Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **409,** 1244–1252 (2010)   doi:10.1111/j.1365-2966.2010.17386.x

Does *Kepler* unveil the mystery of the Blazhko effect? First detection of period doubling in *Kepler* Blazhko RR Lyrae stars

R. Szabó,[1]★ Z. Kolláth,[1] L. Molnár,[1] K. Kolenberg,[2] D. W. Kurtz,[3] S. T. Bryson,[4] J. M. Benkő,[1] J. Christensen-Dalsgaard,[5] H. Kjeldsen,[5] W. J. Borucki,[4] D. Koch,[4] J. D. Twicken,[6] M. Chadid,[7] M. Di Criscienzo,[8] Y-B. Jeon,[9] P. Moskalik,[10] J. M. Nemec[11] and J. Nuspl[1]

[1]*Konkoly Observatory of the Hungarian Academy of Sciences, Konkoly Thege Miklós út 15-17, H-1121 Budapest, Hungary*
[2]*Institut für Astronomie, University of Vienna, Türkenschanzstrasse 17, A-1180 Vienna, Austria*
[3]*Jeremiah Horrocks Institute of Astrophysics, University of Central Lancashire, Preston PR1 2HE*
[4]*NASA Ames Research Center, MS 244-30, Moffet Field, CA 94035, USA*
[5]*Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark*
[6]*SETI Institute/NASA Ames Research Center, Moffett Field, CA 94035, USA*
[7]*Observatoire de la Côte d'Azur, Université Nice, Sophia-Antipolis, UMR 6525, parc Valrose, 06108 Nice Cedex 02, France*
[8]*INAF-Osservatorio Astronomico di Roma, via Frascati 33, I-00040 Monte Porzio Catone, Roma, Italy*
[9]*Korea Astronomy and Space Institute, Daejeon 305-346, Korea*
[10]*Copernicus Astronomical Center, ul. Bartycka 18, 00-716, Warsaw, Poland*
[11]*Department of Physics & Astronomy, Camosun College, Victoria, British Columbia, V8P 5J2, Canada*

Accepted 2010 July 20. Received 2010 July 19; in original form 2010 June 4

ABSTRACT

The first detection of the period doubling phenomenon is reported in the *Kepler* RR Lyrae stars RR Lyr, V808 Cyg and V355 Lyr. Interestingly, all these pulsating stars show Blazhko modulation. The period doubling manifests itself as alternating maxima and minima of the pulsational cycles in the light curve, as well as through the appearance of half-integer frequencies located halfway between the main pulsation period and its harmonics in the frequency spectrum. The effect was found to be stronger during certain phases of the modulation cycle. We were able to reproduce the period-doubling bifurcation in our non-linear RR Lyrae models computed by the Florida–Budapest hydrocode. This enabled us to trace the origin of this instability in RR Lyrae stars to a resonance, namely a 9:2 resonance between the fundamental mode and a high-order (ninth) radial overtone showing strange-mode characteristics. We discuss the connection of this new type of variation to the mysterious Blazhko effect and argue that it may give us fresh insights into solving this century-old enigma.

Key words: stars: individual: RR Lyrae – stars: individual: V808 Cyg – stars individual: V355 Lyr – stars: oscillations – stars: variables: RR Lyrae.

1 INTRODUCTION

The unprecedented power of the *Kepler*[1] space telescope in terms of precision and continuity is poised to deliver major breakthroughs in exoplanet science (Borucki et al. 2010) and stellar photometry (Gilliland et al. 2010a), allowing exploration of territories never tested before. New instruments often reveal surprising new phenomena and lead to new insights into (astro)physical problems. Such an unforeseen feature, *period doubling*[2] (PD), and the corresponding half-integer frequencies (HIFs) were reported by Kolenberg et al. (2010a) in the *Kepler* Q1 data (34 d) of RR Lyr, the prototype, brightest Blazhko-type RR Lyrae star in the sky.

It has been well known for decades that high-luminosity RV Tauri variables show alternating deep and shallow minima in their light and radial velocity curves. Buchler & Kovács (1987) and Kovács & Buchler (1988) carried out the first systematic search of irregular oscillations in radiative and strongly dissipative Pop. II (W Vir)

★E-mail: rszabo@konkoly.hu

[1] http://kepler.nasa.gov

[2] The PD phenomenon is not to be confused with double-mode pulsation, where two radial modes (of low order) are excited simultaneously.

models. They demonstrated that the pulsations in these models undergo a *Feigenbaum cascade* of PD bifurcations by changing the control parameter (effective temperature). That is to say, the instability develops from strict periodic pulsation to period-2, period-4, etc., oscillations resulting in low dimensional chaos.

Moskalik & Buchler (1990, 1991) and Buchler & Moskalik (1992) reported PD bifurcation, as well, in purely radiative Cepheid and BL Her model sequences. By changing the control parameter ($T_{\rm eff}$), their weakly dissipative Pop. I. Cepheid models showed the onset of PD and a subsequent reversion to period-1 oscillations instead of further PD episodes, in contrast to the more dissipative models.

Non-linear stable periodic pulsations (limit cycles) can be made to 'period double' through the destabilization of either a thermal (real) mode or an additional (complex) vibrational mode. Moskalik & Buchler (1990) could trace the origin of the PD to a destabilized low-lying vibrational overtone and that the coupling occurs through an internal resonance of the type $(2n + 1)\,\omega_0 \approx 2\omega_k$, where n is an integer (1 or 2) and the subscripts 0 and k refer to the fundamental and the kth overtone modes, respectively. In this case, the parametric instability of an overtone pulsation mode in half-integer resonance opens up an additional dimension which allows the limit cycle to period double.

In this paper, we describe our discovery of the PD phenomenon in three RR Lyrae variables observed by the *Kepler* space telescope. One of them is RR Lyr (KIC 7198959, Kepler mag: 7.9), the prototype of its class. V808 Cyg (KIC 4484128) and V355 Lyr (KIC 7505345), the two other RRab stars, are of much fainter apparent brightness ($Kp = 15.4$ and 14.1, respectively). Four additional *Kepler* RR Lyrae stars show weak signs of the PD phenomenon. All these objects show the enigmatic Blazhko effect, i.e. amplitude and phase modulation of the regular RR Lyrae pulsation. For the recent *Kepler* findings with respect to RR Lyr itself and an overview of the Blazhko behaviour of *Kepler* RR Lyrae stars, we refer to Kolenberg et al. (2010b) and Benkő et al. (2010), respectively.

Since resonances are known to play much less of a role in RR Lyrae stars than in Cepheids, it is natural to ask whether occurrence of any type of resonance between radial modes can cause this behaviour. Serendipitously, we encountered RR Lyrae models showing the PD bifurcation and subsequently we applied them to the *Kepler* RR Lyrae stars. We were able to demonstrate that in RR Lyrae stars, the physical origin of this instability is a 9:2 resonance between the fundamental mode and a high-order radial overtone. This latter mode is called a *strange mode* (Buchler & Kolláth 2001), because it has no adiabatic counterpart and the energy associated with its pulsation is confined to the outer zones of the star. The goal of this work is to present the first *Kepler* RR Lyrae PD results and our first successful modelling efforts.

The outline of this paper is as follows. In Section 2, we describe *Kepler* observations we use and devote special emphasis to reduction and proper handling of *Kepler* photometry. In Section 3, the observed properties of the PD are presented. We then turn to hydrodynamical models that successfully reproduce the newly discovered phenomenon in Section 4. Finally, the implications of using this transient phenomenon to gain insight into the enigmatic Blazhko effect are explored in Section 5.

2 OBSERVATIONS

Kepler was designed to detect transits of terrestrial planets on Earth-like orbits around solar-like stars. This requires the observation of $\sim 10^5$ main-sequence stars continuously for several years with great accuracy. *Kepler* was launched on 2009 March 6 and observes a 105 deg^2 area of the sky in constellations Cygnus and Lyra, a few degrees above the Galactic plane. After a short commissioning phase, the scientific observations started on May 12. In order to ensure optimal solar irradiation of the solar arrays, a 90° roll of the telescope is performed at the end of each quarter. The first roll lasted only for 33.5 d (Q1). The second roll was the first complete one (Q2). In this work, we use both Q1 and Q2 when available, i.e. 127 d quasi-continuous observations.

The *Kepler* magnitude system (*Kp*) refers to the wide pass band (430–900 nm) transmission of the telescope and detector system. Both long-cadence (LC, 29.4 min; Jenkins et al. 2010), and short-cadence (SC, 58.9 s; Gilliland et al. 2010b) observations are based on the same 6-s integrations which are summed to form the LC and SC data onboard. In this work, we used only LC data. The saturation limit is between $Kp \simeq 11$ and 1 2 mag depending on the particular chip on which the star is observed; brighter than this, accurate photometry can be performed up to $Kp \simeq 7$ mag with judiciously designed apertures.

The Kepler Asteroseismic Science Consortium (KASC) was set up to exploit the potential of *Kepler* in solar-like oscillations as well as in all types of pulsations. KASC Working Group 13 is dedicated to the investigation of RR Lyrae stars. Out of 29 RRab stars that were observed by *Kepler,* 14 were found to be Blazhko stars. Small gaps are seen in their light curves. These are due to an unplanned safe mode and loss of fine point events as well as regular data downlink periods. Excluding these cadences, the Q1 data segment contains 1626 useful data points, while Q2 contains 4097 points.

Our trend and jump filtering algorithm was tested on the *Kepler* data. We found that no detrending was necessary for our targets. Jump corrections were also considered unnecessary for our purposes. We noted that Q1 and Q2 mean brightness and amplitudes of a given pulsating star may differ. For our fainter targets, only the mean brightness had to be adjusted between different rolls. In the case of RR Lyr, however, a more thorough analysis and calibration was needed.

2.1 Accurate photometry of bright *Kepler* targets

As we mentioned before, *Kepler* CCDs saturate between $Kp \simeq 11$ and 12 mag, but the saturated flux is conserved to a very high degree, spilling in the column direction. This allows *Kepler* to perform high-precision photometry on saturated targets like RR Lyr as long as a sufficient number of pixels are captured in the column direction. In Q1 and Q2, a fraction of flux from RR Lyr fell outside the *Kepler* aperture (set of downlinked pixels), so extra care is required to assure that the PD phenomenon is not due to loss of flux in some cadences. We developed an approach that estimates a correction of the RR Lyr flux. Part of the original and corrected light curve is shown in Fig. 1. The details of this method are described in Kolenberg et al. (2010b).

Here we only note that in Q1 the flux corrections in the case of RR Lyr were less than 5 per cent, while in Q2 the necessary flux corrections were larger, ranging from 15 to 35 per cent. The uncertainty in the total corrected flux is about 0.25 per cent. This is to be compared with the flux uncertainty of 8×10^{-6} in cadences where the correction was not applied because the flux was completely captured. For comparison, the uncertainty of the (uncorrected) flux for the two fainter stars showing the PD varies between $3–7 \times 10^{-4}$ for V808 Cyg and $1.5–3.1 \times 10^{-4}$ for V355 Lyr.

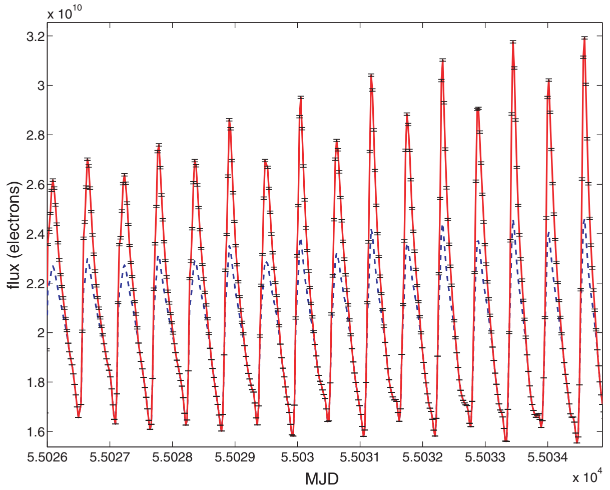

Figure 1. A comparison between the captured flux from RR Lyr (blue dashed) and the corrected flux (red) during Q2, with $\pm 1\,\sigma$ error bars. Note the alternating high/low amplitudes and depths at minimum and maximum light.

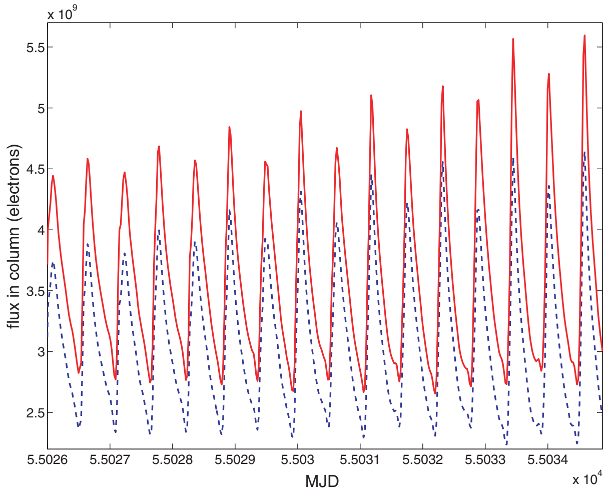

Figure 2. The flux summed along the left-adjacent (blue dashed) and right-adjacent (red) columns to the central column. The incident stellar flux in these adjacent columns was fully captured.

Because these corrections are estimates, we provide the following evidence that the PD phenomenon in Q2 RR Lyr data is not due to loss of flux.

(i) Similar alternating cycles are seen in the flux time series of individual pixels, as well as the sum of pixels along columns with significant flux (other than the central column) from RR Lyr (see Fig. 2). In individual pixels, the PD signal is much larger than the pixel-level noise, which is dominated by pointing jitter and shot noise.

(ii) Saturated pixels induce a faint video cross-talk signal on other CCD channels. At its maximum, cross-talk from the saturated flux from RR Lyr impinged on a faint observed target, and this cross-talk signal is consistent with the reconstructed light curve including PD.

(iii) *Kepler* light curves are created using pixels in a photometrically optimal aperture that maximizes the signal-to-noise ratio for the target (Bryson et al. 2010). *Kepler* downlinks a superset of these pixels (Haas et al. 2010) including at least a 1-pixel halo around the photometrically optimal aperture. In the case of Q1 and Q2 observations of RR Lyr, significantly more pixels were downlinked. Flux light curves created using all downlinked pixels exhibit essentially the same PD phenomenon as the light curve generated from the photometrically optimal pixels. This demonstrates that PD is not due to loss of flux from the (smaller) optimal aperture other than the central saturated column.

(iv) Perhaps the strongest evidence that PD is not due to loss of flux is that the PD phenomenon was seen in Q1 when the flux from RR Lyr was completely captured.

(v) The fact that three stars were found unambiguously showing the PD effect further helps in ruling out external influences. The bright RR Lyr and the two much fainter stars (V808 Cyg and V355 Lyr) demonstrate that the same effect is operational at two different parts of the dynamical range of the CCDs. In addition, all three stars fell on different CCD modules and the modules were rotated between Q1 and Q2, so the PD effect is independent of the CCD modules.

Based on the analysis described in this section, we have high confidence that the PD phenomenon is not due to instrumental effects.

3 THE PERIOD-DOUBLING PHENOMENON

3.1 Alternating extrema and half-integer frequencies

PD is found in three of the Blazhko stars in the *Kepler* field: RR Lyr (KIC 7198959), V808 Cyg (KIC 4484128) and V355 Lyr (KIC 7505345). Some of their properties can be found in Table 1. The pulsational period, the Blazhko period and the amplitude of the first Fourier component (A_1) were derived from the available *Kepler* observations. These numbers will be refined with more *Kepler* data.

The upper panel of Fig. 3 shows the Q1+Q2 light curves for RR Lyr in the *Kp* band. The individual maxima and minima were fitted with a ninth-order polynomial to test the effect of the 29.4-min sampling which may undersample the rapidly changing light curve around the maxima. We find no significant problem arising from the LC sampling. Certain parts of the light curves are marked with dashed line rectangles and are magnified to discern the alternating maxima and minima in Fig. 4. The fitted maxima and minima are plotted as horizontal bars to guide the eye in these figures. In the case of RR Lyr, a difference of 0.1 mag is seen in the brightness of subsequent maxima. This amounts to a few hundredths of magnitude in the case of V808 Cyg. Figs 5 and 6 show the same figures for V808 Cyg.

The alternating maxima and minima in conjunction with the HIFs (i.e. $k/2 \times f_0$, where $k = 1, 3, 5, \ldots$) in the frequency spectrum are typical signs of the PD bifurcation. For the frequency analysis, we used SIGSPEC (Reegen 2007). Where available, Q1 and Q2 data sets were merged. When the spectral significance reached the conservative value of 5 the procedure was stopped, although the traces of HIFs can be followed up to the Nyquist frequency (24.5 c/d). The results were checked by Period04 (Lenz & Breger 2005). Only minor differences were found, mainly in the phase values.

In our third case, V355 Lyr, the PD effect is undoubtedly present, but is rather weak (Fig. 7). The maximum amplitude of the $3/2\,f_0$ frequency is 5 mmag, while it is 25 mmag for the other two targets. This translates to a few hundredths of a magnitude difference in consecutive maxima or minima. Interestingly, the amplitude modulation is also small for this star.

Table 1. Main properties of the observed *Kepler* Blazhko RR Lyrae stars showing the PD effect. Errors are given in parentheses implying variation in the last digit only. The uncertainty of the Blazhko period is estimated to be 0.3 d.

KIC ID	GCVS name	R.A. (J2000)	Dec. (J2000)	*Kp* (mag)	Puls. period (d)	A_1 (mag)	Blazhko period (d)	Runs
7198959	RR Lyr	19 25 27.91	+42 47 03.73	7.862	0.566 9685(8)	0.158(2)	39.6	Q1, Q2
4484128	V808 Cyg	19 45 39.02	+39 30 53.42	15.363	0.547 8721(8)	0.299(3)	90.2	Q1, Q2
7505345	V355 Lyr	18 53 25.90	+43 09 16.45	14.080	0.473 6958(10)	0.374(3)	31.3	Q2

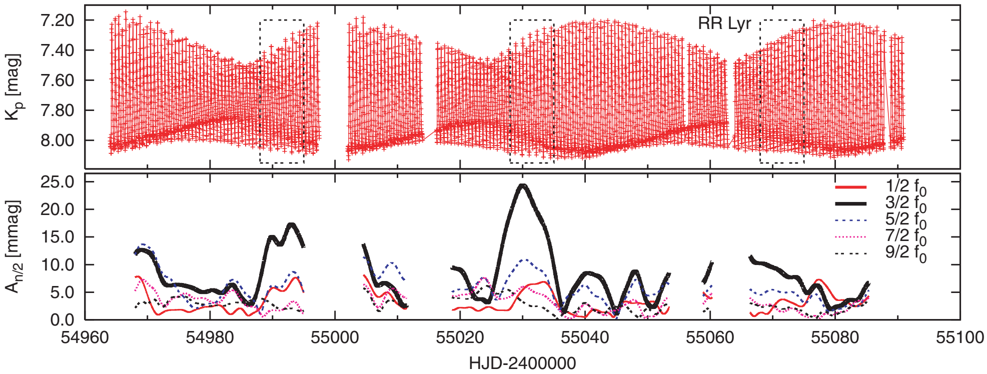

Figure 3. Upper panel: Q1+Q2 light curve of RR Lyr. Note that the individual pulsational cycles are hardly discernible, while the long period (39.6-d) Blazhko modulation clearly stands out. The three dashed boxes are enlarged in Fig. 4. Bottom panel: amplitudes of the HIFs.

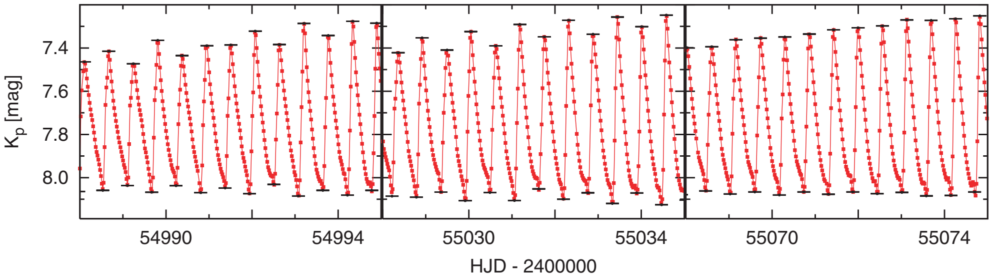

Figure 4. 7-d segments of the *Kepler* light curve of RR Lyr showing different degrees of PD effect at the same Blazhko phase. The maxima and the minima were fitted with a ninth-order polynomial and the small horizontal bars drawn through the extrema are plotted to guide the eye.

Four other Blazhko RR Lyrae stars in the *Kepler* field are seen to possibly exhibit the PD effect: V2178 Cyg (KIC 3864443), V354 Lyr (KIC 6183128), V445 Lyr (KIC 6186029) and V360 Lyr (KIC 9697825). In their frequency spectra, peaks were found close to the predicted HIFs. However, our criteria for the detection of the PD effect were the clear sign of alternating height of the pulsation cycles as well as the simultaneous presence of a large number of HIFs (preferably more than eight). If any of these two requirements were not met by a star, we consider it as a possible PD object only. We note that some of these four stars in this category show additional frequencies, making their frequency spectrum more complex. For more details on these stars, we refer to Benkő et al. (2010).

From now on, we turn to our three stars that show a securely detected PD phenomenon. We plotted the averaged amplitudes of the HIFs of these stars in Fig. 8 taken from their frequency spectra. It is interesting to note that in all three cases, the $3/2\,f_0$ frequency has the highest amplitude among the HIF peaks; next come $5/2\,f_0$ and $1/2\,f_0$. This appears to be a general feature of the PD phenomenon in Blazhko stars. Around the fifth HIF (i.e. $k = 9$), a pronounced bump is seen in the amplitude distribution. The origin of this bump is explained in detail in Section 4. The amplitude of the higher order half-integer peaks is decreasing more or less steadily with the order number k.

3.2 The transient nature of the period doubling

After discussing the time-averaged properties of the HIFs, we now turn to investigate their temporal behaviour. The lower panels of Figs 3 and 5 for RR Lyr and V808 Cyg, respectively, show the temporal behaviour of the amplitude of the most prominent HIFs in the frequency spectra. These were computed using the *analytic signal method* (Kolláth et al. 2002), a powerful method developed to follow time-dependent signals. We note here that the method is superior compared to other time-dependent Fourier methods but has a drawback: in the presence of large gaps, the procedure does not yield reliable results; therefore, we had to cut the neighbourhood of the missing data. We found a 0.25 c/d bandwidth to give the most stable results, and a ~2.5-d long data segment is lost in each side of a gap. The relatively broad bandwidth means that sometimes more than one frequency peak is contained in the computed interval, but the insensitivity to noise compensates for this disadvantage. We tested that the temporal behaviour of the HIFs is not flawed by the chosen bandwidth.

It is obvious from Figs 3 and 5 that the intensity of the PD phenomenon is changing with time. For RR Lyr, it has maximum strength on the ascending branch of the Blazhko envelope on the first two rising branches and is much less visible during the third

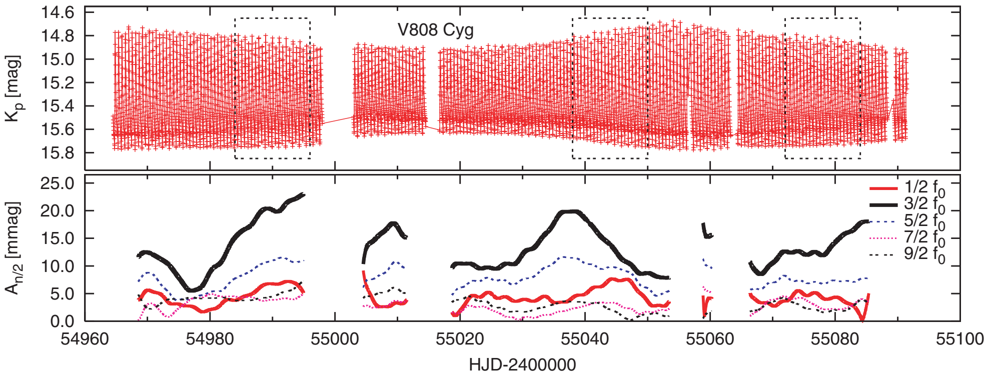

Figure 5. Upper panel: Q1+Q2 light curve of V808 Cyg. The three dashed boxes are enlarged in Fig. 6. Bottom panel: amplitudes of the HIFs.

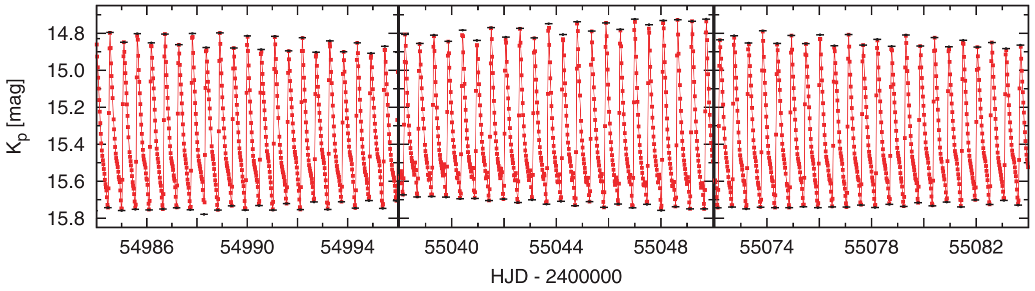

Figure 6. 12-d segments of the *Kepler* light curve of V808 Cyg showing the PD effect at different Blazhko phases.

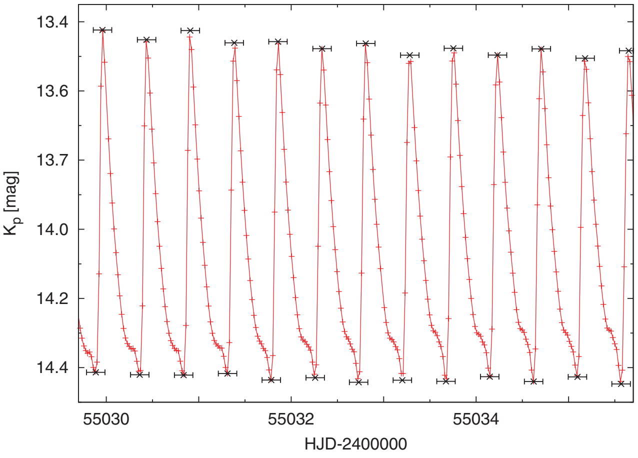

Figure 7. 6-d segment of the V355 Lyr light curve showing small PD effect, i.e. alternating maxima and minima.

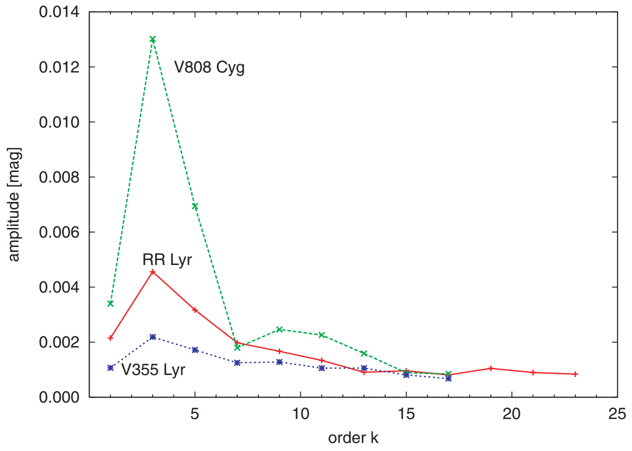

Figure 8. Amplitudes of the HIFs as a function of the order k, where k denotes the $k/2 \times f_0$ frequency. Note the significant bump seen around $k = 9$ as a sign of the 9:2 resonance with the ninth (strange) overtone for V808 Cyg and the change of slope for the other stars.

ascending branch of the Blazhko modulation. The difference in brightness between consecutive maxima reaches as high as 0.1 mag when PD is strongest. The visible alternating extrema can be seen where the amplitude of the HIFs is high, and no significant alternation is found where the amplitude is low. This is true for all our target stars throughout the whole light curve in each case. In RR Lyr, the amplitudes of the HIFs never vanish; in other words, the PD effect is always present. Fig. 9 demonstrates the difference of the strength of the HIFs showing the discrete Fourier transforms of the second and third marked segments of the RR Lyr light curve as shown in Fig. 3. We pre-whitened with the main pulsation frequency and its harmonics (but not with the Blazhko side peaks) for better visibility of the HIFs. It is discernible that the HIFs are present in both data segments, with a factor of 3 difference in the amplitudes. We can conclude that the PD effect is transient and varies with the Blazhko cycle.

In the case of V808 Cyg, the PD effect is practically seen throughout the whole 133-d observational period, albeit with outstanding maxima of the HIFs during the ascending branch of the Blazhko envelope, close to the Blazhko maximum, and during the descending branch. The minimum level of the HIFs is not as low as for RR Lyr, but the maximum height is similar. Again, there is a clear connection of the PD effect to the Blazhko modulation, very similar to the case

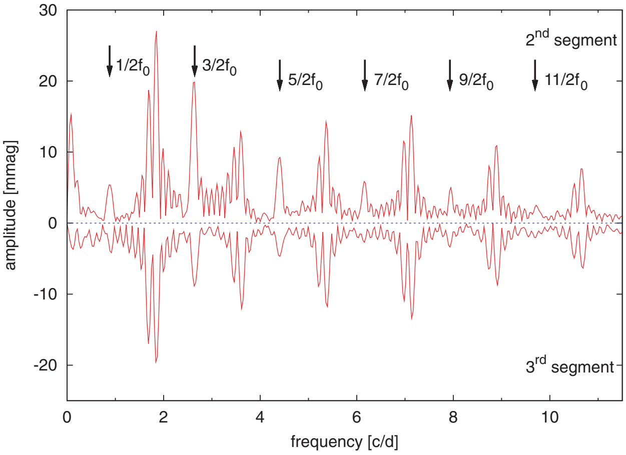

Figure 9. Fourier spectrum of the second (upper panel) and third (lower panel) segments of the *Kepler* light curve of RR Lyr as shown in Fig. 3 (between MJD 55027–55035 and 55068–55076, respectively). We pre-whitened with the main pulsational frequency (f_0) and its harmonics for clarity. The side-peak structure around f_0 and its harmonics are still visible as well as the variable strength of the HIFs.

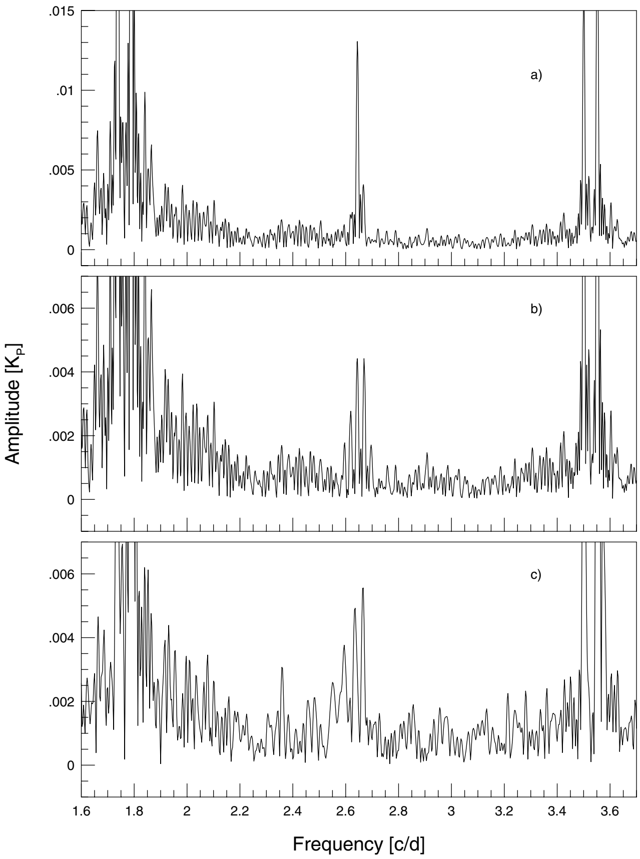

Figure 10. (a) Frequency spectrum between f_0 and $2f_0$ of a synthetic RR Lyr light curve showing the $3/2\,f_0$ frequency peak. The simulation was performed by keeping only f_0, its harmonics and the $k/2f_0$ frequencies ($k = 1, 3, 5, \ldots$), and the same modulation is applied for all these frequencies. (b) The same as in the upper panel, but the HIFs are switched on and off resulting in a bunch of additional frequencies. (c) The frequency spectrum of Q2 *Kepler* RR Lyr data plotted between f_0 and $2f_0$.

of RR Lyr, but with enhanced intensity and an additional maximum height during the descending branch of the Blazhko envelope.

One can see numerous peaks in the vicinity of the HIFs (Fig. 10c). In addition, we noted that the highest frequency peak has a frequency ratio to f_0 that is significantly different from $2/3$, namely 0.662. This is the combined effect of the Blazhko modulation and the temporal onset and disappearance of the HIFs. To test this hypothesis, we performed the following check.

We generated an artificial light curve sampled at the original data points. We took f_0 and its 16 harmonics of RR Lyr, their phases and amplitudes and modulated the amplitude and the phase with a Fourier sum of two and five terms, respectively. The resulting light curve is very similar to the observed one, but we note that the purpose of this simulation was to explain the frequency spectrum in the vicinity of the HIFs and not the reproduction of the light curve.

Then we added the $1/2\,f_0$, $3/2\,f_0$, $5/2\,f_0$, etc., frequency series with small amplitude. The same modulation is applied to these periodic signals, as well. We used $f_0 = 1.762\,989$ c/d in the simulation and applied MUFRAN for the frequency analysis (Kolláth 1990). It resulted in $f'_0 = 1.763\,059$ c/d. We pre-whitened the light curve with f'_0 and its 16 harmonics. The resulting spectrum between f_0 and $2f_0$ is shown in the upper panel of Fig. 10, where additional small peaks appear around $3/2\,f_0$.

The middle panel shows the result of a similar procedure, but here the PD (i.e. the HIFs) was switched on and off periodically. Specifically, the amplitudes of the HIFs were varied as $a = \sin(2\pi f_{\rm m} t + \phi)$ if $a > 0$ and $a = 0$ otherwise, i.e. HIFs are present only when the function is positive. $f_{\rm m}$ denotes the modulation frequency. One can find at least five distinct peaks in the synthetic spectrum; the highest two of them are of similar amplitude. The frequencies of these peaks are $f' = 2.643\,2455$ c/d and $f'' = 2.670\,1858$ c/d; their frequency ratios are $f'/f'_0 = 0.667$ and $f''/f'_0 = 0.660$, respectively.

Our simulation demonstrated convincingly that the large number of frequency peaks and the frequency ratio close to but not equal to $2/3$ are expected consequences of the modulated light curve, and we are dealing with a genuine PD effect.

Summarizing our findings concerning the PD effect in three *Kepler* Blazhko RRab stars, we conclude that this effect occurs in some but not all modulated RR Lyrae stars. For a given star its presence may be continuous, but there are specific phases of the Blazhko cycle, where it gets much stronger (up to five times in amplitude). This is not a strict rule, however. Out of three rising branches of the Blazhko modulation (i.e. envelope) in RR Lyr, we detected a strong presence of PD in the first two cases and much weaker appearance during the third one. The overall dominance of the PD may also vary; for RR Lyr and V808 Cyg it is relatively strong, while for V355 Lyr it is much weaker.

3.3 Detection limit in the *Kepler* sample

Upper limits for the averaged amplitudes of HIFs were established for all the *Kepler* RR Lyrae stars where we did not find PD effect. Frequency spectra were computed for the 14 Blazhko and 15 non-Blazhko *Kepler* RR Lyrae stars using SIGSPEC and pre-whitened successively with the highest frequency peaks. These were the dominant pulsational mode (fundamental in each case), its harmonics up to the Nyquist frequency and modulation side peaks in the case of Blazhko stars. The procedure was stopped when the amplitudes of the remaining peaks reached a spectral significance of 5.0. As the limit for the HIF amplitudes, we accept the amplitude corresponding to this spectral significance limit. We emphasize that

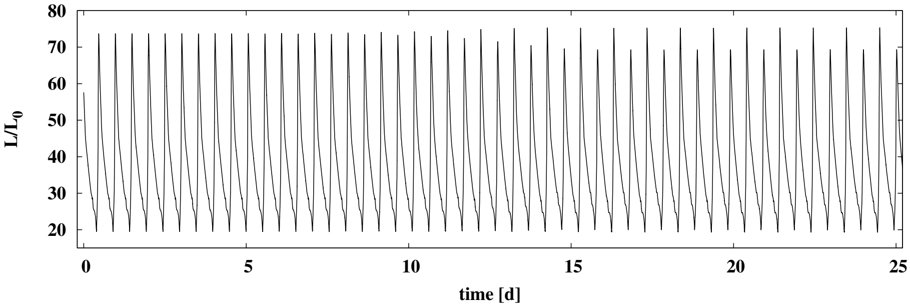

Figure 11. Results of a hydrorun showing the onset of the PD phenomenon.

this choice is conservative, because inspection of the location of the HIFs showed that there were no frequency peaks up to an amplitude two to three times lower than our adopted limit of 5.0.

Two factors affect our detection limit: one is the brightness of the star and the other the complexity of the frequency spectrum (see Benkő et al. 2010). We note in passing that the apparent magnitudes of the *Kepler* RR Lyrae sample are in the range of $Kp \simeq 11$–17, the notable exception being RR Lyr, which is much brighter. Another source of error may be the lack of barycentric correction to the (shorter) Q1 time series. While this correction is important, we argue that it has no effect on our detection limit. First, because it does not affect the alternating maxima and minima. Secondly, although it may cause some systematics in the frequency spectrum, the basic structure of the HIFs is well understood, as we demonstrated in the previous subsection.

As we mentioned in Section 3.1, we found four additional stars showing some HIFs besides RR Lyr, V808 Cyg and V355 Lyr. For the remaining Blazhko stars not showing the PD effect, we find that the upper limit for the HIF amplitudes is between 0.2 and 2.0 mmag. For RR Lyrae stars without modulation, we generally find a smaller upper limit. The detection limit for these objects is between 0.1 and 1. 0 mmag, with more stars lying closer to the 0.1-mmag borderline.

We have not found PD bifurcation in the ultra-high-precision light curve of any of the non-Blazhko stars, in spite of the fact that it would be much easier to discern alternating extrema, as well as HIFs in their frequency spectra. In conclusion, these findings strongly suggest that the PD bifurcation is related to the Blazhko effect.

4 HYDRODYNAMICAL SIMULATIONS

Hydrodynamical modelling has proved that low-order resonance plays an important role in the oscillations of classical pulsating stars. The Hertzsprung progression of bump Cepheids is traced to the $P_0/P_2 = 2$ resonance (e.g. Buchler, Moskalik & Kovács 1990). The sharp features in the Fourier coefficients of s-Cepheids are traced to the $P_1/P_4 = 2$ resonance, despite the very large damping of the fourth overtone (Feuchtinger, Buchler & Kolláth 2000). In the case of BL Her- and Cepheid-type pulsations, it was demonstrated by Moskalik & Buchler (1990) that the 3:2 resonance of the fundamental mode and the first overtone is responsible for PD bifurcation in hydrodynamical models.

However, only the low-order modes up to the fourth overtone were considered when resonances were discussed, and the remaining modes were assumed to have no influence on the asymptotic behaviour of the models since they are strongly damped.

In RR Lyrae stars, effects due to the above-mentioned resonances are not expected because of the different period ratios. However, in some of the RR Lyrae model sequences PD bifurcation was detected, indicating that there indeed exists some mechanism that is able to destabilize the fundamental mode pulsations. To pinpoint the mechanism behind the PD bifurcation, we have performed a systematic survey of RR Lyrae model sequences. The details of these calculations are presented elsewhere (Kolláth, Molnaár & Szabó, in preparation); here, we present only the results relevant to this paper. For our hydrodynamical calculations, we used our standard turbulent convective stellar pulsation hydrocode (Florida–Budapest code; see Kolláth et al. 2002, equations 1–13.) The main model parameters are $M = 0.578\,\mathrm{M}_{\odot}, L = 38.45\,\mathrm{L}_{\odot}, T_{\mathrm{eff}} = 6500\,\mathrm{K}$ and metallicity $Z = 0.0001$.

Standard integration of the models with PD behaviour evolves to the bifurcated solution, and thus the fundamental mode limit cycle cannot be calculated in this way. However, the relaxation method (see Kovács & Buchler 1993) makes it possible to iterate to the limit cycle solution and determine its stability properties. These calculations indicate that the fundamental mode limit cycle is unstable for a wide range of the model parameters. The large value of one of the Floquet exponents ($\lambda_k \approx 0.5$) indicates that perturbations to the limit cycle grow on a time-scale of a few periods. Time integration of the model initiated from the limit cycle solution clearly demonstrates the short time-scale of the transition from the limit cycle to period-2 solution. The luminosity variation during this transition is displayed in Fig. 11. The perturbation to the limit cycle was defined as a 1 per cent increase of eddy viscosity in the model. We have to note, however, that with no direct perturbation the model also evolves to the PD solution on a slightly longer time-scale due to the numerical noise of the computations.

The numerical integration of the model clearly demonstrates the PD bifurcation in our RR Lyrae model. However, it does not provide a direct clue to the destabilizing mechanism of the fundamental mode. The PD bifurcation can occur through the destabilization of either a thermal mode or a vibrational mode. The second case was thoroughly described by Moskalik & Buchler (1990), showing that a half-integer resonance provides the mechanism in PD bifurcation in hydrodynamical models. The Floquet coefficient that gives the instability of the limit cycle is real (the Floquet phase, $\phi_k = \pi$). It indicates that the coupling to a vibrational mode is in effect through a half-integer resonance, but it does not rule out the possibility of a thermal mode behind the PD behaviour. Resonance is not expected in the low-order modes; however, the linear stability analysis of the model sequences shows that some of the higher (eighth to tenth) overtones are unstable for some of the temperatures. This behaviour of the linear models indicates that a strange mode coexists with the normal vibrational modes, suggesting that a resonance with this strange mode can be responsible for the destabilization of the fundamental mode.

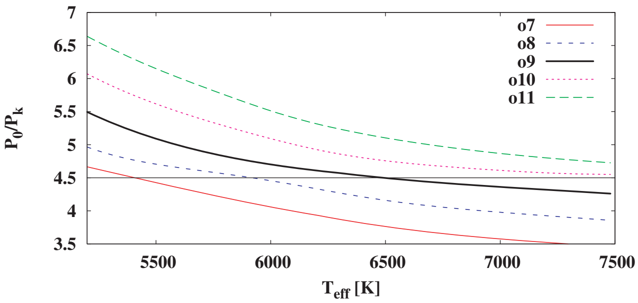

Figure 12. Linear period ratios of high-order radial overtones to the fundamental mode as a function of effective temperature. The 9:2 resonance is shown by a horizontal line.

In Fig. 12, the period ratios P_0/P_k are displayed for the high-order modes. The period ratio curves show signatures of avoided crossings proving the existence of a strange mode (Buchler & Kolláth 2001). Interestingly, it clearly shows a half-integer resonance with P_0:$P_k = 9$:2 in a wide temperature range, which was not expected to provide such a strong influence on fundamental mode pulsation. A thorough non-linear hydrodynamical survey of model sequences demonstrates that this resonance indeed provides the destabilization of the fundamental mode, and it causes the PD bifurcation. Details of these calculations are presented in a parallel paper (Kolláth et al., in preparation).

The PD behaviour strongly depends on the parameters of turbulent convection. Using e.g. the eddy viscosity as a control parameter, the bifurcation from limit cycle to PD is obtained. Similarly, the convective efficiency can play an important role as a control parameter. If one assumes that during the Blazhko cycle the turbulent/convective structure of the star varies (Stothers 2006), it can result in the repeated occurrence of PD similar to the observations.

5 DISCUSSION

The high precision and continuity of the *Kepler* space telescope have enabled us to discover a new phenomenon in Blazhko RR Lyrae stars, namely *PD*. Three stars were found to show this type of behaviour unambiguously. One of them is the brightest representative of its class RR Lyr (KIC 7198959) and the other two are much fainter modulated RRab variables, namely V808 Cyg (KIC 4484128) and V355 Lyr (KIC 7505345). In addition, four other Blazhko RR Lyrae stars in the *Kepler* field may show signs of this new type of instability.

PD manifests itself as alternating maxima and minima of pulsational cycles; sometimes even the shape of the light curve is alternating. As a consequence, the Fourier spectrum contains HIFs, i.e. frequency peaks mid-way between the harmonics of the main pulsational frequency.

Interestingly enough, the intensity of the PD effect is time-dependent. In RR Lyr it is most prominent during the ascending branch of the modulation in two Blazhko cycles, while it is practically missing during the third ascending branch. In V808 Cyg, the PD effect is seen throughout the whole 133-d long data set, albeit with outstanding maxima of the HIFs during the ascending branch close to the Blazhko maximum and during the descending branch. In V355 Lyr the PD effect is present, but it is rather weak; the maximum amplitude of the $3/2 f_0$ frequency is five times less than in the case of the other two targets. Similarly, the amplitude modulation is also rather small for V355 Lyr. This may hint at an intimate connection between the strength of the Blazhko modulation and the PD effect. Also, the fact that no PD effect was found in non-Blazhko *Kepler* RRab stars strongly suggests that this effect is connected to the Blazhko effect.

The structure of the HIFs in the spectrum is found to be rather complex. We successfully demonstrated that the bunch of appearing frequency peaks in the vicinity of the expected HIFs is due to the varying pulsational period throughout the modulation cycle, as well as the transient nature of the PD phenomenon.

Although deviations from regular single-periodic pulsation, i.e. cycle-to-cycle variations in RR Lyrae radial velocity curves (Chadid 2000) and irregularities in photometric observations (Jurcsik et al. 2008), had already been detected from the ground, one might ask why the PD effect was not discovered despite the fact that the difference of the subsequent maxima during the PD episode may well be observable from the ground with accurate CCD photometry. First, $3/2 f_0$ (the highest HIF in all three PD Blazhko stars) grows to 26 mmag in the Fourier spectrum of RR Lyr when it shows maximum power. It may be visible only in well-sampled (essentially continuous) light curves which are very hard to obtain. Before *Microvariability and Oscillations of STars (MOST), CoRoT* and *Kepler,* no continuous RR Lyrae light curve was available. The compact, dedicated single-site observations (Jurcsik et al. 2005, 2008) and the limited multisite campaigns that have been organized for RR Lyrae stars (Kolenberg et al. 2006, 2009) did not yield the required coverage and accuracy to be able to detect the PD phenomenon. Secondly, pulsation periods close to 0.5 d (typical for RR Lyrae stars) may hamper the detection, as from the ground one can follow only even or odd cycles every night and the duration of the maximum strength of the PD phenomenon as we see in *Kepler* data is not long (typically 8–10 d). Finally, PD itself is a transient phenomenon, not seen in every Blazhko cycle. We estimate that the maximum PD strength phase (i.e. possibly observable from the ground) lasts from 10 per cent (RR Lyr) to 22 per cent (V808 Cyg) of the currently available time-span covered by *Kepler* observations. This is where the continuity and longevity of *Kepler* observations have unbeatable advantage. In addition, strong PD occurs only in three stars out of 14 RR Lyrae exhibiting the Blazhko modulation, while four additional modulated RR Lyrae stars show much weaker evidence for PD-like behaviour. We conclude that it is not surprising that the transient PD effect remained unnoticed in decades-long ground-based RR Lyrae observations.

The PD bifurcation was reproduced successfully with the Florida–Budapest hydrocode which accounts for turbulent convection. We emphasize that not only the PD effect occurs naturally in our hydrodynamical models, but the time-scale of the onset and fade-out of the PD effect are excellently reproduced, as well.

Our models of RR Lyrae stars demonstrated that PD is possible in these stars due to a 9:2 resonance of the fundamental and a high-order (ninth overtone) mode. It was not expected that such a high-order mode plays an important role in fundamental mode pulsations. However, it was found that this interacting mode is a strange mode, with a non-normal damping rate and eigenfunction. Normal high-order pulsation modes, with this extreme period ratio (P_0:$P_k = 9$:2), are not able to destabilize the fundamental mode limit cycle and to induce a PD bifurcation. Thus, the observed PD characteristic in the *Kepler* RR Lyrae stars provides a strong indirect evidence for the existence of strange modes in radial stellar pulsation, a phenomenon predicted theoretically by Buchler, Yecko & Kolláth (1997). The significant interaction of the strange mode to the fundamental mode pulsation also suggests that strange modes can play an important role in other phenomena, like three-mode resonances (e.g. among the fundamental, first/second and the strange mode), and perhaps it has an effect in shaping the Blazhko effect as well. In addition, non-radial modes may also be involved in this complex dynamical interplay through resonant or non-resonant interactions, as demonstrated by recent *Kepler* findings in Benkő et al. (2010).

We note that we found the strongest phase (or equivalently period) modulation in the cases of RR Lyr and V808 Cyg among *Kepler* Blazhko stars (Benkő et al. 2010), and these stars show the strongest PD effect. If we assume that during the Blazhko cycle the turbulent/convective structure of the star varies as suggested by Stothers (2006), it seems natural that in certain Blazhko phases (i.e. when the physical conditions are favourable) the PD effect appears, because in our models the PD behaviour strongly depends on the parameters of turbulent convection. This sensitivity, together with the narrow parameter range where the necessary (P_0:$P_k = 9$:2) resonance is at work, makes this new-found phenomenon a precious tool for studying the mysterious Blazhko effect. The understanding of the differences between PD and non-PD Blazhko RR Lyrae stars, as well as the strong and feeble PD phases of a given star, may provide the long-sought insight into the Blazhko mechanism, offering a sensitive way to constrain our models.

With the release of additional *Kepler* data, it will be possible to further study the PD behaviour and learn more about its temporal and transient nature.

ACKNOWLEDGMENTS

Funding for this Discovery mission is provided by NASA's Science Mission Directorate. This project has been supported by the National Office for Research and Technology through the Hungarian Space Office Grant No. URK09350 and the 'Lendület' programme of the Hungarian Academy of Sciences. KK acknowledges the support of Austrian FWF projects T359 and P19962. The authors gratefully acknowledge the entire *Kepler* team, whose outstanding efforts have made these results possible.

REFERENCES

Benkő J. M. et al., 2010, MNRAS, in press
Borucki W. J. et al., 2010, Sci, 327, 977
Bryson S. T. et al., 2010, ApJ, 713, L97
Buchler J. R., Kolláth Z., 2001, ApJ, 555, 961
Buchler J. R., Kovács G., 1987, ApJ, 320, L57
Buchler J. R., Moskalik P., 1992, ApJ, 391, 736
Buchler J. R., Moskalik P., Kovács G., 1990, ApJ, 351, 617
Buchler J. R., Yecko P., Kolláth Z., 1997, A&A, 326, 669
Chadid M., 2000, A&A, 359, 991
Feuchtinger M., Buchler J. R., Kolláth Z., 2000, ApJ, 544, 1056
Gilliland R. L. et al., 2010a, PASP, 122, 131
Gilliland R. L. et al., 2010b, ApJ, 713, L160
Haas M. R. et al., 2010, ApJ, 713, L115
Jenkins J. M. et al., 2010, ApJ, 713, L120
Jurcsik J. et al., 2005, A&A, 430, 1049
Jurcsik J. et al., 2008, MNRAS, 391, 164
Kolenberg K. et al., 2006, A&A, 459, 577
Kolenberg K. et al., 2009, MNRAS, 396, 263
Kolenberg K. et al., 2010a, ApJ, 713, L198
Kolenberg K. et al., 2010b, MNRAS, submitted
Kolláth Z., 1990, Occasional Tech. Notes, Konkoly Obser. No. 1
Kolláth Z., Buchler J. R., Szabó R., Csubry Z., 2002, A&A, 385, 932
Kovács G., Buchler J. R., 1988, ApJ, 334, 971
Kovács G., Buchler J. R., 1993, ApJ, 404, 765
Lenz P., Breger M., 2005, Comm. Asteroseismol., 146, 53
Moskalik P., Buchler J. R., 1990, ApJ, 355, 590
Moskalik P., Buchler J. R., 1991, ApJ, 366, 300
Reegen P., 2007, A&A, 467, 1353
Stothers R. B., 2006, ApJ, 652, 643

This paper has been typeset from a TEX/LATEX file prepared by the author.