman and U. Madhow, "*TCP over wireless with link level error control: analysis and design methodology*", IEEE/ACM Trans. on Networking 7(5), pp. 605-615, October 1999

[6] K. Fall and S. Floyd, "*Simulation-based comparisons of Tahoe, Reno and SACK TCP*", Computer Communication Review 26(3), pp. 5-21, July 1996

[7] G. Hasegawa, M. Murata and H. Miyahara, "*Fairness and stability of congestion control mechanisms in TCP*", Proceedings INFOCOM '99, New York 1999

[8] V. Jacobson and M. J. Karels, "*Congestion avoidance and control*", Computer Communication Review 18(4), pp. 314-329, August 1990, ftp://ftp.ee.lbl.gov/papers/congavoid.ps.Z

[9] J. G. Kim and M. Krunz, "*Bandwidth allocation in wireless networking with guaranteed packet-loss performance*", IEEE/ACM Trans. on Networking 8(3), pp. 337-349, June 2000

[10] T. Laksham and U. Madhow, "*The performance of TCP/IP for networks with high bandwidth-delay products and random loss*", IEEE/ACM Trans. on Networking 5(3), pp. 336-350, 1997

[11] W. R. Stevens, "*TCP/IP illustrated, volume 1: the protocols*", Addison-Wesley, 1994

[12] W. R. Stevens, "*RFC 2001: TCP Slow Start, Congestion Avoidance, Fast Retransmit and Fast Recovery Algorithms*"