Transport tax reform, commuting and endogenous values of time

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Abstract
We consider a model of urban transport with two trip purposes, commuting (assumed perfectly complementary to labour supply) and non-commuting, to analyse the effects of transport tax reform on the value of time and marginal external congestion costs. Higher commuting taxes plausibly reduce time values, but higher non-commuting transport prices will typically raise the value of time. The intuition for this latter finding is that the reduction in congestion that follows from the tax increase itself raises net wages per hour of work (inclusive of commuting time). Empirical illustrations with Belgian data show a potentially large effect of transport tax reform on time values. In quite a few of the tax reforms studied traffic levels are reduced, but the increase in time values implies that marginal external congestion costs actually increase.

JEL: R41, H21, H23
Keywords: transport, congestion, value of time

SUGGESTED RUNNING HEAD: Transport taxes and time values

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1. Introduction
The increase in transport demand over the past few decades has substantially raised congestion in major conurbations around the world. The social costs associated with the time losses in traffic are known to be substantial. A recent US-study of 75 areas estimates that in 2000 the average peak period traveller lost 62 hours per year due to congestion; the corresponding figure for 1982 was 16 hours (Texas Transportation Institute [31]). The same study points out that the average commuter in Washington DC spends approximately two weeks per year stuck in traffic. In European cities a similar evolution is observed. Estimates for Brussels, reported in Van Dender [34], suggest that overall traffic volumes in 2005 are 22% higher than in 1991, leading to a decline in average speed of some 34%. Finally, a comparative study for five large European cities (De Borger and Proost [8]) estimates that by 2005 marginal congestion costs will exceed one Euro per passenger-kilometre in the most congested cities, such as Amsterdam and Brussels.

The concern about congestion and other transport externalities (pollution, noise, accident risks, etc.) has generated a large literature on economic instruments to reduce these effects. Realising that increasing road capacity may not be an effective long-run policy due to latent demand (see, e.g., Small [29]), economists have long advocated the use of pricing instruments to tackle transport externalities. The large number of studies on optimal externality taxes in the transport sector include, among many others, Keeler and Small [19], Glaister and Lewis [12], Small [28], Viton [35], Kraus [20], Arnott et al. [1], De Borger et al. [7], Proost and Van Dender [27], and Small and Yan [30]. Despite economists’ focus on pricing as a solution to congestion externalities, policy makers have only recently started to implement road pricing in practice; the best-known examples are Singapore and the Norwegian cities Trondheim and Oslo (see, e.g., Johansson and Mattson [17]). It is likely that more cities will
introduce some form of road pricing in the near future. For example, cordon pricing has long been considered in the Netherlands, although its actual implementation has been postponed several times. In London, cordon pricing is scheduled for implementation in February 2003. The idea is also gaining support in the US, as witnessed by value pricing projects in California, Florida, New Jersey and Texas.

This paper contributes to the growing literature on congestion pricing by focusing on the impact of transport tax changes on consumers’ time valuation, within the framework of a model with multiple trip purposes.\(^1\) Interestingly, all studies listed above implicitly or explicitly assume that the value of time is unaffected by the proposed tax changes. Moreover, although some studies take account of different transport markets (according to mode, time of day, car type, etc.), they do not distinguish between trip purposes, such as commuting and non-commuting.\(^2\) We consider the implications of relaxing these two assumptions. Specifically, we look at the implications of transport taxes for the value of time in a model with both non-commuting and commuting (directly related to labour supply) transport. It is shown that transport policies may have non-negligible effects on the value of time. As a consequence, realising that in most countries commuting is indeed an important trip purpose during peak hours, the welfare effects suggested by models assuming either constant values of time or single trip purposes may be inaccurate.

Since the seminal papers by Becker [2] and Gronau [13], economists have devoted attention to the determinants of the value of time (for a recent survey, see Jara-Diaz, [16]). Theoretical research and empirical analysis of large-scale surveys

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\(^1\) Other externalities could be easily incorporated into the model. However, as will become clear below, the focus is on congestion.

\(^2\) The few models that do allow for endogenous values of time (Mayeres and Proost [22], with explicit endogeneity, and Parry and Bento [25], where it is implicit) are based on a single trip purpose.
suggest that the value consumers place on time savings not only depends on income or wage levels, but also on socio-economic characteristics. Relevant references include Calfee and Winston [5], Ramjerdi et al. [24] and de Jong and Gunn [9]. Time values also vary with the specific circumstances under which the time saved is actually spent (see, e.g., De Donnea [10], Hague Consulting Group [14], de Jong and Gunn [9]). However, the potential dependency of the value of time on the level of transport prices has received little attention. To our knowledge, it has not explicitly been analysed how a tax reform itself affects the values of time, the marginal congestion costs and the welfare effects of the reform. Using endogenous values of time is especially relevant in optimal tax models and in tax reform exercises in the transport sector, where the prevalence of congestion externalities may require large price adjustments, and where the road is simultaneously used for several trip purposes that differ in their complementarity to leisure.

Using a model with two trip purposes, this paper shows that commuting and labour taxes typically reduce the value of time, but that higher congestion taxes on non-commuting transport are likely to raise time values. The main reason for the latter finding is that the reduction in congestion that follows from the tax increase on non-commuting transport raises the consumer’s evaluation of time, because it raises the (net-of-commuting-costs) wage per (inclusive-of-commuting-time) hour of work. The empirical analysis considers several balanced-budget tax reform policies that combine higher congestion taxes with downward adjustments in labour taxes to reduce the distortions from the tax system (see, e.g., Bovenberg and Goulder [3]). It is found that such policy packages likely increase the marginal value of time. Also, with endogenous time values, a joint tax reform on the transport and labour markets may yield lower traffic levels, less congestion, but higher marginal external congestion.
costs. This contrasts with the popular view that directly associates decreases in traffic levels on a congestible facility with reductions in marginal external congestion costs.

The paper is structured as follows. Section 2 considers the value of time and marginal external congestion costs in a stylised model of consumer choice with two trip purposes. Section 3 illustrates the interactions using a numerical model with two transport modes and two trip purposes, calibrated on Belgian data. The effects of optimal taxes and of various types of tax reform are considered. Section 4 concludes.

2. Transport taxes, the value of time and marginal external congestion costs in a model with two trip purposes

We present the theoretical model and then study the impact of transport and labour tax reforms on the marginal value of time and on marginal external congestion costs.

2.1 A simple model with two trip purposes

Let a representative consumer care about two types of trips, a general consumption good, and leisure. Preferences are given by $u(q_0, q_1, q_2, N)$, where $q_0$ is a composite commodity with price normalised at one, $q_1$ are non-commuting trips, $q_2$ are commuting trips (the journey-to-work), and $N$ is leisure time. The model focuses exclusively on peak period passenger travel, when congestion is worst and both trip purposes are relevant (LRC [21], US-DOT [32]); freight transport is assumed to be negligible. To make the distinction between the two trip motives as transparent as possible, commuting is assumed to be directly proportional to labour supply $L$, i.e. $q_2 = L$. This says that each day of work requires one morning peak trip, assumed for simplicity to be one kilometre long. Labour supply is elastic in terms of the number of days of labour, but the length of each workday is fixed.
The theoretical model assumes that car transport is the only passenger transport mode; also, there is only one car type, and the occupancy rate per car is normalised at one. Transport prices per trip (or per kilometre) are $p_1$ and $p_2$ for non-commuting and commuting, respectively. They capture the monetary expenditures associated with fuel consumption, parking, wear and tear, etc. Note that the unit prices for commuting and non-commuting transport can differ if, for example, commuting expenses are tax deductible (Wrede [36]). In the absence of tax deductibility both prices are identical. Commuting and non-commuting transport share a congestible road network, and therefore jointly determine travel time $a = a(F)$, where $a(.)$ is the (convex) congestion function, $F = n(q_1 + q_2)$ is the total traffic flow, and $n$ is the number of consumers. We normalise $n=1$ throughout.

The consumer maximises utility subject to a budget and a time constraint:

$$q_0 + p_1 q_1 + p_2 q_2 = (1-t_L)L + S \quad [\lambda]$$

$$N + L + (a(F))(q_1 + q_2) = \bar{L} \quad [\gamma]$$

where $t_L$ is the labour tax rate (pre-tax wages are normalised to one without loss of generality), $L$ is labour time, $S$ is a fixed lump sum transfer, and $\bar{L}$ is the time endowment. Finally, $\lambda$ and $\gamma$ are the Lagrange multipliers for the budget and time constraints.

Using $q_2 = L$ and assuming that the representative consumer neglects his own impact on congestion, we obtain the first-order conditions (3), where subscripts indicate partial derivatives

$$u_0 = \dot{\lambda}$$
$$u_1 = \lambda p_1 + a \gamma$$
$$u_2 = -\lambda (1-t_L - p_2) + (1+a) \gamma$$
$$u_n = \gamma$$

(3)
It follows from (3) that, at the optimum, the marginal utility of non-commuting trips is positive. However, the strict proportionality with labour supply implies that the marginal utility of commuting trips, which can also be interpreted as the net marginal utility of time spent working, may take either sign: it is the combination of the positive marginal utility of \( q_2 \) in the absence of the constraint \( q_2 = L \) (this equals, given the specification of the utility function, \( u_z = \lambda p_z + a > 0 \)) and the marginal utility of labour \( (u_L = \gamma - \lambda(1-t_L)) \). The latter can take either sign, although it is often implicitly assumed to be zero in the literature.

Following Becker [2] and Jara-Diaz [16], we define the marginal value of time by \( MVOT \equiv \frac{u}{u_0} \). Using the system of first-order conditions (3), we can write:

\[
MVOT = \frac{u}{u_0} = \frac{\frac{u_z}{\lambda} + (1-t_L - p_z)}{1+a} \tag{4}
\]

The marginal value of time, which at the optimum is independent of the activity in which it is spent, equals the net real wage per unit of time input, corrected for the marginal (dis-)utility of commuting.\(^3\) Note the following two features of (4): first, the net real wage in the numerator corrects for the cost of commuting; second, the time input per hour of work in the denominator includes commuting time.

Although we study the issue more formally below, the potential impact of transport pricing reform on the value of time in this model can be seen from (4). It shows that the value of time directly depends on the congestion level (through commuting time \( a \)) so that, ceteris paribus, a reduction in congestion raises time

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\(^3\) Slightly different expressions for the value of time are obtained depending on the exact specification of the utility function (e.g., explicitly including travel time or labour supply as an extra argument of utility). See Jara-Diaz [16] for an overview. The main point of this paper, viz. that distinguishing trip purposes has implications for the impact of tax changes on the value of time, is not affected.
values. The intuition is that lower congestion levels reduce the time needs associated with an extra hour of labour time. This direct additional effect on time values via congestion drives many of the results of this paper. Since reducing congestion often is a major reason for transport tax reform, and since large transport tax changes are required to cope with external congestion costs, the impact of price changes for non-commuting transport on the value of time through changes in congestion can be large. Of course, a commuting tax has an additional direct effect on time values through the net real wage. Finally note that, since transport prices are also likely to affect labour supply and commuting, the marginal utility of income and of commuting cannot be assumed to be constant.

It is instructive at this point to compare (4) with the value of time that follows from models with a single trip purpose. Assume, as is common in the literature, that all transport were aggregated and treated as non-commuting transport. In other words, let \( q_2 = 0 \), and include labour time \( L \) as an argument in the utility function (see, e.g., Johnson [18], Jara-Diaz [16]). In that case the marginal value of time is easily shown to be \( MVOT = \frac{\gamma}{\lambda} = \frac{u_t}{\lambda} + 1 - t_L \). Transport tax changes then affect time values only through the marginal utilities of labour and income, but not directly via their impact on congestion. Finally, if the marginal (dis-)utility of labour were zero, such a model implies a value of time equal to the net wage \( (1 - t_L) \). In that case, changes in transport prices do not affect values of time at all, and the labour tax reduces the time value on a one-to-one basis.
2.2 Taxes and the value of time with multiple trip purposes

We now consider more formally the impact of price and tax changes on the value of time. To simplify the analysis we impose some extra structure on preferences. Specifically, assume that utility is quasi-linear in the numeraire good and that commuting is additively separable from other consumption goods and leisure:

\[ u(q_0, q_1, q_2, N) = q_0 + U(q_1, N) + g(q_2) \]  

These assumptions imply that the marginal utility of income is constant and equal to one \((u_0 = \lambda = 1)\), and the marginal (dis-)utility of commuting only depends on the level of commuting itself, so that:

\[ MVOT = \gamma = \frac{u_2(q_2) + (1-t_L - p_2)}{1+a} \]

Using these simplifications, simple manipulations immediately yield the effect of an exogenous transport price change on the marginal value of time:

\[ \frac{d\gamma}{dp_1} = \frac{1}{(1+a)} \left[ u_{22} \frac{dq_2}{dp_1} - a' \gamma \left( \frac{dq_2}{dp_1} + \frac{dq_1}{dp_1} \right) \right] \]  

\[ \frac{d\gamma}{dp_2} = \frac{d\gamma}{dt_L} = \frac{1}{(1+a)} \left[ (u_{22} \frac{dq_2}{dp_2} - 1) - a' \gamma \left( \frac{dq_2}{dp_2} + \frac{dq_1}{dp_2} \right) \right] \]

Here the price effects on demand are total effects, incorporating any indirect effects via congestion. Not surprisingly, both the effects of commuting and non-commuting transport prices on time values are ambiguous in general; they depend on the impact of the price changes on the total traffic volume and therefore on congestion, and on their effect on commuting and labour supply.

Although (7) and (8) can be used to speculate on the sign of the effect of price changes on time values, more precise information can be obtained as follows.
Substitute $q_0$ from the budget constraint into the utility function and use $\lambda = 1$, so that the system of first-order conditions now reads:

\[
\begin{align*}
    u_1 &= p_1 + a' \\
    u_2 &= -(1-t_L - p_2) + (1+a') \\
    u_N &= \gamma \\
    \bar{L} &= N + aq_1 + (1+a)q_2
\end{align*}
\] (9)

The final equation is the time restriction. Differentiating (9), capturing price effects on $a$ via demand changes, and using matrix notation then yields:

\[
\begin{bmatrix}
    u_{11} - \gamma a' & -\gamma a' & u_{1N} & -a \\
    -\gamma a' & u_{22} - \gamma a' & 0 & -(1+a) \\
    u_{N1} & 0 & u_{NN} & -1 \\
    -(a + a'F) & -(1+a + a'F) & -1 & 0
\end{bmatrix}
\begin{bmatrix}
    dq_1 \\
    dq_2 \\
    dN \\
    d\gamma
\end{bmatrix}
= \begin{bmatrix}
    dp_1 \\
    dp_2 \\
    0 \\
    0
\end{bmatrix}
\] (10)

As before, $F = q_1 + q_2$ is total transport demand and $a' = \frac{da(F)}{dF}$ is the slope of the congestion function.

This system can be solved for $dq_1, dq_2, dN, d\gamma$ as functions of the exogenous price changes (see the Appendix for more details). Here we directly proceed to the results. To facilitate the interpretation of the price effects for the value of time, it will be useful to first briefly consider the impact of transport price changes on demand. These can be written as follows:

\[
\frac{dq_1}{dp_1} = -\frac{1}{\Delta} \{u_{22} - \gamma a' + u_{NN}(1+a)(1+a + a'F)\} \] (11)

\[
\frac{dq_1}{dt_L} = \frac{dq_1}{dp_2} = \frac{1}{\Delta} \{(a)u_{NN} - u_{1N})(1+a + a'F) - \gamma a'\} \] (12)

\[
\frac{dq_2}{dp_1} = \frac{1}{\Delta} \{(1+a)[(a)u_{NN} - u_{1N}) + a'Fu_{NN}] - \gamma a'\} \] (13)

\[^4\text{In general, the results depend on all second derivatives of the utility function. This obscures the interpretation, because many of the cross-effects of the marginal utilities are difficult to sign a priori.}\]
\[
\frac{dq_2}{dt_1} = \frac{dq_2}{dp_1} = \frac{-1}{\Delta} \left\{ (a u_{NN} - u_{1N})(a + a' F) - (a u_{1N} - u_{11} + \gamma a') \right\}
\] (14)

Here $\Delta$ is the determinant associated with equation system (10). In the Appendix it is shown that a mild restriction on the feedback effects of congestion on demand guarantees that it is negative. We assume this condition to hold; in addition, we assume throughout that marginal utility is non-increasing, i.e., $u_{NN} \leq 0$, $u_{11} \leq 0$ and $u_{22} \leq 0$. Also note that labour and commuting taxes have the same effect on transport demand, since they identically affect the net real wage.\(^5\)

Under these assumptions it immediately follows from (11) that the overall own price effect of non-commuting transport, taking into account all indirect congestion effects, is negative.\(^6\) While the signs of the cross-price effects in (12) and (13) are theoretically ambiguous, note that $a u_{NN} - u_{1N} < 0$ is a sufficient condition for them to be positive. Loosely speaking, this will hold as long as the marginal utility of non-commuting transport does not decline strongly at higher levels of leisure. Finally, (14) implies that the sign of the effect of the labour (commuting) tax on labour supply (demand for commuting) is ambiguous. Again, however, it will be negative (i.e., the labour supply curve is upward sloping) provided $u_{1N}$ is not too negative.

We now turn to the impact of transport prices on the value of time. The solution of (10) implies:\(^7\)

\[
\frac{d\gamma}{dp_1} = \frac{1}{\Delta} \left\{ u_{22} \left[ (a u_{NN} - u_{1N}) + a' Fu_{NN} \right] + \gamma a' (u_{1N} + u_{NN}) \right\}
\] (15)

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\(^5\) This is no longer true in the empirical model, where two transport modes are considered.

\(^6\) This provides a clear interpretation for the stability condition $\Delta < 0$. It guarantees that the overall effect of an increase in $p_1$ on non-commuting transport demand, including all feedback effects of congestion on both the commuting and non-commuting markets, is negative. See below for details.

\(^7\) Using (11)-(14), simple algebra shows that expressions (15) and (16) are indeed consistent with (7) and (8).
First, consider increases in the price of non-commuting transport \( (p_1) \). To interpret the result, recall the earlier discussion that the price change affects \( \gamma \) via two channels (see also (6)). It potentially has an impact on labour supply and commuting demand \( (L = q_2) \) and hence on the marginal utility of commuting, and it affects time values through implied changes in overall congestion \( a \). The two effects are clearly illustrated in (15). If there were no effects via congestion \( (a' = 0) \), then (13) and (15) together imply that a higher price would reduce the value of time as long as the cross-price effect on labour supply is positive. Congestion effects, however, may drastically affect the ultimate outcome. Note that the effect of raising the price of non-commuting on total traffic demand is given by, combining (11) and (13):

\[
\frac{dF}{dp_1} = \frac{dq_1}{dp_1} + \frac{dq_2}{dp_1} = \frac{-1}{\Delta} \left[ u_{22} + (1 + a)(u_{1N} + u_{NN}) \right]
\]  

(17)

To see the impact of congestion on the value of time most clearly, suppose that the marginal disutility of labour is constant \( (u_{22} = 0) \). It then immediately follows from (15) and (17) that an increase in the price of non-commuting will raise the value of time as long as it reduces overall traffic demand.

In sum, the ultimate impact of the tax change on time values is ambiguous. Which of the two effects dominates is an empirical matter. Taxing non-commuting transport will reduce the value of time if the tax increase raises labour supply and commuting, but is not very successful in reducing total congestion. However, if congestion does substantially decline, and either the labour supply effects of non-commuting transport taxes are relatively small or the marginal utility of labour does not substantially vary with labour supply \( (u_{22} \text{ small}) \), then the effect of the tax
increase is to raise time values. Although little empirical evidence is available on the labour supply effects of non-commuting transport taxes, intuition suggests that they may be relatively limited. Recent evidence on the price and time elasticities of transport demand (de Jong and Gunn [9]) on the other hand indicates that the overall congestion effects of price increases are indeed negative. If these statements reasonably describe reality it should not come as a surprise to find that non-commuting tax increases raise the value of time. We investigate this issue further using numerical techniques in the next section.

Turning to the impact of labour or commuting taxes on time values, (16) suggests that an increase in the labour or commuting tax plausibly reduces time values. Since \( u_{NN} u_{11} - u_{N1}^2 > 0 \) by the strict concavity of the utility function, they will reduce the value of time unless \( u_{11} \) is positive and very large.

It is important to stress that the results derived in (15) and (16) would have been substantially different in a model with a single trip purpose. To see this most clearly, assume that \( u_{22} = 0 \). We then showed that non-commuting transport taxes plausibly raise the value of time, whereas labour taxes reduce time values. For a single trip purpose model with non-commuting transport only, however, the value of time would be given by \( \gamma = 1 - t_L \), so that \( \frac{d \gamma}{dp} = 0 \) and \( \frac{d \gamma}{dt} = -1 \). In other words, under the conditions specified this would imply that transport prices would not affect time values at all, whereas the impact of a labour tax equals precisely \(-1\).

Finally, consider the case where commuting and non-commuting transport prices cannot be differentiated, either for technical reasons or because of political constraints. The results for \( \frac{d \gamma}{dt} \) remain unaffected. However, since the common transport price directly affects the net real wage, the impact of a higher transport price
on time values does change. Specifically, we find (see the Appendix for more details on the demand effects):

\[ \frac{d\gamma}{dp} = \frac{1}{\Delta} \left( \left[ u_{NN}(u_{22} + u_{11}) - u_{N1} \right] (1 + a + a'F) - (u_{1N} + u_{NN})u_{22} \right) \]  

(18)

Interestingly, since the term between square brackets is positive, assuming a constant marginal utility of commuting \((u_{22} = 0)\) now implies that a price increase reduces the value of time. This is precisely the opposite of the outcome we obtained for the price of non-commuting transport in the case with price differentiation (see (16)). The reason is obvious: a transport price increase is now a simultaneous price increase for non-commuting transport and a reduction in the net real wage. Even if the former reduces congestion (raising time values), this is more than compensated by the direct reduction in the real net wage. If \(u_{22} < 0\), the negative effect is counteracted by a positive effect that will be larger the larger the reduction in labour supply. The ultimate sign is indeterminate. Note also that we expect the price effect on the value of time to be smaller here than the price effect for non-commuting transport in the tax differentiation case.

2.3 Taxes and marginal external costs with endogenous time values

In this model, marginal external congestion costs \(MECC\) are the same for commuting and non-commuting. Driving one extra kilometre raises \(F\), reduces travel speed and increases travel time per kilometre \(a(F)\). The time losses apply to all kilometres driven and are evaluated at the value of time per time unit. So we can write:

\[ MECC = (MVOT) \cdot (a') \cdot (q_1 + q_2) \]  

(19)
Taxes and transport prices affect congestion costs through traffic flows, the slope of the congestion function, and the value of time. Differentiating (19) and rearranging, we obtain the following expressions:

\[
\frac{d(MECC)}{dt_i} = \left[ MVOT \left( F \frac{da_i}{dF} + a_i \right) \right] \left( \frac{dq_1 + dq_2}{dt_i} \right) + \left[ (a') \frac{dMVOT}{dt_i} \right] \]

where \(i=1,2\). The first term on the right-hand-side of (20)-(21) measures the impact of the tax or price change via its effect on the traffic flow \(F\). At constant values of time, the change in traffic flow influences the number of users affected by a marginal traffic increase, and it affects the slope of the congestion function. Since both effects are plausibly positive, the first term indicates that a price or tax increase reduces marginal external congestion costs as long as it reduces congestion. The second term is the impact of tax changes on the marginal congestion cost via their effect on the value of time. These effects were derived before for a simplified setting.

It is clear, then, that ignoring changes in time values introduces errors in determining marginal external costs. For example, suppose an increase in the price of non-commuting transport reduces congestion and raises the value of time. Assuming exogenous values of time then leads to overestimating the reduction in MECC. Analogously, if an increase in the commuting tax reduces congestion but reduces the value of time then the reduction in MECC will be underestimated if exogenous time values are assumed. Note that if the value of time substantially rises, marginal congestion costs may actually increase despite the reduction in congestion brought about by the price increase. For the same reason it is conceivable that a joint
transport-labour tax reform increases MECC even when traffic flow declines, due to its positive impact on the value of time.

Three implications of the above discussion are worth mentioning. First, exogenous time values will lead to inaccurate assessments of the welfare effects of transport tax reforms and of optimal congestion taxes, when a nontrivial fraction of transport flows refer to commuting. A model that implicitly treats all transport as non-commuting and that imposes constant time values is likely to overestimate the reduction in the MECC as well as the welfare effects. For the same reason, optimal taxation exercises focusing on the transport market but using exogenous time values are likely to underestimate optimal congestion taxes. Second, even with endogenous values of time, we expect combined transport and labour tax reforms to lead to quite different implications in models with multiple trip purposes as compared to models that treat all transport as non-commuting. Suppose a transport tax reform is accompanied by measures to reduce the distortion on the labour market (raising transport taxes rise but reducing labour taxes). In models with a single trip purpose this reform is likely to raise time values. Within the framework of our model with two types of trips, this is not at all obvious. Higher non-commuting transport prices may also raise time values, as previously shown. However, to the extent that the combined change in commuting and labour taxes ultimately reduces the net real wage (i.e., the commuting tax is not fully compensated by the labour tax reduction), this second effect counteracts the first and reduces the value of time. The impact on marginal congestion costs will therefore be dampened. Third, larger shifts in time values and external costs are expected if tax reforms allow differentiation between non-commuting and commuting taxes.
Although generalising these findings is difficult because of the simplicity of the model (for example, the assumption of strict proportionality between labour supply and commuting), there is no reason to expect the interactions will disappear in more general models. Moreover, there is no reason to expect time values to be unaffected by a transport tax reform itself. Whether the issue is empirically sufficiently important to be worried about is something to be found out, and this is the purpose of the numerical exercise in the next section.

3. A numerical application

We illustrate some of the theoretical results using a numerical model with two trip purposes and endogenous time values (see Van Dender [33] for a more detailed description). The model is calibrated using data representing peak period traffic flows and congestion for an average workday in a typical Belgian mid-size city.

3.1 Overview of the model

The applied model generalises the theoretical analysis in two respects. First, instead of a quasi-linear utility function, a nested-CES representation of preferences is used. Second, we allow for multi-modality by distinguishing car and bus trips. The separability assumption for commuting transport is retained, as in Parry and Bento [25]. The consumer’s problem then becomes:

\[
\begin{align*}
\text{Max } & \quad u = U(q_0, (q_1, q_2, N)) + g(q_3, q_4) \\
\text{subject to } & \quad (1-t_i)L + S = q_0 + \sum_{i=1}^{4} p_i q_i \\
& \quad L = N + L + a \sum_{i=1}^{4} q_i
\end{align*}
\]

Here \(q_0\) is the composite commodity (untaxed numeraire), \(q_1\) are non-commuting car trips, \(q_2\) are non-commuting bus trips, \(q_3\) is car commuting, \(q_4\) is bus commuting and \(N\) is leisure. Also, \(a=a(F)=a(q_1+q_3+a(q_2+q_4))\), as the congestion function is adjusted.
for the presence of buses. The parameter $\alpha < 1$ indicates that an extra bus trip per passenger contributes less to congestion than a car trip.\footnote{Note the assumption that the bus occupancy rates are fixed, which is reasonable for peak hours.} One simplification of the numerical model is that a linear congestion function is used, so that a change in the traffic flow does not affect its slope.\footnote{When the real congestion function is convex, using a linear approximation will overestimate the travel time reductions associated to decreases in traffic flow. In order to moderate this overestimate, the linear approximation was made at traffic levels below the reference flows. Newbery and Santos} This allows us to focus on the effect of tax changes on marginal values of time and congestion levels, see (20) and (21). Finally, the proportionality between commuting and labour supply now implies that $q_3 + q_4 = L$.

Traffic flow data and congestion technology are derived from a network model for the city of Namur (Cornélis and Van Dender [6]). The reference peak period speed is 30km/h, half the free flow speed. Reference transport prices are based on Proost and Van Dender [27]. According to a national survey (Pollet [26]) and a survey for Brussels (IRIS [15]), 67% of commuting trips and 75% of non-commuting trips use the car mode. These shares reflect the modal split when the public transport mode is easily accessible. The same sources say that the share of commuting trips in all peak hour trips is 53%. This is in line with a survey for London (LRC [21]), according to which 60% of morning peak trips, and 50% of evening peak trips are commuting trips. Survey evidence for the US (US-DOT [32]: 7) suggests that only some 40% of morning rush hour trips are work-related, with an even lower share for evening rush hours.

In view of the above discussion, we take the 53% share of commuting trips as the reference value in the central scenario. However, it is clear from the theoretical analysis that the relative importance of commuting trips is a key determinant of the effects of tax reform on time values and congestion costs. Moreover, some sources
suggest that commuting shares obtained from surveys may well underestimate the true share of commuting trips, and they report higher shares\textsuperscript{10}. For example, Giuliano [11] finds that at least 70\% of all trips on Californian highways between 6 and 10 am are work-related similarly, the US Value Pricing website www.valuepricing.org states that at least 25\% of peak period trips are non-work trips. Given the range of estimates of the share in commuting trips in the literature, in the next section we therefore also report on some sensitivity analysis with respect to the value of this crucial parameter in the reference situation.

3.2 Results

3.2.1 The central scenario

We report empirical results for a number of tax reform and optimal tax exercises. The reference equilibrium (REF), representing the initial situation in Belgium, is described in the left-most column of Table 2. The labour tax is 40\%, and both commuting and non-commuting car traffic are taxed at less than marginal external cost: taxes amount to 4.24 Euro (per round trip) as compared with marginal external congestion costs (MECC) of 6.87 Euro. For public transport, note that the model assumes, consistent with current practice in Belgium, that bus transport is government-supplied and that the production costs are financed out of general public funds. With the reference subsidy of 2.7 Euro/trip, the consumer price amounts to 0.53 Euro/trip.

The calibrated marginal value of time in the reference equilibrium is 7.67 Euro/hour, or 47\% and 78\% of the gross and the net hourly wage, respectively.

\textsuperscript{10} We are grateful to an anonymous referee for pointing us to this information. The referee also suggests why commute trip shares may be underestimated in surveys: trip chains consisting of, e.g., a network-derived linear congestion functions perform well for an analysis of cordon pricing schemes on a network.
The absolute and the relative levels are in line with the literature (e.g. Small [29]). Other information (on traffic flows, labour supply, etc.) is presented in index or percentage form.

The tax reforms are balanced-budget from the government’s perspective: optimal transport taxes are computed for several arbitrary labour tax changes, and the lump-sum transfer remains constant.\textsuperscript{11} We look at the implications of balanced-budget labour tax reductions by 1\% and 10\% (leading to new labour tax rates of 39.6\% and 36\% respectively), allowing transport taxes to be optimally adjusted and taking account of external congestion costs. Bus fares and car prices are restricted to be non-negative, implying maximal subsidies of 3.22 Euro and 8.08 Euro, respectively.\textsuperscript{12} For each tax reform exercise we distinguish the cases of tax differentiation between commuting and non-commuting, and of uniform taxes across trip purposes. For all tax reforms analysed, the impact on welfare is measured in monetary terms by calculating the equivalent variation of the change. The latter is expressed in Euros per year per consumer.

Table 2 summarises the results. First consider scenarios A to D, which refer to the tax reform exercises. All experiments reported in the table lead to higher welfare (row (1)). Not surprisingly, welfare improvements are much higher for differentiated transport taxes. For example, the equivalent variation of a labour tax reduction from 40\% to 36\% and optimal differentiated transport taxes amounts to some 230 Euros;
with uniform taxes the corresponding figure is 145 Euros. In view of the theoretical discussion in the previous section, it is interesting to observe that the value of time and the marginal external congestion costs increase in all scenarios. The increases in MECC occur *despite* the decrease in the aggregate peak period traffic flow in three out of four scenarios. In other words, the tax reforms often reduce congestion while marginal external congestion costs increase.

The tax adjustments and therefore the changes in the value of time strongly vary between scenarios. In the case of uniform taxes across trip purposes, the optimal response to a 1% reduction in labour tax is to increase car taxes, but to reduce bus prices. A 10% labour tax reduction raises car prices more substantially, and it implies higher bus fares. As a consequence of these adjustments, labour supply rises slightly. Increasing non-commuting transport taxes and minor reductions in the net real wage lead to rising time values (by 0.5% and 10% for the 1% and 10% labour tax reductions, respectively).

The corresponding outcomes are quite different when differentiated taxes are introduced. In those cases, all non-commuting trips become much more expensive. Since the labour tax reductions of 1% and 10% fall short of the optimal labour tax adjustment (see below), there is strong pressure on commuting transport prices not to reduce the net real wage. The bus commuting fare reaches its lower limit of zero, and at the 39.6% labour tax rate, car commuting is heavily subsidized (at ca. 50% of its

---

12 This restriction becomes binding for bus fares if labour taxes are substantially above optimal levels. Since commuting is proportional to labour supply, the optimal transport tax reform will ‘correct’ excessive labour taxes by heavily subsidizing commuting transport.

13 Only the 1% decrease in the labour tax, financed by differentiated transport tax changes, leads to an increase in the traffic flow. The reason is that the insufficiently large labour tax reduction combined with tax differentiation strongly reduces (mainly) commuting by bus. The increase in car commuting raises traffic flow expressed in PCU’s and hence congestion.
Labour supply rises by some 6%. The strong increases in non-commuting transport taxes and the reduction in the labour tax together imply higher values of time: they increase by 5% and 10%, respectively. Note that the larger effects on time values in the case of differentiated taxes are consistent with the theoretical discussion.

Finally, consider the results for the optimal tax exercises. We only report values for differentiated taxes (see scenario E), as the welfare changes and the marginal values of time are not affected by the uniformity constraint (because of the strict complementarity between commuting and labour supply). The optimal labour tax reduction turns out to be 8%-point, reflecting the high initial labour tax. Optimal non-commuting transport taxes are extremely high. Commuting transport taxes also rise relative to the reference situation. The marginal value of time is 14.3% higher than in the reference equilibrium. This is not a trivial change, and it is much larger than the impacts predicted by Mayeres and Proost [22], where the time value is endogenous but there is only one trip purpose.

The numerical model suggests that the endogeneity of the value of time is important when changes in marginal external congestion costs and in welfare are assessed. In all scenarios but one, it affects the size and the direction of the change in MECC. With constant values of time, marginal external congestion costs would have declined substantially because of the reduction in traffic demand after the tax changes. With endogeneity, marginal external congestion costs rise by up to 3%. Using fixed values of time hence leads to erroneous estimates of the adaptations of travel demand and modal split to transport tax changes.

---

14 Calthrop [4] and Wrede [36] report optimal commuting subsidies of 50% and more than 100% of the resource cost, respectively.
3.2.2 Varying the importance of non-commuting trips
The reference share of commuting trips in the previous section (53%) was chosen in order to accord with available evidence for Belgium. However, as previously suggested, there is considerable uncertainty surrounding this parameter, and several authors have suggested higher values. This section therefore provides some insight into the sensitivity of the results on the reference share of commuting trips.\(^{15}\) We first give a detailed comparison of the results of the central scenario with an alternative for which, in the reference situation, commuting trips stand for 70% of all trips. Next we more briefly consider the effects on time values and congestion costs of even further increasing the importance of commuting.

Table 3 compares the central scenario to the case where 70% of peak period trips are commuting trips. Only tax reforms that allow for differentiation of taxes across trip purposes are considered. Three observations are worth making. First, as expected, with a higher reference share of commuting trips, welfare gains and increases in the marginal value of time are considerably smaller. The changes nevertheless remain significant: the value of time at the optimum is 7% higher than the reference value (instead of 14.3%). Second, note that smaller reductions in the labour tax now are required in order to obtain the maximal possible welfare gain. In the central scenario, the optimal labour tax was 32%, while here a reduction from the reference tax of 40% to 36% is sufficient. Also, the combination of a limited labour tax decrease (to 39.6%) with optimal differentiation of transport taxes, now generates

\(^{15}\) We also varied the elasticity of substitution between the composite commodity and leisure activities. This mainly affects the elasticity of labour supply. When the elasticity of labour supply is low, transport taxes will mainly be used to reduce congestion levels by affecting the modal split, and changes in the MVOT are small. When it is high, the welfare gains from (differentiated) transport taxes and increases in MVOT become larger. Further discussion of this exercise is omitted for reasons of brevity. Sensitivity analysis on the slope of the congestion function, the reference modal split, and
73% of the maximal attainable welfare gain. Third, given the higher reference share of commuting trips, the increases in the marginal value of time are now insufficient to raise marginal external congestion costs: in several of the tax reforms considered time values rise, congestion declines and marginal congestion costs go down. This illustrates that the effect on time values is quite robust, but that the ultimate impact on marginal congestion costs determined in the theory can play out in different ways. Importantly, it also suggests that accurate information on trip purpose shares is essential for correct policy assessments.

Table 4 shows the results of further increasing the reference share of commuting trips. For various shares of commuting trips, it gives the percentage changes in the marginal value of time, and the percentage change in welfare associated with optimal labour and transport taxes. The first row in Table 4 refers to the central scenario analysed before (Table 2), the second one to the alternative scenario of Table 3.

the remaining elasticities of substitution was carried out as well. They were found to be less important for the problem at hand, justifying omission of their discussion.
Table 2 Impacts from simultaneously decreasing labour taxes and optimally adapting transport taxes for a constant government budget

<table>
<thead>
<tr>
<th>Reference</th>
<th>40% labour tax</th>
<th>39.6% labour tax</th>
<th>36% labour tax</th>
<th>Optimal labour tax</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit</strong></td>
<td>Uniform transport taxes</td>
<td>Uniform transport taxes</td>
<td>Differentiated transport taxes</td>
<td>Uniform transport taxes</td>
</tr>
<tr>
<td><strong>REF</strong></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>1 Equivalent variation</td>
<td>Euro/year**</td>
<td>0</td>
<td>30.1</td>
<td>140.3</td>
</tr>
<tr>
<td>2 % of maximal welfare gain</td>
<td>%</td>
<td>0</td>
<td>10.4</td>
<td>64.0</td>
</tr>
<tr>
<td>3 MVOT</td>
<td>Euro/h</td>
<td>7.67</td>
<td>7.71</td>
<td>8.44</td>
</tr>
<tr>
<td>4 MECC</td>
<td>Euro/round trip</td>
<td>6.87</td>
<td>6.92</td>
<td>6.93</td>
</tr>
<tr>
<td>5 Traffic flow in PCU</td>
<td>Index</td>
<td>1</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>6 Q1 level</td>
<td>Index</td>
<td>1</td>
<td>0.98</td>
<td>0.75</td>
</tr>
<tr>
<td>7 Q2 level</td>
<td>Index</td>
<td>1</td>
<td>1.05</td>
<td>0.78</td>
</tr>
<tr>
<td>8 Q3 level</td>
<td>Index</td>
<td>1</td>
<td>0.82</td>
<td>1.36</td>
</tr>
<tr>
<td>9 Q4 level</td>
<td>Index</td>
<td>1</td>
<td>1.37</td>
<td>0.45</td>
</tr>
<tr>
<td>10 Q1 market share</td>
<td>%</td>
<td>35.3</td>
<td>34.5</td>
<td>28.9</td>
</tr>
<tr>
<td>11 Q2 market share</td>
<td>%</td>
<td>11.8</td>
<td>12.4</td>
<td>10.0</td>
</tr>
<tr>
<td>12 Q3 market share</td>
<td>%</td>
<td>35.3</td>
<td>28.9</td>
<td>52.4</td>
</tr>
<tr>
<td>13 Q4 market share</td>
<td>%</td>
<td>17.6</td>
<td>24.2</td>
<td>8.6</td>
</tr>
<tr>
<td>14 Labour supply</td>
<td>Index</td>
<td>1</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>15 T1: car tax non-commuting</td>
<td>Euro/round trip</td>
<td>4.24</td>
<td>5.73</td>
<td>12.61</td>
</tr>
<tr>
<td>16 T2: bus tax non-commuting</td>
<td>Euro/round trip</td>
<td>0.53</td>
<td>0.37</td>
<td>5.61</td>
</tr>
<tr>
<td>17 T3: car tax commuting</td>
<td>Euro/round trip</td>
<td>4.24</td>
<td>5.73</td>
<td>-4.05</td>
</tr>
<tr>
<td>18 T4: bus tax commuting</td>
<td>Euro/round trip</td>
<td>0.53</td>
<td>0.37</td>
<td>0</td>
</tr>
</tbody>
</table>

* The optimal labour tax decrease with uniform transport taxes is approximately 18%.
** Daily values have been converted to yearly values by assuming 220 workdays per year.
<table>
<thead>
<tr>
<th></th>
<th>Equivalent variation</th>
<th>Unit</th>
<th>40% labour tax</th>
<th>39.6% labour tax</th>
<th>36% labour tax</th>
<th>32% labour tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>REF1</td>
<td>REF2</td>
<td>A1</td>
<td>A2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>53%</td>
<td>70%</td>
<td>53%</td>
<td>70%</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>Euro/year*</td>
<td>0</td>
<td>0</td>
<td>140.3</td>
<td>99.1</td>
</tr>
<tr>
<td>2</td>
<td>% of maximal welfare gain</td>
<td>%</td>
<td>0</td>
<td>0</td>
<td>64.0</td>
<td>73.5</td>
</tr>
<tr>
<td>3</td>
<td>MVOT</td>
<td>Euro/h</td>
<td>7.67</td>
<td>7.78</td>
<td>8.44</td>
<td>8.19</td>
</tr>
<tr>
<td>4</td>
<td>MECC</td>
<td>Euro/round trip</td>
<td>6.87</td>
<td>5.34</td>
<td>6.93</td>
<td>5.28</td>
</tr>
<tr>
<td>5</td>
<td>Traffic flow in PCU</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>1.05</td>
<td>1.06</td>
</tr>
<tr>
<td>6</td>
<td>Q1 level</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>7</td>
<td>Q2 level</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>0.78</td>
<td>0.73</td>
</tr>
<tr>
<td>8</td>
<td>Q3 level</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>1.36</td>
<td>1.24</td>
</tr>
<tr>
<td>9</td>
<td>Q4 level</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>0.45</td>
<td>0.61</td>
</tr>
<tr>
<td>10</td>
<td>Q1 market share</td>
<td>%</td>
<td>35.3</td>
<td>23.1</td>
<td>28.9</td>
<td>18.1</td>
</tr>
<tr>
<td>11</td>
<td>Q2 market share</td>
<td>%</td>
<td>11.8</td>
<td>7.2</td>
<td>10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>12</td>
<td>Q3 market share</td>
<td>%</td>
<td>35.3</td>
<td>46.6</td>
<td>52.4</td>
<td>60.8</td>
</tr>
<tr>
<td>13</td>
<td>Q4 market share</td>
<td>%</td>
<td>17.6</td>
<td>23.1</td>
<td>8.6</td>
<td>15.1</td>
</tr>
<tr>
<td>14</td>
<td>Labour supply</td>
<td>Index</td>
<td>1</td>
<td>1</td>
<td>1.06</td>
<td>1.03</td>
</tr>
<tr>
<td>16</td>
<td>T2: bus tax non-commuting</td>
<td>Euro/round trip</td>
<td>0.53</td>
<td>0.53</td>
<td>5.61</td>
<td>6.702</td>
</tr>
<tr>
<td>17</td>
<td>T3: car tax commuting</td>
<td>Euro/round trip</td>
<td>4.24</td>
<td>4.24</td>
<td>-4.05</td>
<td>0.025</td>
</tr>
<tr>
<td>18</td>
<td>T4: bus tax commuting</td>
<td>Euro/round trip</td>
<td>0.53</td>
<td>0.53</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* Daily values have been converted to yearly values by assuming 220 workdays per year.
Table 4 The dependence of changes in welfare and in the marginal value of time on the reference level and share of non-commuting trips, with tax differentiation between trip purposes

<table>
<thead>
<tr>
<th>Share of commuting trips</th>
<th>Optimal labour tax</th>
<th>% MVOT change</th>
<th>Index of welfare change</th>
</tr>
</thead>
<tbody>
<tr>
<td>53% (central sc.)</td>
<td></td>
<td>14.3%</td>
<td>100</td>
</tr>
<tr>
<td>70%</td>
<td></td>
<td>6.9 %</td>
<td>42</td>
</tr>
<tr>
<td>82%</td>
<td></td>
<td>3.4 %</td>
<td>26</td>
</tr>
<tr>
<td>99.99%</td>
<td></td>
<td>0.37 %</td>
<td>7</td>
</tr>
</tbody>
</table>

The table shows that as the importance of non-commuting transport decreases, the impact of tax adjustments on time values and the potential welfare gains declines. When the share of non-commuting trips is negligible (bottom row), the impact on time values is very small because transport taxes now are pure commuting taxes, which have the same effect on time values as labour taxes. The labour tax reduction and transport tax increases then hardly affect time values, and the only source of the welfare gain is the improvement of the modal split through modal tax differentiation.

The basic message from this experiment is that, as predicted by the theoretical analysis, the size of the impact of transport tax changes on the marginal value of time crucially depends on the presence and the quantitative importance of at least two trip purposes. The results also clarify that there is a close connection between the size of the welfare gain and the size of the change in marginal values of time.

4. **Summary and conclusion**

The analysis suggests that transport tax reform to cope with external congestion costs in a second-best setting tends to increase consumers’ marginal value of time savings, if account is taken of the simultaneous presence of commuting and non-commuting trips on the road. The fundamental reason is that the tax reform allows shifting part of the tax burden to relative complements to leisure, thereby increasing the opportunity cost of leisure. A numerical illustration for a prototype mid-sized Belgian city suggests that the effect of transport tax changes on the marginal value of time is significant. In our central scenario, marginal external congestion costs after the
reform are higher than in the pre-reform equilibrium, because the increase in the marginal value of time more than compensates for the reduction in congestion costs due to lower traffic demand. It should be stressed that this increase is accompanied by a welfare increase. The effect also holds when optimal transport and labour taxes are considered. The increase in the value of time is smaller, but positive, when transport taxes cannot be differentiated across trip purposes. Increased reference shares of commuting trips tend to weaken the effects described above. In particular, while marginal values of time still increase, marginal external congestion costs decrease in our alternative scenario.

Hence, assuming constant values of time in an analysis of transport tax reform may produce inaccurate results whenever the traffic flow consists of commuting and non-commuting transport. Traffic flows that are homogenous in terms of trip purpose will display a smaller sensitivity of the marginal time value to transport tax changes.

The analysis is subject to some caveats. First, the assumption of strict complementarity between peak-hour commuting trips and labour supply is restrictive. Relaxing it will affect the optimal values of the tax instruments and the resulting value of time, but the direction of the change can be expected to remain as discussed here. Assuming strict complementarity probably is to be preferred above treating transport as a standard commodity, when transport tax reform is analysed in a context of distortionary taxes on labour. Second, the analysis has abstracted from distributional considerations. Taking these into account may imply that alternative types of revenue use (e.g. increasing lump sum transfers instead of reducing labour taxes) become relatively more attractive. Finally, the numerical results are exploratory. The goal here is to illustrate the mechanisms at work in the theoretical analysis, using realistic
orders of magnitude for the parameters. More realistic policy analysis would require more investment in the transport data.

References

[18] M. Johnson, Travel time and the price of leisure, Western Economic Journal


Appendix

As an example, let us start from (10) and use Cramer’s rule to solve for $dq_i$. We then have:
\[
 dq_1 = \frac{1}{\Delta} \begin{vmatrix}
 dp_1 & -\gamma a' & u_{1N} & -a \\
 dp_2 + dt_L & u_{22} - \gamma a' & 0 & -(1+a) \\
 0 & 0 & u_{NN} & -1 \\
 0 & -(1+a+a'F) & -1 & 0 \\
\end{vmatrix}
\]

where:

\[
\Delta = \begin{vmatrix}
 u_{11} - \gamma a' & -\gamma a' & u_{1N} & -a \\
 -\gamma a' & u_{22} - \gamma a' & 0 & -(1+a) \\
 u_{N1} & 0 & u_{NN} & -1 \\
-(a+a'F) & -(1+a+a'F) & -1 & 0 \\
\end{vmatrix}
\]

is the determinant associated with (10). We show below that \( \Delta \) is plausibly negative and explain the interpretation of the condition \( \Delta < 0 \). Some matrix algebra yields the effect of an exogenous price or tax changes on non-commuting transport demand:

\[
\frac{dq_1}{dp_1} = \frac{1}{\Delta} \begin{vmatrix}
 u_{22} - \gamma a' & 0' & -(1+a) \\
 0 & u_{NN} & -1 \\
-(1+a+a'F) & -1 & 0 \\
\end{vmatrix} = \frac{-1}{\Delta} \{(u_{22} - \gamma a') + u_{NN}(1+a)(1+a+a'F)\}
\]

\[
\frac{dq_1}{dp_2} = \frac{-1}{\Delta} \begin{vmatrix}
 -\gamma a' & u_{1N} & -a \\
 0 & u_{NN} & -1 \\
-(1+a+a'F) & -1 & 0 \\
\end{vmatrix} = \frac{1}{\Delta} \{(au_{NN} - u_{1N})(1+a+a'F) - \gamma a'\}
\]

In a similar fashion, the impact of tax or price changes on labour supply (equal to commuting), on leisure demand and on the value of time \( \gamma \) can be derived. The results for commuting demand and the value of time are reported in the paper. The effects on leisure demand are given by:

\[
\frac{dN}{dp_1} = \frac{1}{\Delta} \left\{ u_{1N}(1+a)(1+a+a'F) + u_{22}(a+a'F) + \gamma a' \right\}
\]

\[
\frac{dN}{dt_L} = \frac{dN}{dp_2} = \frac{-1}{\Delta} \left\{ (u_{1N}a - u_{1N})(1+a+a'F) + \gamma a' \right\}
\]

The sign of the price effects on leisure demand crucially depends on the sign and the size of the impact of leisure on the marginal utility of transport, \( u_{1N} \). The larger this
effect, the more likely it is that non-commuting transport prices reduce leisure demand. A sufficient condition for a labour tax increase to raise leisure demand is that \( a u_{i_N} - u_{i_1} > 0 \).

Now consider the case where transport prices cannot or are not allowed to differ between commuting and non-commuting (i.e., \( p_1 = p_2 = p \)). All effects of the labour tax remain as before, and the price effects are given by:

\[
\frac{dq_1}{dp} = \frac{-1}{\Delta} \left\{ (u_{NN} + u_{i_N})(1 + a + a' F) + u_{22} \right\}
\]

\[
\frac{dq_2}{dp} = \frac{-1}{\Delta} \left\{ (u_{NN} - u_{i_N})(a + a' F) + (u_{i_N} + u_{i_1}) \right\}
\]

\[
\frac{dN}{dp} = \frac{-1}{\Delta} \left\{ (u_{i_1} - u_{i_N})(1 + a + a' F) - u_{22}(a + a' F) \right\}
\]

The impact of a common transport price increase is the sum of the effects of the labour tax and the non-commuting transport price of the price differentiation case. Interpretation is as before. The impact on the value of time is reported in the paper.

Finally, we return briefly to the meaning of the condition \( \Delta < 0 \). By developing the relevant determinant one shows:

\[
\Delta = \Delta_{a' = 0} + u_{NN} \left\{ \gamma a' - a a' F u_{22} \right\} = (u_{NN} u_{i_1} - u_{i_N}^2) a'(1 + a) + \gamma a'(u_{i_1} + u_{22})
\]

\[
+ u_{i_N} \left\{ a' F u_{22} + 2 \gamma a' \right\}
\]

Here \( \Delta_{a' = 0} = -(u_{NN} u_{i_1} - u_{i_N}^2)(1 + a)^2 - u_{22}(u_{i_1} + a^2 u_{NN} - 2a) \) is the value of the determinant at constant congestion \( (a' = 0) \). This is negative by the second-order conditions of the consumer’s optimisation problem. All terms in the definition of \( \Delta \) are negative except the last one. Without a mild condition, therefore, it cannot be guaranteed that \( \Delta < 0 \) and, as a consequence, that \( \frac{dq_1}{dp_1} < 0 \). We assume throughout the condition is satisfied. Intuitively, it is a restriction on the size of feedback effects.
To see this, note that the demand functions resulting from the consumer’s problem can be written in general as functions of prices, the lump-sum transfer $S$ and, since the consumer treats congestion as given, the congestion level (as captured by $a$):

\[
q_0 = q_0(p_1, 1-t_L - p_2, a, S) \\
q_1 = q_1(p_1, 1-t_L - p_2, a, S) \\
q_2 = q_2(p_1, 1-t_L - p_2, a, S) \\
N = N(p_1, 1-t_L - p_2, a, S)
\]

Differentiating this system, taking account of the definition of $a = a(F) = a(q_1 + q_2)$, the full effect of a price change $dp_1$ on non-commuting demand can be written as:

\[
\frac{dq_1}{dp_1} = \frac{\partial q_1}{\partial p_1} (1-a'*\frac{\partial q_1}{\partial a}) + a\frac{\partial q_1}{\partial p_1} \frac{\partial q_1}{\partial a} \\
\frac{dq_2}{dp_1} = \frac{1-a'*\frac{\partial q_2}{\partial a}}{1-a'} - a\frac{\partial q_2}{\partial a}
\]

If congestion is constant ($a'=0$), the total effect equals the partial effect. If it is not, there are feedback effects on demand, implying deviations between the partial and total effect. The denominator is plausibly positive because one expects more congestion to reduce travel demand. The numerator is negative unless the cross-price effect \(\frac{\partial q_2}{\partial p_1}\) is negative and large, so that the final term in the numerator more than offsets the (negative) first term. The economic intuition of this extreme situation is clear. Suppose a price increase of non-commuting trips at constant congestion levels reduces non-commuting trips. But assume that the price increase also implies a large reduction in commuting demand which itself reduces congestion, raising the demand for non-commuting transport again. If this latter effect more than offsets the initial negative impact, the numerator of the above expression becomes positive and the ultimate outcome is a positive price effect.