DEPARTMENT OF ECONOMICS

Household and firm leverage, capital flows and monetary policy in a small open economy

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Household and firm leverage, capital flows and monetary policy in a small open economy

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Abstract

This paper presents a framework to analyze the interplay between financial frictions at the household and firm level, liability dollarization and monetary policy in a small open economy subject to productivity and capital inflow shocks. Optimized monetary policy rules are calculated under several specifications (inflation targeting, exchange rate targeting, fixed exchange rate, credit growth targeting) and for two central bank’s objectives (macroeconomic stability and macroeconomic plus financial stability). I find that, first, adding financial stability to the central bank’s objectives results in more inertial monetary policy rules. Second, the optimized Taylor rules under the financial stability objective achieve a lower volatility of inflation and of credit growth at the same time. However, this comes at the expense of a higher standard deviation of production. Third, when financial stability is included among the central bank’s objectives, engaging in exchange rate smoothing delivers the smallest value of the central bank’s loss function, mainly arising through a much reduced volatility of the credit aggregate. In the considered economy, credit growth targeting is suboptimal because of the effect of stronger interest rate increase on currency fluctuations, which reinforce the financial accelerator. Finally, for the considered shocks, the extent of co-movement of financial variables pertaining to entrepreneurs and homeowners crucially depends on the degree of exchange rate flexibility.

JEL classification:

Keywords: optimal monetary policy, financial frictions, liability dollarization, financial stability.

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1 Introduction

The recent financial crisis demonstrated that stable economic growth and an environment of low inflation are not sufficient to guarantee financial stability. Imbalances built up in good times can deepen the magnitude of slowdowns and amplify the effect of adverse shocks. This was particularly evident in the Emerging European countries where a remarkable economic performance since the mid-1990s was dramatically reversed by the global credit crunch and exacerbated by existing imbalances. In the years preceding the financial crisis, Emerging Europe saw a prolonged period of steady economic growth, driven by a demand boom (increased consumption and investment) accompanied by high credit growth, fostered to a large extent by the massive waves of foreign capital flowing in the region. As shown in Figure 1, the first years of the new millennium were characterized by large capital inflows, particularly in the form of foreign direct investment and other inflows (mainly bank loans), which intensified with prospects of EU membership of the central and eastern European countries. In the Baltic countries and Romania, capital inflows fostered credit growth in both the household and the corporate sector (Figure 2).

Figure 1: Capital Inflows to Central and Eastern Europe (average)

Note: The considered countries are Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia and Slovenia.
Source: Pirovano et al. (2011)

The availability of cheaper mortgage loans and the consequent increased demand for real estate inflated housing prices, with a positive effect on consumption through wealth effects. At the same time, investment in the corporate sector increased to several times its level at the beginning of the decade, fueling asset prices growth. On the other hand, in countries like Bulgaria credit to households in the boom years was small relative to corporate credit and
hence GDP growth was mainly driven by productive investment (Figure 3). Independent of the sectorial destination, capital flows in the region led to credit growth and to an overall increase in leverage in the economy. Economic overheating manifested through rising inflation, inflating asset and real estate prices and loss of competitiveness due to exchange rate appreciations. On the external side, widening current account deficits were accompanied by increasingly negative international investment positions. Although empirical evidence (Kiss et al. (2006)) reveals that credit growth in the new EU member states was largely justified by economic fundamentals and was a natural result of the deepening of the financial sector, credit dynamics in Emerging Europe posed a challenge to macro-financial stability on three grounds. First, the increased dependence on foreign financing exposed countries to risks of contagion from external developments. Secondly, rapid growth in bank credit and asset prices significantly contributed to financial fragility, by increasing leverage and hampering the resilience of the economy during downturns. Third, liability dollarization and resulting currency mismatches exacerbated existing imbalances. In fact, in Eastern European countries, a large share of credit to the private sector was denominated in foreign currency.

Figure 2: Credit Growth in Central and Eastern Europe in boom years (2003-2008

![Credit Growth in Central and Eastern Europe in boom years (2003-2008)](image)

Source: Bakker and Gulde (2010)

Sirtaine and Skamnelos (2007) document that, in 2005, foreign currency lending as a share of total lending amounted to more than 70% in Estonia and Latvia, while Lithuania, Hungary, Poland, Slovenia and Romania settled on values ranging from 50% to 70%. By 2008, foreign currency loans reached values greater than 80% of the total in Estonia and Latvia and doubled in Bulgaria, exceeding 60% of total loans (Rancière et al. 2010). This
resulted in an increased exposure of borrowers to currency mismatches, with assets denominated in domestic currency and liabilities in foreign currency, posing a serious threat to macroeconomic and financial stability in case of a currency devaluation. When the financial crisis hit and foreign capital inflows dried up, Emerging Europe was dragged into the spiral and suffered large losses in terms of GDP growth. However, heterogeneities in economic performance are noticeable. Specifically, countries which experienced the strongest credit boom before the crisis and large macroeconomic and financial imbalances (notably, the Baltic countries) faced the largest contraction in GDP growth (Bakker and Gulde (2010)).

Three main lessons can be drawn from the recent financial crisis. First, imbalances built-up in good times magnify the intensity of a slowdown. Second, financial instability can arise even in an environment of stable inflation and overall macroeconomic stability. Third, developments in the real estate market can be important impulses to business fluctuations, and were of utmost importance in aggravating the crisis. From a monetary policy perspective, this evidence led, on one side, to reexamine the appropriateness of the traditional objectives of monetary policy, i.e. inflation and output stability, on the grounds that they might not be necessarily conducive of financial stability. On the other hand, it spurred a debate on the implementation of monetary policy, reconsidering the effectiveness of inflation targeting regimes whereby central banks set the policy rate reacting to inflation and a measure of economic activity. Hence, the dispute on monetary policy conduct in the aftermath of the crisis evolves around two main questions. Should central banks be concerned about financial, in addition to macroeconomic, stability? And, if so, should central banks react to indicators of financial overheating when setting the monetary policy rate? From a macroeconomic research perspective, the recognition of the importance of developments in the housing market in shaping recent economic events led to a growing number of studies incorporating credit frictions in mortgage credit in mainstream macroeconomic models. However, these studies largely focus on developed economies, and scarce consideration is given to characteristics,

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1In a recent study, Mitra (2011) argues that sectoral destination of capital flows is an important determinant of the impact of the crisis. She observes that countries that suffered the largest swings in GDP growth coincide with those where capital inflows were heavily channeled to real estate and to the household sector. The heaviest impact of the crisis on these countries might be related to two aspects. On one side, to the credit crunch that severely limited the credit supply that fostered consumption and hence GDP growth. In comparison, countries where credit was channeled to the production sector could rely on higher productivity brought about by newly installed capital, better competitiveness and thus a better export performance, which limited the extent of the downturn. On the other hand, the three Baltic Republics committed to a fixed exchange rate regime, thereby constraining the room for manoeuvre of monetary policy authorities and renouncing to the benefits of a currency depreciation that would have followed the sudden stop in capital flows.

such as financial dollarization and susceptibility to capital inflows, typical to emerging markets. The interplay between credit frictions and liability dollarization has been the subject of a large number of studies, which tackled the question in a small open economy DSGE framework. These papers particularly highlight the balance sheet effect of exchange rate fluctuations and examine the issue as to whether a fixed exchange rate regime might be more desirable than a flexible currency in isolating small open dollarized economies against foreign shocks. While these studies are insightful in providing evidence in support of the superiority of a flexible currency even in the presence of liability dollarization, they abstract from three issues, which are of particular relevance in light of the experience of emerging European economies in the run-up to the crisis. First, they do not consider the housing market and the interplay with financial frictions at the household and firm level. Second, they discard potential financial stability objectives of the small open economy’s central bank. Their conclusions concerning the superiority of flexible exchange rate regimes relies on the volatility of output and inflation, without consideration of the volatility of financial variables. Third, they model capital inflow shocks as an exogenous increase in the foreign interest rate or in the country’s risk premium. However, the experience of many emerging economies revealed that capital flows are largely influenced by waves of optimism and pessimism of international investors, often unrelated to country risk premia or interest rate differentials.

This paper presents a framework to analyze the interplay between financial frictions at the household and firm level, liability dollarization and monetary policy in a small open economy DSGE model reflecting the aforementioned characteristics of emerging European economies. In particular, I focus on the interaction of firm and household leverage in the transmission of shocks to domestic technology and capital inflows, under optimized monetary policy rules. In particular, I consider three specifications of the monetary policy rule that have been widely considered in the literature for emerging economies (i.e. inflation targeting, exchange rate targeting and fixed exchange rate) and a Taylor rule reacting to credit growth. Furthermore, I analyze the optimized monetary policy rules under two specifications of the central banks’ objectives, namely macroeconomic stability and macroeconomic plus financial stability.

I find that, adding financial stability to the central bank’s objectives results in more inertial monetary policy rules. Furthermore, compared to the case where the central bank

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4Capital inflows and their implications for monetary policy are considered in studies by Curdia (2006 and 2007), Ozkan and Unsal (2012) and Unsal (2013). In both cases, capital inflows (or sudden stops in Ozkan and Unsal (2012)) are considered in the financing of capital investment and intermediate goods purchases only. Furthermore, the monetary policy implications are studied assuming the central bank has macroeconomic stability as its sole objective and credit to households is not considered.
only cares about macroeconomic stability, the optimized Taylor rules under the financial stability objective achieve simultaneously a lower volatility of inflation and of credit growth. However, this comes at the expense of a higher standard deviation of production. Finally, regardless of the central bank’s objective, a fixed exchange rate delivers the highest value of the loss function. In particular, while it implies a much reduced variability of inflation (mainly through the effect of exchange rate movements on the price of imported goods), output and credit are much more volatile. Concerning the optimized rules more specifically, when macroeconomic stability is the only central bank’s concern, my findings largely mirror those of the literature, suggesting that the monetary policy should not engage in either exchange rate nor credit growth targeting. On the contrary, when financial stability is included among the central bank’s objectives, engaging in exchange rate smoothing delivers the smallest value of the central bank’s loss function, mainly arising through a much reduced volatility of the credit aggregate. In the considered economy, reacting to the credit aggregate is suboptimal because of the effect of stronger interest rate increase on currency fluctuations, which reinforces, instead of dampening, the financial accelerator. Finally, in the case of both technology and capital inflow shocks, the extent of co-movement of financial variables pertaining to entrepreneurs and homeowners crucially depends on whether the exchange rate is flexible or pegged. Specifically, under a fixed exchange rate regime, a negative correlation arises, i.e. stronger balance sheet conditions of entrepreneurs lead to weakened or virtually unchanged balance sheet conditions for homeowners. Under a fixed exchange rate regime, a positive correlation of financial variables of the two types of borrowers arises, mainly operating through the balance sheet effect of exchange rate fluctuations. In case of capital inflow shocks, similar conclusions can be drawn concerning the interaction between the monetary policy regime and the dynamics of financial variables across sectors. Furthermore, the analysis reveals that sectorial capital inflow shocks spill over to the other sector mainly through their effect on domestic production through increased demand of domestic goods used for investment purposes, and through balance sheet effects of currency appreciation.

2 Literature Review

2.1 Monetary policy and financial stability

Contrary to previous episodes, the undisputed protagonist of the recent global crisis were the financial sector and its underlying instability. One of the most important lessons recent events have taught academics and policymakers is that financial imbalances can arise even in an environment of low inflation and overall macroeconomic stability. In particular, it has
been argued that asset price bubbles and excessive credit growth might occur as a consequence of stable inflation expectations and optimistic prospects about the future economic outlook, which encourage risk taking and financial fragility. The crisis led, on one side, to reexamine the appropriateness of the traditional objectives of monetary policy, i.e. inflation and output stability, on the grounds that they might not be necessarily conducive of financial stability. On the other hand, it spurred a debate on the implementation of monetary policy, reconsidering the effectiveness of inflation targeting regimes whereby central banks set the policy rate reacting to inflation and a measure of economic activity. Hence, the dispute on monetary policy conduct in the aftermath of the crisis evolves around two main questions. Should central banks be concerned about financial, in addition to macroeconomic, stability? And, if so, should central banks react to financial variables when setting the monetary policy rate?

On one hand, proponents of the inflation targeting regime argue that, to the extent that asset price inflation and credit growth lead to an expansion of aggregate demand through their effect on wealth and spending, a monetary policy reacting to inflation and output will automatically counteract financial imbalances. The main analytical contribution to this view is offered by Bernanke and Gertler (2001) who use a small scale macroeconomic model to show both theoretically and empirically that, to the extent that central banks react to inflation in the pursuit of price stability, a reaction to changes in asset prices is warranted only as far as they contain useful information about inflationary or deflationary pressures.\(^5\)

Of a similar opinion are Bullard and Schaling (2002), who consider a macroeconomic model where the central bank adds asset prices as a target in an otherwise traditional Taylor rule. Their analysis suggests that asset price targeting can interfere with the minimization of inflation and output volatility, leading to suboptimal levels of inflation and the output gap. Faia and Monacelli (2003) focus on the welfare implications of asset prices targeting and dismiss such a monetary policy strategy on the grounds that it is not conducive to higher welfare. The view against asset prices targeting has been widely embraced by central bankers in the United States\(^6\): in a recent speech, the Philadelphia Fed President Charles Plosser asserted that "Financial stability should not be an explicit objective of monetary policy per se" and that, in his opinion, "we need to resist the temptation of adding the financial stability goal to the burdens of monetary policy".\(^7\)

On the other hand, proponents of a "leaning against the wind" approach of central banks

\(^5\) A similar conclusion is reached, a few years later, by Distayat (2005). In his model, the nonfundamental component of real asset returns appears in the central bank’s optimal monetary policy because it helps to predict future output and inflation dynamics.


\(^7\) Speech held in occasion of the American Economic Association Annual Meeting, on January 4th 2013.
to inflating asset prices and credit growth argue that output and inflation stability might not be sufficient to induce stable growth in asset prices and credit, with destabilizing consequences on the financial sector and, ultimately, on the real economy. Among the early advocates of this view are Cecchetti et al. (2000), Borio and Lowe (2002 and 2004), Bordo and Jeanne (2002) and White (2006). Their argument rests on the claim that setting monetary policy only considering developments in inflation and the output gap might be a too narrow approach, and that better results in terms of stabilization could be achieved by explicitly targeting unsustainable increases in asset prices and excessive credit growth, even at the cost of increased variability in inflation and output. In particular, it is argued that ensuring a stable path of credit growth is conducive of both financial stability (through reduced swings in asset prices and sustainable leverage dynamics) and macroeconomic stability (hampering excessive fluctuations in consumption and investment). More recently, Curdia and Woodford (2010), use a new Keynesian DSGE model with credit frictions and financial intermediaries to conclude that it is optimal to include a spread adjustment term in the Taylor rule. Woodford (2012) strongly encourages central banks to acknowledge the influence of monetary policy on financial stability, and he argues that the monetary policy trade-off between inflation and financial stability is very similar to that between inflation and output stabilization. In the same way as central banks strike a balance between price stability and output gap stabilization engaging in a so-called "flexible inflation targeting regime", they may very well be able to find a short-run path for the economy balancing inflation stability against output gap and financial stabilization. The optimal target criterion resulting from his model reveals that it is appropriate to set the monetary policy instrument to “lean against” a credit boom, even if this requires a temporary sacrifice in terms of inflation and output gap. The validity of a central bank’s financial stability objective from a welfare standpoint has been emphasized by Angeloni and Faia (2013). By making a quantitative comparison of welfare under different central bank’s objectives, they conclude for the appropriateness of financial stability being included as one of such objectives. Finally, contrary to their American counterparts, European central bankers seem more keen to consider financial stability concerns in their monetary policy conduct and to amend the implementation of monetary policy to include financial variables. In a recent speech at the Czech National Bank, Yves Mersch, a member of the ECB’s Executive Board, stated that, although the primary objective of the ECB is to maintain price stability, "it may be desirable to incorporate in the decision-making process of monetary policy certain financial variables, which, over the medium to longer term, may

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8 However, Woodford underlines how his analysis does not imply that financial stability should be the primary responsibility of monetary policy. As his analysis reveals the presence of trade-offs between financial stability and traditional monetary policy objectives, the development of additional tools (i.e. macroprudential policy) is of utmost importance.
influence inflationary developments (e.g. excessive credit growth, asset bubbles etc.).

How is this debate relevant to emerging economies? Middle income countries can be ascribed characteristics that distinguish them from mature, high income countries. First of all, financial markets are less developed, in particular capital markets. As equity issuance remains limited, firms rely more heavily on bank credit in order to finance their investment projects. Second, emerging economies, and among them particularly Eastern European countries, are heavily financially integrated, which makes them vulnerable to external developments and to international financial cycles. Finally, middle income countries have endured several crises in the past decades, mainly associated with swings in capital inflows, and preceded by the accumulation of large imbalances (currency mismatches, excessive credit growth, asset price inflation). Hence, in this context, it might be worth for monetary authorities to adapt their monetary policy strategy in order to prevent the build up of imbalances, with potentially large effects on welfare. In addition to the pros and cons to financial stability concerns in monetary policy making put forward by the literature, some considerations are particularly relevant for emerging economies (Agénor and Pereira da Silva (2011)). On one side, monetary policy might have undesirable side effects in the event of capital inflows which are accompanied by exchange rate appreciations, credit growth and inflationary pressures. In particular, a tightening of monetary policy which increases the interest rate differential of the small open economy might reinforce the capital inflow. In this circumstance, counter-

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9Speech by Yves Mersch, member of the Executive Board of the ECB in occasion of the seminar: “Financial Stability Policies in a Post-Crisis World” held at the Czech National Bank, 4 March 2013.

10Data on stock market capitalization reveal that, in 2007, the size of Eastern European stock markets was much inferior to the average in Euro Area countries. While Estonia, Lithuania, Romania and the Slovak Republic settled on values below 30% of GDP, the average of Bulgaria, the Czech Republic and Poland amounted to 50% of GDP. Only in Slovenia, stock market capitalization reached a value as high as 60% of GDP. In comparison, the average of Euro Area countries exceeded 80% of GDP (Pirovano et al. (2011)).

11As reported by Lane and Milesi-Ferretti (2007), the degree of international financial integration measured through the capital account of the Central and Eastern European economies has doubled since the beginning of the transition process. The sum of external assets and liabilities as a percentage of GDP was on average 80% in 1994 and rose to 160% by 2004.

12The arguments opposing the inclusion of financial stability considerations in monetary policy’s objectives and implementation relate mainly to the adequacy of the short-term interest rate in dealing with financial imbalances. It is argued, first, that multiple objectives call for multiple instruments (the Tinbergen principle) and, second, that macroeconomic and financial stability necessarily imply a trade-off for the central bank. Third, reacting to financial variables might induce too large swings in the monetary policy rate exerting a destabilizing effect on the real economy. The case for a more proactive role of monetary policy in pursuit of financial stability objectives rests on the following arguments. First, if monetary policy had not been so accommodative before the crisis and took leverage and credit developments into account, it could have mitigated the impact of the crisis. Second, monetary policy might be very effective in deflating credit-financed bubbles. Third, macroprudential policy as implemented before the crisis did not prove to be very successful. For a thorough review of the arguments in favor and against financial consideration in monetary policy conduct, see Kohn (2006) and Agénor and Pereira da Silva (2011).
cyclical policies such as capital controls might be a more suitable alternative. On the other hand, relying too heavily on macroprudential regulation by limiting credit availability and increasing borrowing costs might encourage the shadow banking sector, making it even more difficult to maintain financial stability. Furthermore, as confirmed by the recent experience, in the run-up to a financial crisis both macroeconomic and financial instability increase, justifying a preemptive intervention of the central bank in normal times to offset the growing financial vulnerability.

2.2 Credit frictions, housing and capital flows in DSGE models

In recent years, a great research effort has been devoted to introducing an explicit role for asset prices and leverage in shaping credit developments in general equilibrium macroeconomic models. In the literature, financial market imperfections have been introduced either in the form of collateral constraints (Kiyotaki and Moore (1997)) or assuming an asymmetric information problem between borrowers and lenders, leading to the existence of a premium on external finance (Carlstrom and Fuerst (1997), Bernanke et al. (1999)). The difference between the two approaches lies in the channel through which imperfections in financial markets impact on credit supply. In the Kiyotaki and Moore’s model, credit is tied to the value of collateral: hence, credit is constrained by quantity restrictions. On the contrary, Bernanke and Gertler’s approach hinges on asymmetric information problems between borrowers and lenders, leading to a higher price of credit relative to the case where credit markets are perfect. Furthermore, as the cost of external finance positively depends on leverage, the model allows for a nontrivial role of net worth in the propagation of disturbances. Since these seminal contributions, a vast literature has flourished analyzing the role of financial frictions in originating and propagating macroeconomic fluctuations.

In the same years, the recognition of the importance of housing as a source of collateral, and of the negative implications of housing price bubbles for economic and financial stability has spurred the attention of many macroeconomic researchers. Therefore, a growing number of studies have begun to appear, modeling collateral constraints in housing investment, following the financial accelerator framework of Kiyotaki and Moore (1997). The study

13 Brzoza-Brezina et al. (2011b) compare the properties of the two approaches in a consistent way, and find that the business cycle properties of the external finance premium framework are more in line with empirical evidence.

14 Estimated DSGE models with financial frictions confirm the importance of the latter in better capturing the dynamics observed in the data, both in developed (Christensen and Dib (2008), Christiano et al. (2010)) and emerging economies (Elekdag et al. (2005), Saxegaard et al. (2010), Gelain and Kulikov (2011)).

15 I refer to Iacoviello (2010) for a review of the stylized facts on the housing sector, and its relevance for macroeconomic fluctuations.

16 A notable seminal contribution on housing in macroeconomic models is by Davis and Heathcote (2003).
that paved the way for research in the field is by Iacoviello (2005), who reproduces the positive correlation between spending and house price shocks introducing collateral constraints in real estate and capital investment. Furthermore, he finds that, allowing the central bank to respond to asset prices does not improve its performance in stabilizing output and inflation. Christensen et al. (2007) extend the Iacoviello framework to an open economy setting where collateral constraints apply to both real estate and capital investment financing and estimate the model with Canadian data. Their analysis provides an empirical validation to the model’s ability to reproduce the positive response of consumption to increases in house prices. However, comparing the model with one without collateral constraints, they do not find large differences. The empirical relevance of housing is further confirmed by Pariès and Notarpietro (2008) in a two country model with collateral constraints on housing investment estimated with U.S. and Euro Area data. Furthermore, their analysis of optimal monetary policy in this setting reveals a welfare gain in allowing the central bank to respond to house price movements. The role of developments in the housing market as a driving force of business cycle fluctuations is explored by Iacoviello and Neri (2010) in a model estimated for the U.S. economy, asserting the fit of the model in explaining trends in U.S. housing prices and investment. Furthermore, they find that collateral constraints on household borrowing amplify the response of non-housing consumption. Kannan et al. (2009) reexamine the interplay between house price booms and monetary policy. In particular, they find that a stronger monetary policy reaction to signs of overheating in the economy (i.e. credit growth and house price booms) is successful in dampening the acceleration effect of collateral constraints. Furthermore, they examine the appropriateness of a specific macroprudential policy designed to offset credit developments, and conclude for its usefulness. While these studies largely focused on developed economies, particularly the United States, fewer studies looked at housing developments in emerging economies. Two exceptions are constituted by Brzoza-Brezina and Makarski (2011a) and Ajevskis and Vitola (2011). In the former, a small open economy DSGE model with collateral constraints on housing and capital financing and a banking sector is estimated for Poland, in order to gauge the role played by the banking sector in generating the slowdown during the financial crisis. The second study estimates a small open economy with Latvian data to examine the role of financial intermediation in the business cycle, and the implications for monetary and macroprudential policies.

The authors construct a growth model with a production and a construction sector and succeed at reproducing important features of U.S. data (namely the positive correlation between consumption, GDP and investment in the two sectors, and the larger volatility of investment in the real estate, compared to capital, sector).

17 However, note that, in his model, firms are constrained by the value of their real estate holdings, not by that of their assets (capital stock).
Only few studies in the literature adopt the asymmetric information framework in introducing financial frictions to mortgage credit (Aoki et al. (2004), Solomon (2010) and Forlati and Lambertini (2011)). Aoki et al. (2004) focus on the implications of credit frictions at the household level for the transmission of monetary shocks. They conclude that the presence of asymmetric information in the credit contract between financial intermediaries and households financing housing purchases amplifies the transmission of changes in the interest rate to housing investment, house prices and consumption. The objective of Forlati and Lambertini’s (2011) study is to examine the impact of shocks to mortgage default rates on the macroeconomy, and to evaluate different parametrizations of the central bank’s policy rule. They conclude for the superiority of low-inertial rules in stabilizing the economy after an exogenous increase in mortgage defaults. In particular, as inertial rules imply smoother reductions in the nominal interest rate, they imply larger output contractions. Finally, Solomon (2010) examines the interaction between consumer debt and firm debt over the business cycle, focussing on the quantitative importance of feedback effects between the debt levels in the two sectors. His model abstracts from rigidities in price and wage setting, and from monetary policy considerations. His estimation of the model with U.S. data reveals that, while credit frictions at the firm level significantly amplify the response of investment to shocks, they do not amplify output responses. Furthermore, tighter borrowing conditions for households contribute to ease those for firms, leading to a negative co-movement of financial variables across sectors. However, all three studies are set in a closed economy context, and their conclusions are of limited relevance to small, open, dollarized economies. In particular, exchange rate fluctuations have nontrivial consequences for domestic production and prices, thereby influencing consumption and housing demand. Furthermore, balance sheet effects of currency movements at both the household and firm level considerably enrich the dynamics, possibly reverting the monetary policy implications drawn for developed countries. Specifically, Forlati and Lambertini’s (2011) prescription for non-inertial rules might be reverted in an open economy context, as large interest rate responses imply exchange rate fluctuations that have large repercussions on trade and balance sheets.

While the collateral constraint approach of Kiyotaki and Moore has been mainly applied to characterize credit market imperfections in real estate mortgages, the asymmetric information framework by Bernanke, Gertler and Gilchrist (1999) was mainly used in the context of firms’ capital financing. In particular, DSGE models of this kind have been largely used to explore the interplay between financial frictions and liability dollarization, together with their monetary policy consequences, in small open emerging economies. A particular attention had been devoted to the analysis of alternative exchange rate regimes, and their performance in stabilizing the economy following foreign shocks. In particular, the main argument for
exchange rate stabilization in this context relies on the fact that, when debt is denominated in foreign currency, exchange rate fluctuations affect the economy not only through trade, but also through balance sheet effects on borrowers. In this context, an exchange rate appreciation that, on one side, reduces exports with negative effects on aggregate demand, relaxes credit conditions of indebted agents, thereby stimulating further borrowing. Studies in this field largely focus on the impact of negative foreign shocks on the small open dollarized economy. Cespedes, Chiang and Velasco (2004) explore the stabilization properties of fixed and flexible exchange rate regimes in a dynamic general equilibrium model of a small open economy characterized by a financial accelerator and liability dollarization, concluding that, although balance sheet effects magnify the effect of external disturbances, a flexible exchange rate is still successful in insulating the economy from external shocks. The superior stabilization properties of flexible exchange rates are confirmed by Devereux et al. (2006), which subject their small open economy to foreign interest rate and terms of trade shocks. However, their conclusion crucially hinges on the degree of exchange rate pass-through. With high pass-through, stabilizing the exchange rate implies a high trade-off between output and inflation volatility, since it requires a stronger interest rate response; when pass-through is low, exchange rate movements do not have a strong destabilizing effect on the price level and it is better for the central bank to focus on stabilizing inflation, while allowing for the currency to float. In a similar framework, Gertler et al. (2007) explore the issue of whether the exchange rate regime influences a country’s response to a financial crisis, defined as an exogenous increase in the country’s risk premium. They find that while the financial accelerator amplifies the effect of the shock, it does not alter the ranking between fixed and flexible exchange rate regimes: in particular, they find that the effect of the financial accelerator is more muted with a floating currency. Concerning liability dollarization, they conclude that, although it lowers the attractiveness of a flexible exchange rate, this still leads to a smaller output drop. While these papers treat the foreign economy as exogenous, Batini et al. (2007) study the monetary policy implications of increased degrees of financial frictions and dollarization in a small open economy obtained as the limit case of a two-country DSGE model and characterize the optimal monetary policy in this setting. They conclude that the financial accelerator has a larger impact on the performance of monetary policy rules than the presence of liability dollarization: in particular, targeting the exchange rate is not optimal, as exchange rate movements attenuate the effect of financial frictions.

These studies, however, abstract from three important issues. First, they do not consider the housing market and the interplay with financial frictions at the household and firm level. Second, they discard potential financial stability objectives of the small open econ-

\footnote{Krugman (1999), Aghion, Bacchetta, and Banerjee (2001).}
omy’s central bank. Their conclusions concerning the superiority of flexible exchange rate regimes relies on the volatility of output and inflation, without consideration of the volatility of financial variables. Third, they model capital inflow shocks as an exogenous increase in the foreign interest rate or in the country’s risk premium. However, the experience of many emerging economies revealed that capital flows are largely influenced by waves of optimism and pessimism of international investors, without any relation to country risk premia or interest rate differentials. Curdia (2006 and 2007) models capital inflows as exogenous shifts in foreign investors’ perceptions of domestic borrowers’ productivity, in a DSGE setting where financial frictions in the spirit of Bernanke at al. (1999) are introduced on the financing of intermediate goods used for production. In particular, foreign lenders have a distorted perception of borrowers’ idiosyncratic productivity and, when they perceive it to be higher, they loosen credit conditions, leading to a decrease in lending rates. As the financial accelerator kicks in, the country experiences a reinforcing spiral of improved lending conditions, rising asset prices and increased net worth, that further lower borrowing costs through a positive effect on leverage. A similar modeling choice is adopted in a series of papers by Ozkan and Unsal (2010, 2011), in order to analyze the transmission of a financial crisis to a small open economy and evaluate monetary policy responses. However, their studies abstract from credit frictions at the household level and financial stability objectives of the monetary authority.

3 The Model

The small open economy is populated by six agents: households, entrepreneurs, firms, capital and housing producers, and the central bank.

The model features financial frictions affecting the credit relationships of both households and firms in a New Keynesian small open economy model. In particular, credit frictions are modeled following Bernanke, Gertler and Gilchrist (1999), postulating the existence of an asymmetric information problem between borrowers and lenders implying costly state verification and generating an external finance premium directly linked to borrowers’ leverage. Hence, in such a context, fluctuations in asset prices affect agents’ ability to borrow and contribute to their overall leverage position. Furthermore, debt is denominated in foreign currency, therefore, exchange rate movements impact on borrowers’ balance sheets.

The capital inflow shock is embedded into the asymmetric information set-up following Curdia (2007, 2008). In particular, it is assumed that foreign lenders have a distorted perception of borrowers’ creditworthiness. In good times, lenders become optimistic about borrowers’ productivity, leading them to enforce looser credit conditions to borrowers. Lower
lending rates strengthen the balance sheet position of borrowers, encouraging them to undertake more projects, thereby increasing investment and asset prices and leading to a self-fulfilling virtuous cycle of economic expansion. This mechanism, albeit stylized, is able to replicate the credit and investment increases observed in the Central and Eastern European economies during capital inflow surges. In order to introduce the capital inflow shock consistently in the two sectors (real estate and capital investment), I assume the existence of capital and housing producers that buy final goods and convert them in new housing and capital stock. In the production sector, entrepreneurs invest in new capital goods using their own net worth and borrowing from foreign financial intermediaries, who face a costly state verification problem and charge an external finance premium dependent on leverage. They rent their capital to production firms who produce for the domestic and foreign market, and are subject to staggered price setting. In order to keep consistency and tractability, the housing market is modeled following Aoki, Proudman and Vlieghe (2004). In particular, it is assumed that households are composed of two behavioral types: homeowners and consumers. Homeowners are analogous to entrepreneurs: they use own net worth and borrowed funds to finance housing investment. The credit relationship is characterized by the same asymmetric information problem faced by entrepreneurs. Homeowners then rent the housing stock to consumers, which also consume domestic and imported goods and supply labor to domestic firms. In order to capture wealth effects in the household sector deriving from real estate price fluctuations, it is assumed that, at the end of each period, homeowners perform a transfer to consumers within the household. This simple modeling framework captures the fact that households can use their housing equity to finance consumption. More specifically, the transfer is related to the household’s leverage: when equity rises, the household can either accumulate the extra net worth thereby easing her credit condition in the following period, or use it to increase current consumption by increasing the transfer. Finally, the central bank sets the nominal interest rate according to a policy rule.

3.1 Households

In order to introduce borrowing and saving behavior within households in a tractable way, I follow the modeling approach of Aoki, Proudman and Vlieghe (2004). In particular, households are composed of two behavioral types, homeowners and consumers. While the former undertake housing investment and own the housing stock, the latter rent housing services and consume consumption goods. The two types are linked by a transfer scheme from homeowners to consumers, allowing for wealth effects of housing price fluctuations. Furthermore,

\[19\] In other terms, households are able to borrow against the value of their home to finance consumption, a practice often named mortgage equity withdrawal.
consumers are also divided in two types. A fraction $n$ of consumers is Ricardian (R), has access to domestic and foreign assets and is able to smooth consumption over time. The remaining $(1 - n)$ consumers are non-Ricardian (NR), and consume their current income in each period. Both types of consumers supply differentiated labor services to unions, which act as wage setters in monopolistically competitive markets. Income of NR consumers is then made up of wage income plus a transfer received from homeowners. Finally, as the economy is open, the consumption bundle is composed of domestic and imported goods.

### 3.1.1 Consumers

There exists a continuum of consumers of measure 1, deriving utility from consumption of a bundle $C_t$ and disutility from supplying labor services ($N_t$). The utility function common to all consumers is the following:

$$U(C_t, N_t) = \frac{(C_t - \tau C_{t-1})^{1-\sigma}}{1 - \sigma} - \chi_N (N_t)^{1+\varphi}$$

Where $\tau$ is the habit formation parameter, $\sigma$ and $\varphi$ are respectively the elasticity of intertemporal substitution and the elasticity of labor supply, and $\chi_N$ is a scaling parameter for the disutility of working hours. The consumption bundle $C_t$ is composed of consumption goods $c_t$ and housing services $h_t$:

$$C_t = \left[ \gamma_c^{\frac{1}{\varsigma}} (c_t)^{\frac{\varsigma - 1}{\varsigma}} + (1 - \gamma_c)^{\frac{1}{\varsigma}} (h_t)^{\frac{\varsigma - 1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma - 1}}$$

Where $\varsigma$ is the elasticity of substitution between consumption and housing services, and $\gamma_c$ is the weight of goods consumption in the overall basket. Furthermore, consumers allocate consumption between domestically produced and imported goods as follows:

$$c_t = \left[ \gamma_h^{\frac{1}{\eta}} (c_t^H)^{\frac{\eta - 1}{\eta}} + (1 - \gamma_h)^{\frac{1}{\eta}} (c_t^F)^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}$$

From which it follows that the consumer price index $P_t$ is defined as:

$$P_t = \left[ \gamma_h (P_t^H)^{1-\eta} + (1 - \gamma_h) (P_t^F)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

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20In the context of this study, this modeling choice is dictated by the necessity to incorporate a transfer from homeowners to consumers, in order to obtain wealth effects from investment. In general, the introduction of Non-Ricardian households in DSGE models is motivated by the empirical evidence suggesting a high dependence of consumption from current income, which cannot be obtained when households satisfy the permanent income hypothesis (Gali’, Lopez-Salido and Vallès (2007). Campbell and Mankiw (1989) and Mankiw (2000) provide empirical evidence on the relationship between consumption and income in advanced economies).
Where $P_t^{H}$ and $P_t^{F}$ are respectively the price of domestically produced and imported goods. As all consumers share the same preferences and face the same prices, intratemporal optimization results in the following demand schedules for consumption, housing services, domestic and imported goods:

$$\frac{c_t}{h_t} = \frac{\gamma_c}{1 - \gamma_c} \left( \frac{1}{P_t^{H}} \right)^{-\zeta}$$

(1)

$$c_t^H = \gamma_h \left( P_t^{H} \right)^{-\phi} c_t$$

(2)

$$c_t^F = (1 - \gamma_h) \left( P_t^{F} \right)^{-\phi} c_t$$

(3)

Furthermore, I assume that housing services are a constant fraction $s$ of the housing stock:

$$h_t = s H_t$$

(4)

While intratemporal choices are analogous for the two types of consumers, only Ricardian consumers (whose proportion in the total population is equal to $n$) face an intertemporal choice problem. In particular, Ricardian consumers maximize the discounted value of their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U (C_t^R, N_t^R)$$

subject to the budget constraint:

$$C_t^R + \frac{R_t B_t}{P_t} + \frac{R_t^{*} S_t B_t^{*}}{P_t} = \frac{W_t}{P_t} N_t^R + \frac{B_{t+1}}{P_t} + \frac{S_t B_{t+1}}{P_t} + \Pi_t - T_t^R$$

Besides consuming and supplying labor services, Ricardian consumers have access to domestic ($B_t$) and foreign ($B_t^{*}$) borrowing. While both assets are of a risk-free nature, and hence bear the risk-free rate of the relevant country, access to the international financial market is subject to small transaction costs $\Psi_t$. Hence, while the domestic asset carries the gross domestic risk-free interest rate $R_t$, the cost of foreign borrowing is $R_t^{*} \Psi_t$, where the portfolio adjustment cost $\Psi_t$ is defined as:

$$\Psi_t = \exp \left[ -\phi_B \left( \frac{S_t (B_{t+1}^* + L_t^F + L_t^H)}{Y_t} - \frac{S (B^* + L^F + L^H)}{Y} \right) \right]$$

(5)

Where $(B_{t+1}^* + L_t^F + L_t^H)$ represents the aggregate foreign debt of the small open
economy, and variables without time subscript denote steady state values.\footnote{This specification of the portfolio adjustment cost implies that the cost of foreign borrowing is higher the higher the net indebtness of the economy. While the coefficient $\phi_B$ is so small that it does not affect the dynamics of the model, introducing a portfolio adjustment cost in small open economy models guarantees the existence of a well defined steady state and delivers a stationary path for net foreign assets and consumption (Schmitt-Grohé and Uribe (2003)).} Furthermore, Ricardian households receive profits from the ownership of firms ($\Pi_t$) and pay lump-sum taxes to the government ($T_t^R$). The first order conditions of the Ricardian consumers’ maximization problem with respect to consumption and bond holdings are the following:

$$\lambda_t = \left( C_t^R - \tau C_{t-1}^R \right)^{-\sigma}$$  \hfill (6)

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right\}$$  \hfill (7)

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{S_{t+1}}{S_t} \Psi_t \frac{R^*_t}{\pi_{t+1}} \right\}$$  \hfill (8)

Non-Ricardian consumers are of measure $(1 - n)$ and are assumed to fully consume their income in every period. Hence, they do not optimally allocate consumption intertemporally following interest rate fluctuations, nor they smooth consumption following changes in the real wage. Consumption of NR households is then determined by their wage income, dividends from homeowners\footnote{Details on the specification of the transfer follow in section 3.1.2.} ($D_t$) and lump-sum taxes ($T_t^{NR}$), as follows:

$$C_t^R = \frac{W_t}{P_i} N_t^{NR} + D_t - T_t^{NR}$$  \hfill (9)

Total consumption is obtained aggregating over R and NR consumers:

$$C_t = nC_t^R + (1 - n)C_t^{NR}$$  \hfill (10)

The wage setting process and the consequent labor supply decision are governed by unions. Both R and NR consumers supply differentiated labor services to a continuum of monopolistically competitive unions\footnote{See Conen and Straub (2004) for this specification in the context of Ricardian and Non-Ricardian consumers.}, which act as wage setters taking the aggregate wage $W_t$ and the aggregate labor demand $N_t^d$ as given. Unions pool the wage income of all consumers and then distribute the aggregate wage income in equal proportion among the latter. Consumers, in turn, are assumed to supply sufficient labor to satisfy demand, equal to:
\[ N_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t^d \]

Where \( W_t(i) \) and \( N_t(i) \) are respectively the wage and labor demand for labor type \( i \) and \( \varepsilon_w \) is the elasticity of substitution between different labor types. This condition, coupled with the requirement that the total labor supply satisfies the market clearing condition \( N_t = \int_0^1 N_t(i) di \), yields:

\[ N_t = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t^d \]

The union then takes \( W_t \) and \( N_t^d \) as given and sets the optimal wage \( \tilde{W}_t(i) \) to equate the union’s expected average marginal return and the marginal cost of supplying labor. However, in doing so the union faces nominal rigidities in the Calvo fashion. Specifically, in each period the wage can be optimized only in a fraction \( (1 - \theta^w) \) of labor markets. In the remaining fraction \( \theta^w \) the real wage is indexed to past inflation and it is therefore given by:

\[ W_t(i) = W_{t-1}(i) \pi_{t-1} \]

The reoptimizing union sets the optimal wage \( \tilde{W}_t(i) \) to maximize:

\[
E_t \sum_{k=0}^{\infty} (\beta \theta^w)^k \Lambda_{t+k} \left( \frac{W_t \prod_{i=1}^{k} \left( \frac{\pi_{t+i-1}}{\pi_{t+i}} \right)}{W_{t+k}} \right)^{-\varepsilon_w} N_t^d \left[ \tilde{W}_t \prod_{i=1}^{k} \left( \frac{\pi_{t+i-1}}{\pi_{t+i}} \right) - \frac{U_t^N}{\Lambda_{t+k}} \right]
\]

Where the first term in parenthesis represents the marginal gain for the union of supplying an extra unit of labor, and the second term represents the marginal disutility of doing so.  

\[ K^w_t = \left( \frac{\varepsilon_w}{\varepsilon_w} \right) \tilde{W}_t \Lambda_t N_t \left( \frac{W_t}{W_t} \right)^{\varepsilon_w} + \beta \theta^w \left( \frac{\pi_{t+1}}{\pi_t} \tilde{W}_{t+1} \right)^{\varepsilon_w} K^w_{t+1} \quad (11) \]

\[ F^w_t = \chi_H (N_t^d)^{\varepsilon_w} \left( \frac{W_t}{W_t} \right)^{\varepsilon_w} N_t + \beta \theta^w \left( \frac{\pi_{t+1}}{\pi_t} \tilde{W}_{t+1} \right)^{\varepsilon_w} F^w_{t+1} \quad (12) \]

\[ K^w_t = F^w_t \quad (13) \]
It follows that the law of motion of the aggregate wage is:

\[ W_t = \left( 1 - \theta^u \right) \frac{1}{1 - \varepsilon_w} \left[ \frac{1}{1 - \varepsilon_w} \right] + \theta^u (W_{t-1}(i)\pi_{t-1})^{1 - \varepsilon_w} \]

(14)

Note that the aggregate labor supply of consumers is given by:

\[ N_t = nN_t^R + (1 - n)N_t^{NR} \]

However, given the hypothesis that unions pool wage incomes of R and NR consumers, labor market equilibrium requires:

\[ N_t = N_t^R = N_t^{NR} \]

In order to ensure that the wage rate is the same for the two consumer types, hours worked must be equalized.\(^{27}\)

### 3.1.2 Homeowners

Housing investment decisions are made by homeowners, who act like entrepreneurs in the model. Homeowners are risk neutral, they purchase housing from housing producers, transform it into homogeneous rentable units and rent them to consumers. At the end of period \( t \) the \( i \)-th homeowner has available net worth equal to \( NW_{t+1}^H(i) \). At time \( t \) she purchases unfinished housing \( (H_{t+1}(i)) \) from housing producers at a unit price \( Q_{h,t} \) and finances the part of investment in excess of her net worth by stipulating foreign currency loans \( L_{t+1}^H(i) \).\(^{28}\)

In the next period, she will use unfinished housing to produce rentable units, which will be rented to consumers at a rental price \( P_{h,t+1} \). Homeowners borrow from a competitive foreign financial intermediary whose relevant opportunity cost is the gross risk-free rate prevailing in the foreign country, \( R_{t+1}^* \). The typical homeowner faces the following budget constraint, expressed in domestic currency:

\[ \Delta_{w,t} = (1 - \theta_w) \left( \frac{W_t}{W_t^H} \right)^{\varepsilon_w} + \theta_w \left( \frac{W_t}{W_{t-1}} \frac{\pi_{H,t}}{\pi_{H,t-1}} \right)^{\varepsilon_w} \Delta_{w,t-1} \]

So that the effective labor supply is \( H_t = \frac{H_t^H}{\Delta_{w,t}} \).

\(^{26}\)Staggered wage setting implies an inefficient wage dispersion, arising from the fact that wages are not set simultaneously. The law of motion of such wage dispersion, defined as \( \Delta_{w,t} = \int_0^1 (\frac{W_t(i)}{W_1})^{-\varepsilon_w} di \), is given by:

\[ \Delta_{w,t} = (1 - \theta_w) \left( \frac{W_t}{W_t^H} \right)^{\varepsilon_w} + \theta_w \left( \frac{W_t}{W_{t-1}} \frac{\pi_{H,t}}{\pi_{H,t-1}} \right)^{\varepsilon_w} \Delta_{w,t-1} \]

\(^{27}\)This also arises as a result of the fact that firms allocate their labor demand uniformly across labor varieties, independently of their consumer type (R or NR).

\(^{28}\)Note that loans are stipulated in period \( t \) but will be repayed at \( t + 1 \), hence the choice of subscript. Similarly, housing purchased at time \( t \) will be used in the next period, hence the time subscript.
The expected gross return of a unit of housing investment is composed of the return from renting houses to consumers (i.e. the rental price of houses, $P_{h,t}$) and the value of the undepreciated housing stock, adjusted for the change in price:

$$W_{t+1}^H(i) = Q_{h,t}H_{t+1}(i) - S_t L_{t+1}^H(i)$$

Where $\delta_h$ is the depreciation rate of the housing stock.

Each homeowner has access to a stochastic technology that transforms $H_{t+1}(i)$ units of unfinished housing into $H_{t+1}(i) = \omega_{t+1}^H(i)H_{t+1}(i)$ rentable units. The idiosyncratic productivity shock $\omega_{t+1}^H(i)$ is iid across homeowners and time and it is assumed to follow a lognormal distribution with density $f(\omega^H)$ and $E \{ \omega^H \} = 1^{29}$. The realization of productivity is freely observed by homeowners, but lenders can only observe it by incurring a monitoring cost proportional to the gross payoff to the homeowner’s project $(\mu^H(\omega_{t+1}^H(i))R_{t+1}^H Q_{h,t} H_{t+1}(i)))$: this asymmetric information is at the core of the external finance premium. Furthermore, following Curdia (2007), I assume that lenders have a distorted perception of the productivity parameter. In particular, the lenders’ perception of productivity is $\omega_{t+1}^{H*}(i) = \omega_{t+1}^H(i)H_{t+1}(i)$ where $\omega_{t+1}^H(i) \in [0, 1]$ is the misperception factor which evolves according to $\ln(\omega_t^H) = \omega_{t-1}^H + \xi_t^H$. $\xi_t^H$ is a shock to lenders perceptions of homeowners’ productivity and it is the origin of capital inflows in the model. When $\omega_t^H$ increases, lenders perceive homeowners’ to be more productive or, in other words, they perceive their default probability to be lower. Hence, they will charge a lower premium, allowing borrowers to expand their balance sheet.

The optimal credit contract between financial intermediaries and homeowners specifies a fixed payment to the lender whenever the project’s nominal gross return exceeds the nominal value of the debt obligations. Otherwise, the homeowner defaults on her debt and the lender seizes the remaining value of the project, after paying the monitoring cost.

I denote $R_{L,t+1}^H$ the interest rate on loans to homeowners contracted in period $t$. The threshold productivity level $\tilde{\omega}_{t+1}^H(i)$ at which the homeowner does not default is the one equating the homeowner’s receipts with the repayment of the loan$^{30}$:

$$\tilde{\omega}_{t+1}^H = \ln(\frac{s P_{t+1}^h + (1 - \delta_h)Q_{h,t+1}}{Q_{h,t}})$$

Where $\delta_h$ is the depreciation rate of the housing stock.
\[ \frac{\bar{\omega}_{t+1}^H(i)}{S_{t+1}} (Q_{h,t}H_{t+1}(i)R_{t+1}^H) = R_{L,t+1}^H L_{t+1}^H(i) = \frac{R_{L,t+1}^H (Q_{h,t}H_{t+1}(i) - NW_{t+1}^H(i))}{S_t} \]

The term \( \bar{\omega}_{t+1}^H(i)Q_{h,t}H_{t+1}(i)R_{t+1}^H \) represents the minimum gross return of the produced housing necessary to repay the loan and the corresponding interests. If \( \bar{\omega}_{t+1}^H \) the homeowner does not default, pays to the lender the contracted amount \( (\bar{\omega}_{t+1}^H(i)Q_{h,t}H_{t+1}(i)R_{t+1}^H) \) and keeps the extra profit. Lenders receive their payment and do not pay the verification cost. If \( \bar{\omega}_{t+1}^H < \bar{\omega}_{t+1}^H \) the homeowner defaults: in this case, the lender pays the verification cost and takes possession of the homeowner’s remaining assets (hence, the homeowner’s payoff is 0). \(^{31}\)

Taking as given \( Q_{h,t}, R_{t+1}^H \) and net worth \( NW_{t+1}^H(i) \), the optimal contract is fully specified in terms of the threshold productivity level \( \bar{\omega}_{t+1}^H(i) \) and demand for initial investment \( H_{t+1}(i) \). \(^{32}\)

In what follows, it is assumed that both homeowners and lenders are risk neutral, implying that they will engage in the contract if it guarantees an expected payoff at least equal to what they would obtain by investing in the risk-free asset.

The expected payoff of the homeowner is:

\[
E_t \left[ \left( \int_{\bar{\omega}_{t+1}^H(i)}^{\infty} \omega_{t+1}^H(i) f(\omega^H) d\omega^H \right) (Q_{h,t}H_{t+1}(i)R_{t+1}^H) - \left( \int_{\omega_{t+1}^H(i)}^{\infty} f(\omega^H) d\omega^H \right) R_{L,t+1}^H L_{t+1}^H(i) \right] \\
= E_t \left[ (Q_{h,t}H_{t+1}(i)R_{t+1}^H) \left( \int_{\omega_{t+1}^H(i)}^{\infty} \omega_{t+1}^H(i) f(\omega^H) d\omega^H - \bar{\omega}_{t+1}^H(i) \int_{\omega_{t+1}^H(i)}^{\infty} f(\omega^H) d\omega^H \right) \right] \\
= E_t \left[ (Q_{h,t}H_{t+1}(i)R_{t+1}^H) \left( A^H(\bar{\omega}_{t+1}^H(i)) \right) \right]
\]

Where \( A^H(\bar{\omega}_{t+1}^H(i)) \) represents the fraction of the expected payoff captured by homeowners.

The expected payoff of the lender is:

\(^{31}\)Lenders verify the borrowers’ output only when the latter declares default. In fact, under asymmetric information, the borrower might declare default in order not to pay back the lender even if the project is profitable.

\(^{32}\)Recall that \( P_{t+1}^h \) is a market price, and as such it will be determined by the equilibrium between demand and supply of rentable houses.
for un… nished housing by homeowners is positively related to the rental price of housing and an optimal demand of houses \( (\text{unknowns:}) \) to the lenders’ participation constraints (20). As shocks in the model, homeowners are identical by investing in a risk-free asset. Hence, they will participate if the following condition holds:

\[
R_{t+1}^H Q_{h,t} H_{t+1}(i) \left[ \tilde{\omega}_{t+1}^H(i) \int_{\omega_{t+1}^H(i)}^\infty f(\omega^H) d\omega^H + (1 - \mu^H) \int_0^{\omega_{t+1}^H(i)} f(\omega^H) d\omega^H \right] = R_{t+1}^H L_{t+1}^H(i)
\]

Where \( B^H (\tilde{\omega}_{t+1}^H, v_t^H) \) is the fraction of homeowner’s payoff captured by the lender (recall the definition \( \omega_{t+1}^H = \omega_t^H v_t^H \)), net of monitoring cost. As foreign lenders are risk neutral, they participate in the debt contract only if the expected payoff is equal to that they would obtain by investing in a risk-free asset. Hence, they will participate if the following condition holds:

\[
\frac{R_{t+1}^H Q_{h,t} H_{t+1}(i)}{S_{t+1}} \left[ B^H (\tilde{\omega}_{t+1}^H, v_t^H) \right] = R_{t+1}^H L_{t+1}^H(i)
\]  

Note that the lending rate \( R_{t+1}^H \) is state contingent, and will be set to satisfy the participation constraint of lenders. The optimal contract specifies a threshold value \( \tilde{\omega}_{t+1}^H \) and an optimal demand of houses \( (H_{t+1}^H) \) maximizing the homeowner’s payoff (18) subject to the lenders’ participation constraints (20). As \( \omega_{t+1}^H \) is \( \text{iid} \) and independent of all other shocks in the model, homeowners are identical \( \text{ex – ante} \), face the same contract and will be charged the same lending rate. Hence, I can drop the index \( i \). The first order conditions of the optimal credit contract are \(^{33}\):

\[
E_t \left( R_{t+1}^h \right) = R_{t+1}^* \left[ \frac{A^h(\tilde{\omega}_{t+1})}{B^H (\tilde{\omega}_{t+1}, v_t^H) A^h(\tilde{\omega}_{t+1}) - B^H (\tilde{\omega}_{t+1}, v_t^H) A^H(\tilde{\omega}_{t+1})} \right] E_t \left\{ \frac{S_{t+1}}{S_t} \right\}
\]

\[
\frac{Q_{h,t} H_{t+1}}{N W_{t+1}^H} = \frac{1}{\left( 1 - \frac{R_{t+1}^H}{S_{t+1}} R_{t+1}^H \right) B^H (\tilde{\omega}_{t+1}, v_t^H)}
\]

These equations, together with equation (17) form a systems of three equations in three unknowns: \( \tilde{\omega}_{t+1}, R_{t+1}^H \) and \( H_{t+1} \). In particular, equation (22) implies that the demand for unfinished housing by homeowners is positively related to the rental price of housing \( P_{t+1}^h \), which enters \( R_{t+1}^h \), and inversely related to the price of housing good, \( Q_{h,t} \). Equation

\(^{33}\)The derivation of the optimal contract is described in the Appendix. Here, \( A'(\omega) = \frac{\partial A(\omega)}{\partial \omega} \)
(21) is the basis of the financial accelerator in the model. It links the cost of external finance to homeowners’ financial position and, hence, to their demand for housing good. In fact, risk premia are a positive function of $\bar{\omega}_{t+1}^H$ which is, in turn, a positive function of the homeowner’s leverage. Hence, lower leverage implies lower probability of default and hence a lower risk premium. Furthermore, as borrowing is denominated in foreign currency, exchange rate movements also affect the risk premium: a domestic currency appreciation (decrease in $S_{t+1}$) lowers the risk premium both directly and indirectly by decreasing the value of outstanding debt and thereby lowering leverage.

To complete the description of homeowners’ behavior, it is necessary to define the evolution of their net worth. At the end of each period, non-defaulting homeowners keep their payoff net of loan repayment, which is going to increment their stock of equity. Furthermore, I follow Aoki, Proudman and Vlieghe (2004) in assuming that homeowners perform a transfer to consumers within the household. Here, the transfer is not fully microfounded. Rather, it is simply assumed that the transfer rule is given by:

$$D_t = \chi_D \left( \frac{NW_{t+1}^H}{Q_{h,t} H_{t+1}} \right)$$

(23)

Where $\frac{NW_{t+1}^H}{Q_{h,t} H_{t+1}}$ is the inverse leverage ratio and $\chi_D$ represents the elasticity of the transfer to the equity-to-assets ratio. This simple rule captures the concept that, following a rise in real estate prices, homeowners are faced with two choices. Either they keep the transfer constant and they accumulate more net worth (thereby increasing their equity and enjoy looser credit conditions in the future), or they increase the transfer to consumers leading to an increase of current household consumption. Hence, this is a simple way to generate wealth effects of real estate prices, and a positive correlation between housing prices and consumption observed in the data (see Iacoviello (2010)). Furthermore, homeowners are assumed to be endowed with a unit of labor, which they supply inelastically to domestic firms. Hence, the evolution of homeowners’ net worth can be represented as:

$$NW_{t+1}^H = A^H(\bar{\omega}_{t}^H)R_t^H Q_{h,t-1} H_{t-1}^H - D_t + W_t^H N_t^H$$

(24)

Where $A^H(\bar{\omega}_{t}^H)R_t^H Q_{h,t-1} H_{t-1}^H$ represents the share of payoff captured by homeowners in the previous period, after fulfilling their debt obligations.
3.2 Housing and Capital Producers

Housing and capital producers operate in a regime of perfect competition. In each period, they combine investment goods \((I_{j,t}, \text{ with price } P^I_{j,t}, j = k, h)\) and the old undepreciated capital (housing) stock to produce new capital (housing) goods, which will be sold at the real price \(Q_{k,t} (Q_{h,t})\). Investment is subject to adjustment costs, represented by the function \(\Phi_{j,t} = \frac{\kappa_j}{2} \left( \frac{I_{j,t}}{I_{j,t-1}} - 1 \right)^2\) (Smets and Wouters (2003)). Capital producers choose the optimal amount of investment so as to maximize the following real profit function:

\[
E_t \left\{ \sum_{s=0}^{\infty} \beta^t \lambda_{t+s} \frac{1}{P_{t+s}} [q_{k,t} P_{t+s} ((1 - \delta_k) K_{t+s} + (1 - \Phi_{k,t}) I_{k,t+s} - K_{t+s}) - P_{t+s} I_{k,t+s}] \right\}
\]

The first order condition with respect to \(I_{k,t}\) yields:

\[
1 = q_{k,t} \left[ 1 - \frac{\kappa_t}{2} \left( \frac{I_{k,t}}{I_{k,t-1}} - 1 \right)^2 - \kappa_t \left( \frac{I_{k,t}}{I_{k,t-1}} - 1 \right) \left( \frac{I_{k,t}}{I_{k,t-1}} \right) \right] + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} q_{k,t+1} \left[ \kappa \left( \frac{I_{k,t+1}}{I_{k,t}} - 1 \right) \left( \frac{I_{k,t+1}}{I_{k,t}} \right)^2 \right] \right\}
\]

Where \(q_{k,t}\) is the real price of the capital stock defined as \(\frac{Q_{k,t}}{P_t}\). The law of motion of the economy wide capital stock is:

\[
K_{t+1} = \left[ 1 - \frac{\kappa_t}{2} \left( \frac{I_{k,t}}{I_{k,t-1}} - 1 \right)^2 \right] I_{k,t} + (1 - \delta_k) K_t
\]

Analogous expressions hold for housing producers, with the subscript \(k\) replaced by \(h\).

---

34Here, I denote with subscript \(k\) (\(h\)) variables pertaining to capital (housing) producers.

35The investment bundle for both producers has a similar composition of the consumption bundle, and can therefore be defined as:

\[
I_{k,t} = \left[ \gamma_{k,h} \left( I_{k,t}^{H} \right)^{\frac{n-1}{n}} + (1 - \gamma_{k,h}) \left( I_{k,t}^{P} \right)^{\frac{n-1}{n}} \right]^{\frac{n}{n-1}}
\]

an analogous expression holds for investment in the housing production sector. Furthermore, note that the relevant discount factor, \(\lambda_t\) is that pertain to ricardian consumers, as they own productive activities in the economy.
3.3 Entrepreneurs

The behavior of entrepreneurs is modeled as in Bernanke, Gertler and Gilchrist (1999) and closely mirrors that of homeowners. Entrepreneurs engage in capital investment, and in each period they purchase capital from capital producers \((Q_{k,t}K_{t+1})\) using their net worth \((NW_{t+1}^E)\) and borrowing from foreign financial intermediaries \((L_{t+1}^E)\) with a lending rate \(R_{L,t+1}^E\). The return from capital investment, \(R_{t+1}^E\), is given by the return from renting capital to firms \(r_t^K\) and the capital gain:

\[
R_{t+1}^E = \frac{x_{t+1}^K + (1 - \delta_t)Q_{t+1}^E}{Q_{t,t}}
\]

Furthermore, each entrepreneur has a stochastic technology \(\omega_{t+1}^E(i) \sim \log N(-\frac{\sigma^2_E}{2}, \sigma^2_E)\), the realization of which determines the profitability of their investment and, then, their default probability. The threshold productivity level that discriminates between defaulting and non-defaulting entrepreneurs is given by:

\[
\omega_{t+1}^E(i) \frac{Q_{k,t}K_{t+1}(i)R_{t+1}^E}{S_{t+1}} = R_{L,t+1}^E L_{t+1}^E(i) = R_{L,t+1}^E \frac{(Q_{k,t}K_{t+1}(i) - NW_{t+1}^E(i))}{S_t}
\]

Finally, as in the case of homeowners, lenders have a distorted perception of entrepreneurial productivity, given by \(\omega_{t+1}^E = \omega_{t+1}^E v_t^E\) where \(v_t^E \in [0, 1]\) is the misperception factor which evolves according to \(\ln(v_t^E) = \rho_v \ln(v_{t-1}^E) + \xi_t^E\). The optimal financial contract is identical to that faced by homeowners and the first order conditions result in:

\[
E_t (R_{t+1}^E) = R_{t+1}^E \left[ \frac{A^E(\omega_{t+1}^E)}{B^E(\omega_{t+1}^E, v_t^E) A^E(\omega_{t+1}^E) - B^E(\omega_{t+1}^E, v_t^E) A^E(\omega_{t+1}^E)} E_t \left\{ \frac{S_{t+1}}{S_t} \right\} \right]
\]

\[
\frac{Q_{k,t}K_{t+1}}{NW_{t+1}^E} = \frac{1}{\left(1 - \frac{S_{t+1}R_{t+1}^E}{S_t R_{t+1}^E} B^H (\omega_{t+1}^E, v_t^E)\right)}
\]

Contrary to the case of homeowners, entrepreneurs do not pay a transfer. In order to characterize the evolution of their net worth it is assumed that entrepreneurs have finite horizon: in particular, a proportion \((1 - \tilde{\pi})\) of entrepreneurs die in each period but are immediately replaced by newcomers, so that the total population is constant. This is necessary to guarantee that the net worth of entrepreneurs does not grow to the point they can finance their investment using their equity only. Furthermore, entrepreneurs are endowed with a unit of labor that they supply inelastically to firms, paying a wage \(W_t^E\). At the end of period \(t\), entrepreneurs collect their investment payoff and honour the debt obligations contracted.
in the previous period. Net worth of surviving entrepreneurs is then composed of the profits from investment and wage income:

\[ NW_{E,t+1} = \tilde{\pi} \left[ A^E (\tilde{\omega}_t^E) R_t^E Q_{k,t-1} K_t \right] + W_t^E \]  

(32)

Entrepreneurs exiting the market consume their remaining equity:

\[ P_t C_t^E = (1 - \tilde{\pi}) A^E (\tilde{\omega}_t^E) R_t^E Q_{k,t-1} K_{t-1} \]  

(33)

Entrepreneurs consume domestic and import good in the same mix as consumers. Hence, the demand functions for the two goods of the two types of entrepreneurs are:

\[ C_H^E = \gamma_h \left( P_t^H \right)^{-\phi} C_t^E \]  

(34)

\[ C_F^E = (1 - \gamma_h) \left( P_t^F \right)^{-\phi} C_t^E \]  

(35)

3.4 Firms

There exist two types of firms in the economy. A continuum of intermediate producers indexed by \( f \in [0, 1] \) operates in a monopolistically competitive environment and produce differentiated goods employing capital and labor. Furthermore, these firms face price rigidities à la Calvo, implying staggered priced setting. Then, a set of perfectly competitive final goods producers aggregate costlessly the differentiated intermediate goods into a single final good, which is then sold to consumers (both domestically and abroad).

3.4.1 Final good producers

Final good producers operate in a perfectly competitive environment. They purchase intermediate goods and aggregate them, using the following CES technology:

\[ Y_t = \left[ \int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon \gamma}} df \right]^\frac{\varepsilon}{\varepsilon - 1} \]

Where \( \varepsilon \) is the elasticity of substitution between varieties of domestic goods. Cost minimization yields the following demand schedule for each variety:

\[ Y_t(f) = Y_t \left( \frac{P_t^H(f)}{P_t^H} \right)^{-\mu} \]  

(36)

Where \( P_t^H = \left( \int_0^1 P_t^H(f)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}} \) is the price of the final, domestically produced good.
The final good is sold both domestically and abroad. In particular, the export good is produced one-for-one by a representative competitive producer, using the domestic final good as input. The foreign demand for the domestic good is given by:

$$X_t = \gamma^* \left( \frac{P^x_t}{P^*_t} \right)^{-\mu_x} Y^*_t$$

(37)

Where $P^*_t$ is the foreign price index and $Y^*_t$ is foreign output. $\mu_x$ represents the elasticity between domestically produced and imported goods in the foreign country. Finally, $\gamma^*$ is the share of imports in the foreign country’s consumption basket. I assume that the law of one price holds in the export market, implying that the domestic good sells for the same price on the two markets when converted to the same currency. Hence, defining the nominal exchange rate $S_t$ as the price of the foreign currency in terms of domestic currency, the price of exports in foreign currency ($P^x_t$) is given by:

$$P^x_t = \frac{P^x_{H,t}}{S_t}$$

(38)

### 3.4.2 Intermediate goods producers

There exist a continuum of monopolistic intermediate good producers indexed by $f$. Each producer operates under monopolistic competition and is owned by Ricardian households, with the demand for its products given by (36). Producers use capital and three types of labor inputs ($N_t$, $N^E_t$ and $N^H_t$, supplied respectively by consumers, entrepreneurs and homeowners) to produce differentiated goods. The production function for domestic intermediate good producers is:

$$Y_t (f) = e^{A_t} K^\alpha_t (f) N_t (f)^{(1-\alpha)(1-\Omega_E-\Omega_H)} N^E_t (f)^{(1-\alpha)\Omega_E} N^H_t (f)^{(1-\alpha)\Omega_H}$$

(39)

Where $\alpha$ is the share of capital in production, $\Omega_E$ and $\Omega_H$ are the shares of entrepreneurial and homeowners’ labor in production. Cost minimization implies the following standard factor demand functions, where $r_{k,t}$ denotes the rental rate of capital:

$$W_t = MC_t (1 - \alpha) (1 - \Omega_E - \Omega_H) \frac{Y_t (f)}{N_t (f)}$$

(40)

$$W^E_t = MC_t (1 - \alpha) \Omega_E \frac{Y_t (f)}{N^E_t (f)}$$

(41)

$$W^H_t = MC_t (1 - \alpha) \Omega_F \frac{Y_t (f)}{N^H_t (f)}$$

(42)
3.4.3 Price setting

Price setting is staggered. In each period, only a fraction \((1 - \theta)\) of firms are allowed to reset their price optimally. The fraction \(\theta\) that is not allowed to optimize sets the price equal to that prevailing in the previous period, indexing it to past inflation at a rate \(\gamma_p\) and to the steady state inflation rate at rate \((1 - \gamma_p)\). As all firms allowed to optimize set the same price, denoted as \(\bar{P}_{H,t}\), the law of motion of the domestic good price is:

\[
P_{H,t} = \left[ \theta \left( P_{H,t-1}^{H} \gamma_p \left( \pi^H \right)^{1-\gamma_p} \right)^{1-\varepsilon} + (1 - \theta) \left( \bar{P}_{H,t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{44}
\]

The optimal price \(\bar{P}_{H,t}^{H}\) is set in order to maximize the discounted sum of future real profits:

\[
E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} \left[ \left( \frac{\bar{P}_{H,t}^{H}(f)}{P_{H,t+k}^H} \prod_{i=1}^{k} \left( \pi^H_{t+i-1} \right)^{\gamma_p} \left( \pi^H \right)^{1-\gamma_p} \right)^{1-\varepsilon} - m_{ct+k} \left( \frac{\bar{P}_{H,t}^{H}(f)}{P_{H,t+k}^H} \prod_{i=1}^{k} \left( \pi^H_{t+i-1} \right)^{\gamma_p} \left( \pi^H \right)^{1-\gamma_p} \right)^{-\varepsilon} \right] Y_{t+k} \right\}
\]

The first order conditions can be written recursively as follows:

\[
\frac{\bar{P}_{H,t}^{H}}{P_{H,t}^H} = \frac{\varepsilon}{\varepsilon - 1} F_{H,t} \tag{45}
\]

\[
F_{H,t} = \lambda_t m_{ct} Y_t + \beta \theta E_t \left\{ \left( \pi^H_{t+1} \right)^{\varepsilon} F_{H,t+1} \right\} \tag{46}
\]

\[
D_{H,t} = \lambda_t Y_t + \beta \theta E_t \left\{ \left( \pi^H_{t+1} \right)^{\varepsilon-1} D_{H,t+1} \right\} \tag{47}
\]

3.4.4 Import firms

I introduce imperfect exchange rate pass-through in import prices following Monacelli (2003). In particular, I assume the existence of monopolistically competitive importers who sell foreign goods in the domestic market. Importers purchase the foreign differentiated good at the (domestic currency) price \(S_t P_t^*\) and set the optimal domestic price of the imported good as to maximize real discounted profits. Furthermore, importers face a staggered price setting à la Calvo: in each period, they can optimally reset prices with probability \(\theta_m\). The price index of imported goods is given by:
\[ p_t^f = \left( (1 - \theta_m) (p_{ft}^{new})^{1-\mu_m} + \theta_m (p_{t-1}^f)^{1-\mu_m} \right)^{\frac{1}{\mu_m}} \] (48)

Where \( \mu_m \) is the elasticity of substitution between different varieties of import goods. Each firm in the import sector chooses the optimal price as to maximize discounted profits:

\[
\max_{p_t^f(j)} E_t \left[ \sum_{k=0}^{\infty} (\beta \theta_f)^k \Lambda_{t,t+k} \left( \frac{p_t^f(j)}{p_{t+k}^f} - \frac{S_{t+k}p_{t+k}^s}{p_{t+k}^f} \right) Y_{t+k}^M(j) \right] \]

Subject to the demand function:

\[
Y_{t+k}^M(j) = \left( \frac{p_t^f(j)}{p_{t+k}^f} \right)^{-\mu_m} Y_t^M \]

Where \( p_t^f = \left( \int_0^1 p_t^f(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \) and \( Y_t^M \) denotes aggregate imports demand. In the symmetric equilibrium, all firms allowed to reset price will set it at the same level, equal to a markup over current and expected future marginal costs:

\[
\frac{p_t^f(j)}{p_t^f} = \frac{\mu_m}{\mu_m - 1} E_t \left[ \sum_{k=0}^{\infty} (\beta \theta_m)^k \Lambda_{t,t+k} Y_{t+k}^M \left( \frac{\pi_{t,t+k}^f}{p_{t+k}^f} \right)^{\mu_m} \frac{S_{t+k}p_{t+k}^s}{p_{t+k}^f} \right] \]

Subject to the demand function:

\[
Y_{t+k}^M(j) = \left( \frac{p_t^f(j)}{p_{t+k}^f} \right)^{-\mu_m} Y_t^M \]

Where \( p_t^f = \left( \int_0^1 p_t^f(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \) and \( Y_t^M \) denotes aggregate imports demand. In the symmetric equilibrium, all firms allowed to reset price will set it at the same level, equal to a markup over current and expected future marginal costs:

\[
\frac{p_t^f(j)}{p_t^f} = \frac{\mu_m}{\mu_m - 1} E_t \left[ \sum_{k=0}^{\infty} (\beta \theta_m)^k \Lambda_{t,t+k} Y_{t+k}^M \left( \frac{\pi_{t,t+k}^f}{p_{t+k}^f} \right)^{\mu_m - 1} \right] \]

### 3.5 Aggregate demand and balance of payments

Domestically produced goods are used for domestic consumption by consumers and entrepreneurs, investment by housing and capital goods producers, government expenditure (\( G_t \)), exports (\( X_t \)) and to pay monitoring costs arising from imperfect information in the credit relationships between financial intermediaries and homeowners and entrepreneurs (\( M_{t}^H \) and \( M_{t}^E \)). Hence, the national accounting identity reads:

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36 As import firms are owned by Ricardian consumers, \( \Lambda_{t,t+k} = \frac{C_{t+k}^{\mu_m}}{C_{t}^{\mu_m}} \) is the consumers’ stochastic discount factor.

37 Given the distribution of \( \omega_t^H \) and \( \omega_t^E \), the fraction of payoff used to monitor borrowers in the two sectors amounts to:

\[
M_{t}^H = \mu^H \cdot F \left( \frac{\ln \omega_t^H - 0.5\sigma_h^2}{\sigma_h} \right) \cdot R_{t}^H Q_{h,t-1} H_t
\]

\[
M_{t}^E = \mu^E \cdot F \left( \frac{\ln \omega_t^E - 0.5\sigma_e^2}{\sigma_e} \right) \cdot R_{t}^E Q_{k,t-1} K_t
\]

---
\[ Y_t = c_t^H + C_{H,t}^E + I_{k,t}^H + I_{h,t}^H + G_t + X_t + M_t^H + M_t^E \]  

(50)

Imported goods are used for consumption and investment, hence total imports \( (Y_t^M) \) are defined as:

\[ Y_t^M = c_t^F + C_{F,t}^E + I_{k,t}^F + I_{h,t}^F \]  

(51)

Finally, the balance of payments of the small open economy is obtained by aggregating the budget constraints of consumers, homeowners and entrepreneurs, and results in the following expression:

\[ S_t R_t^* (\Psi_t B_t^* + I_t^E + L_t^H) - S_t (B_{t+1} + L_{t+1}^E + L_t^H) = S_t P_t^* X_t - S_t P_t^* Y_t^M \]  

(52)

Where the nominal foreign interest rate, \( R_t^* \), is taken as given by the small open economy.

3.6 Monetary and fiscal policy

Government in this setting is in charge of conducting monetary and fiscal policy. As for the latter, the government simply aims at maintaining fiscal balance:

\[ G_t = T_t^R + T_t^{NR} \]  

(53)

The general form of the rule used by the central bank to conduct monetary policy is:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\frac{\rho_{\pi}}{\rho_{\pi}}} \left( \frac{Y_t}{\bar{Y}} \right)^{\rho_{\pi}} \left( \frac{S_t}{S_{t-1}} \right)^{\rho_{\pi}} \left( \frac{L_t}{L_{t-1}} \right)^{\rho_{L}} \exp \left( \xi_{R,t} \right) \]  

(54)

Where \( \rho_{\pi} \) and \( \rho_Y \) are, respectively, the weights that the monetary authority places on deviations of inflation and output from the steady state and \( \xi_{R,t} \) is a monetary policy shock. Furthermore, it is assumed that the central bank adjusts the nominal interest rate also in response to changes in the exchange rate and credit growth, defined as the total loans contracted by homeowners and entrepreneurs. However, setting the coefficients of the last two terms to zero, it is possible to obtain a standard Taylor Rule. Furthermore, when \( \rho_S \to \infty \), the central bank follows a pegged exchange rate. Finally, the monetary authority engages in interest rate smoothing whenever \( \rho_R > 0 \).
3.7 Exogenous processes

There are six exogenous processes in the model: aggregate technology \((A_t)\), perception of lenders on homeowners and entrepreneurial productivity \((v_t^H \text{ and } v_t^E)\), and variables pertaining to the foreign country namely foreign interest rate \((R_t^*)\), foreign output \((Y_t^*)\) and foreign price level \((P_t^*)\).

Exogenous variables obey the following autoregressive processes:

\[
\begin{align*}
\log(A_t) &= \rho_A \log(A_{t-1}) + \xi_{A,t} \\
\ln(v_t^H) &= \rho_v \ln(v_t^H) + \xi_{v,t}^H \\
\ln(v_t^E) &= \rho_v \ln(v_t^E) + \xi_{v,t}^E \\
\log(R_t^*) &= \rho_{R^*} \log(R_{t-1}^*) + \xi_{R,t}^* \\
\log(Y_t^*) &= \rho_{Y^*} \log(Y_{t-1}^*) + \xi_{Y,t}^* \\
\log(P_t^*) &= \rho_{ps} \log(P_{t-1}^*) + \xi_{ps,t}
\end{align*}
\]

3.8 Calibration

The calibration of the model parameters is largely drawn from existing studies on small open economies. In particular, I set the discount factor \(\beta = 0.99\), implying an annual risk-free interest rate of 4%. The intertemporal elasticity of substitution \((\sigma)\) is set to 1, so as the elasticity of labor supply \((\varphi)\) following Christiano, Eichenbaum, and Evans (1997). In order to obtain a steady state labor supply of 0.33 the coefficient on labor in the utility function \((\chi_N)\) is calibrated at 8.8394. Regarding the composition of consumption, I set the share of imported goods in the consumption basket at 0.4, consistent with the value set for Latvia by Ajevskis and Vitola (2011), which implies some degree of home bias. Furthermore, I set the consumption habit parameter at 0.8, following the estimates for Estonia by Gelain and Kulikov (2009). As in Aoki, Proudman and Vlieghe (2004) and Forlati and Lambertini (2011), the elasticity of substitution between consumption and housing services is set to 1. The same value is chosen for the elasticity of substitution between domestic and foreign goods in the consumption basket, following Gertler, Gali’ and Natalucci (2003). Furthermore, I set the share of housing services in the consumption bundle \((1 - \gamma_c)\) to 0.0950, so that in steady state, the imputed rents to consumption ratio is equal to 10.5, which is consistent with pre-crisis data of Central and Eastern European countries.\(^{38}\) Setting a depreciation

\(^{38}\)The ratio has been calculated dividing imputed rents by total consumption expenditures, for the period 2003-2007 (Eurostat data).
rate for the housing stock \((\delta_h)\) to 1% annually results to a steady state housing investment to output ratio of 1%, which is consistent with the average of 1.06% observed in the data.\(^{39}\)

Turning to the production side of the economy, I set the elasticity of substitution between different varieties of domestic goods to 6, implying a price markup of 20%. Following Ajevskis and Vitola (2011) and Merola (2010), I set the same elasticity of substitution for different varieties of labor. Furthermore, I set the price and wage stickiness parameters to 0.75, implying that prices and wages are adjusted, on average, every 4 quarters. The share of capital in production, \(\alpha\), is set to 0.35. Furthermore, the share of homeowners’ and entrepreneurial labor in production is set to 0.01.

The parameters in the benchmark model calibration are set following Bernanke, Gertler and Gilchrist (1999). In particular, the standard deviation of the idiosyncratic productivity shock of homeowners and entrepreneurs \((\sigma^H\text{ and }\sigma^E)\) are set to 0.28. The monitoring cost parameters are calibrated at 0.12, implying a quarterly default probability of homeowners and entrepreneurs of 0.87% (3.48% annually). This results in an external finance premium equal to 228 basis points on an annual basis and in a steady state leverage ratio of 0.5. I can then back out the survival probability of entrepreneurs, which is calibrated at 0.98. Finally, the elasticity of the transfer from homeowners to consumers is calibrated at 0.0526.\(^{40}\)

4 Simulation Results

4.1 Optimal Monetary Policy

In what follows, I consider the optimal monetary policy of a small open economy faced by technology and capital inflow shocks. Specifically, I consider different scenarios according to the objectives of the central bank, and the specific rule used to set the nominal interest rate. The central bank’s objective is represented by a loss function, which the monetary authority attempts to minimize. I first consider a setting in which the central bank is only concerned about stabilizing the real economy, and attempts to avoid excessive fluctuations in output and inflation. Furthermore, the central bank considers desirable to limit the volatility of the

\(^{39}\)Here, I used Eurostat data on gross capital formation in the construction sector as a proxy for investment in the real estate sector. Again, the average is computed over the period 2003-2007.

\(^{40}\)While, for firms, this calibration is largely consistent with the values set by Ajevskis and Vitola (2011) for Latvia, they report much higher leverage ratios for the household sector. Hence, I also calibrate the model in order to deliver a higher leverage ratio (equal to 2.5) of homeowners. This amounts to setting the monitoring cost and the idiosyncratic volatility parameters to 0.18 and 0.2053 respectively. While the steady state default probability is unchanged, the steady state external finance premium rises to 340 basis points annually. This, however, does qualitatively alter the results.
domestic interest rate, i.e. it will try to smooth changes in the monetary policy instrument. Hence, the loss function is defined as follows:

\[ L^{MS} = E_t \left[ \pi^2_t + \lambda_y \hat{\pi}_t^2 + \lambda_r \hat{R}_t^2 \right] \]

Where variables with a hat denote log deviations from steady state values. Furthermore, \( \lambda_y \) represents the relative weight the central bank places on output stability relative to inflation stability, and \( \lambda_r \) denotes the relative weight of interest rate smoothing.

In a second scenario, I consider the possibility that the central bank is also concerned with financial stability. In this setting, I specify the central bank’s loss function as being a positive function of the volatility of aggregate credit in addition to output, inflation and interest rate volatility. In this case, the loss function is defined as:\(^{41}\)

\[ L^{FS} = E_t \left[ \pi^2_t + \lambda_y \hat{\pi}_t^2 + \lambda_r \hat{R}_t^2 + \lambda_L \hat{L}_t^2 \right] \]

The central bank will then set the nominal interest rate as to minimize the relevant loss function. The implementation of monetary policy involves the policy instrument and the key variables the central bank monitors in order to achieve its objective. The instrument of monetary policy is the short-term interest rate, \( R_t \), which can be set according to different rules depending on the monetary policy strategy of the central bank. I consider four different policy rules.

In the first scenario, the central bank sets the interest rate according to a standard Taylor rule, reacting to deviations of output and inflation:

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \left( \frac{Y_t}{Y} \right)^{\rho_Y} \right]^{(1-\rho_R)} \exp(\xi_{R,t}) \quad (55) \]

Where variables without time subscript refer to steady state values.

The second rule I consider is one in which the central bank sets the nominal interest rate reacting to a financial aggregate. The issue is then to choose what financial indicator is more appropriate for inclusion in the central bank’s Taylor rule. Evidence presented by the IMF (2009) finds common patterns in economic variables in the period preceding an asset price bust. In particular, significant expansions in domestic credit and investment accompanied by current account deficits have been found to be recurrent in the run-up to a bust. Agénor and Pereira da Silva (2011) argue that in the context of middle income countries, central banks should conduct monetary policy by reacting to the economy’s credit growth gap. They claim that, in so doing, the central bank can offset the acceleration mechanism that leads to credit

\(^{41}\)In particular, I set \( \lambda_y = 0.1, \lambda_L = 0.1, \lambda_r = 0.05. \)
growth and asset price inflation that is at the heart of financial imbalances. In particular, during upturns, informational asymmetries between borrowers and lenders are enhanced, and the prevailing loosening of lending standards erodes the resilience of the country to financial distress. Furthermore, studies as Claessens et al. (2011) and Calderón and Fuentes (2011) affirm that credit aggregates are useful leading indicators of asset price bubbles. In particular, while credit booms are not necessarily conducive of a crisis, the evidence suggests that almost all crises are preceded by a credit boom. Hence, I consider a scenario where the central bank monitors the growth in loans in addition to output and inflation when setting the policy rate:

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \frac{\pi_t}{\pi} \right]^{\rho_\pi} \left( \frac{Y_t}{Y} \right)^{\rho_Y} \left( \frac{L_t}{L_{t-1}} \right)^{\rho_L} \exp (\xi_{R,t}) \right]^{(1-\rho_R)}$$ (56)

The third policy rule considered represents a situation where the central bank reacts to exchange rate movements. Rules of this kind have been widely considered for small open economies with a high degrees of dollarization, especially in light of the fact that many emerging economies engaged in exchange rate stabilization or opted for a fixed exchange rate regime. In particular, the main argument for exchange rate stabilization in this context relies on the fact that, when debt is denominated in foreign currency, exchange rate fluctuations affect the economy not only through trade, but also through balance sheet effects on borrowers.\(^{42}\) In this context, an exchange rate appreciation that, on one side, reduces exports with negative effects on aggregate demand, relaxes credit conditions of indebted agents, thereby stimulating further borrowing. Studies in this field\(^ {43}\) find that the suboptimality of exchange rate stabilization as a monetary policy strategy is strictly connected with the degree of openness of the economy (Devereux, Lane and Xu (2006)) and the source of the shock (Faia (2010)).

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \frac{\pi_t}{\pi} \right]^{\rho_\pi} \left( \frac{Y_t}{Y} \right)^{\rho_Y} \left( \frac{S_t}{S_{t-1}} \right)^{\rho_S} \exp (\xi_{R,t}) \right]^{(1-\rho_R)}$$ (57)

Finally, I consider the case in which the central bank pursues a fixed exchange rate regime:

$$\Delta S_t = 0$$ (58)

In particular, the fixed exchange rate case results as a special case of equation (57), where \(\rho_S \to \infty\).


Tables 1 and 2 summarize the results of the optimal monetary policy exercise. Table 1 shows the optimized Taylor rule coefficients and the corresponding value of the loss function in the case where the central bank’s objective is defined solely in terms of macroeconomic stability, while Table 2 refers to the case where the central bank’s loss function assigns a weight to credit growth, and hence the monetary policy authority is also concerned about financial stability. Furthermore, the optimized coefficients are calculated for three specifications of the Taylor rule: the inflation targeting case, corresponding to equation (55), inflation targeting and exchange rate stabilization (equation (57)) and inflation targeting augmented with credit growth (equation (56)). In the fixed exchange rate regime case, as the sole objective of the central bank is to keep the value of the currency unaltered, coefficients are not optimized, and I report the value of the loss function implied by the model. Columns 8, 9 and 10 report the standard deviations of inflation, output and loans corresponding to the relevant scenario.

Table 1: Optimal Taylor Rule coefficients with macroeconomic stability objective

<table>
<thead>
<tr>
<th>Inflation targeting</th>
<th>Inflation targeting with exchange rate smoothing</th>
<th>Inflation targeting with credit growth</th>
<th>Fixed exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_r )</td>
<td>( \phi_r )</td>
<td>( \phi_r )</td>
<td>( \phi_r )</td>
</tr>
<tr>
<td>0.752</td>
<td>0.755</td>
<td>0.7520</td>
<td>-</td>
</tr>
<tr>
<td>1.628</td>
<td>1.627</td>
<td>1.6285</td>
<td>-</td>
</tr>
<tr>
<td>0.954</td>
<td>0.959</td>
<td>0.954</td>
<td>-</td>
</tr>
<tr>
<td>na</td>
<td>0.084</td>
<td>na</td>
<td>-</td>
</tr>
<tr>
<td>na</td>
<td>na</td>
<td>na</td>
<td>-</td>
</tr>
<tr>
<td>0.4513</td>
<td>0.4516</td>
<td>0.4516</td>
<td>-</td>
</tr>
<tr>
<td>0.1981</td>
<td>0.1982</td>
<td>0.1982</td>
<td>-</td>
</tr>
<tr>
<td>0.2222</td>
<td>0.2218</td>
<td>0.2222</td>
<td>-</td>
</tr>
<tr>
<td>1.4531</td>
<td>1.4494</td>
<td>1.4530</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma_\pi )</td>
<td>( \sigma_\pi )</td>
<td>( \sigma_\pi )</td>
<td>( \sigma_\pi )</td>
</tr>
<tr>
<td>0.4513</td>
<td>0.4516</td>
<td>0.4516</td>
<td>-</td>
</tr>
<tr>
<td>0.1981</td>
<td>0.1982</td>
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<tr>
<td>0.2222</td>
<td>0.2218</td>
<td>0.2222</td>
<td>-</td>
</tr>
<tr>
<td>1.4531</td>
<td>1.4494</td>
<td>1.4530</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma_Y )</td>
<td>( \sigma_Y )</td>
<td>( \sigma_Y )</td>
<td>( \sigma_Y )</td>
</tr>
<tr>
<td>0.1981</td>
<td>0.1982</td>
<td>0.1982</td>
<td>-</td>
</tr>
<tr>
<td>0.2222</td>
<td>0.2218</td>
<td>0.2222</td>
<td>-</td>
</tr>
<tr>
<td>1.4531</td>
<td>1.4494</td>
<td>1.4530</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma_L )</td>
<td>( \sigma_L )</td>
<td>( \sigma_L )</td>
<td>( \sigma_L )</td>
</tr>
<tr>
<td>1.4531</td>
<td>1.4494</td>
<td>1.4530</td>
<td>-</td>
</tr>
</tbody>
</table>

Before examining the results in detail, three general remarks emerge from comparing tables 1 and 2. First, adding financial stability to the central bank’s objectives leads to optimal policy rules characterized by higher inertia. Given that the relevant risk free rate for foreign lenders is the foreign one, this result might be puzzling. However, in a small open economy, changes in the nominal interest rate are mirrored by exchange rate fluctuations, which impact the balance sheet of borrowers with foreign currency debt, leading to more volatility in financial variables, including credit growth. Therefore, a central bank concerned with financial stability finds it optimal to smooth movements in the monetary policy rate. Second, regardless of the specific Taylor rule, the optimal monetary policy with a financial
stability objective implies a stronger response to inflation and a more muted response to output. This allows to decrease simultaneously the volatility of inflation and credit growth, at the expense of increased output volatility.

Table 2: Optimal Taylor Rule coefficients with financial stability objective

<table>
<thead>
<tr>
<th></th>
<th>( \varphi_r )</th>
<th>( \varphi_\pi )</th>
<th>( \varphi_y )</th>
<th>( \varphi_S )</th>
<th>( \varphi_L )</th>
<th>Loss</th>
<th>( \sigma_\pi )</th>
<th>( \sigma_Y )</th>
<th>( \sigma_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation targeting</td>
<td>0.765</td>
<td>1.872</td>
<td>0.873</td>
<td>0.873</td>
<td>na</td>
<td>na</td>
<td>0.5398</td>
<td>0.1910</td>
<td>0.28</td>
</tr>
<tr>
<td>Inflation targeting with exchange rate targeting</td>
<td>0.839</td>
<td>1.848</td>
<td>0.996</td>
<td>2.998</td>
<td>na</td>
<td>na</td>
<td>0.5364</td>
<td>0.1944</td>
<td>0.2802</td>
</tr>
<tr>
<td>Inflation targeting with credit growth</td>
<td>0.773</td>
<td>1.863</td>
<td>0.898</td>
<td>1.704</td>
<td>0.5385</td>
<td>0.1923</td>
<td>0.2755</td>
<td>1.3234</td>
<td></td>
</tr>
<tr>
<td>Fixed exchange rate</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \infty )</td>
<td>-</td>
<td>3.917</td>
<td>0.0682</td>
<td>1.4767</td>
</tr>
</tbody>
</table>

Third, the fixed exchange rate regime delivers the highest value of the loss function. In spite inflation volatility is much lower when the central bank pegs the currency, the volatility of output is almost seven times as large. As I will illustrate in the next section, a fixed exchange rate delivers higher inflation stability by neutralizing the effect of exchange rate movements on import prices. However, as the currency is prevented from appreciating, export demand boosts production and yields a higher output volatility.

Looking more specifically at the case where the central bank focusses on macroeconomic stability (Table 1), it becomes immediately evident that engaging in either exchange rate smoothing or credit growth targeting does not lead to improvements in the loss function. As a matter of fact, the loss function even decreases when the central bank targets exchange rate movements. Furthermore, in both cases, the optimized coefficient is very small, implying that the inflation targeting taylor rule is the best among the considered alternatives. The reason why the central bank might not find it optimal to smooth currency fluctuations and react to credit developments is because such variables do not convey any additional information to the monetary authority than already incorporated in the dynamics of output and inflation. In fact, changes in the currency value directly affect inflation to the extent that the economy is open to trade and imports foreign goods. An exchange rate depreciation increases the domestic currency price of imports, thereby increasing inflation. However, as the central bank enacts a monetary tightening in the event of inflationary pressures, it also counteracts
the currency depreciation. In models with financial frictions and liability dollarization, an argument for exchange rate targeting has been proposed on the grounds that exchange rate fluctuations affect borrowers’ balance sheets, effect that can potentially overcome the positive effect of a devaluation on the trade balance. However, the majority of studies in the field concluded against exchange rate stabilization, claiming that the effect of the financial accelerator is more muted with a floating currency. A similar argument holds for credit growth: as loans increase, firms and households start investing and consuming more, leading to an expansion of domestic output and inflationary pressures. As such, increased credit growth does not add to the information set relevant for macroeconomic stability.

The results for the case in which the central bank includes financial stability among its objectives are shown in Table 2. Three results are worth highlighting. First, going from a simple inflation targeting to both alternative regimes leads to a reduction in the loss. Second, the optimized coefficients on currency depreciation and on credit growth are large, implying a strong optimal response of the central bank to developments in credit and currency markets. Third, the Taylor rule augmented with exchange rate stabilization is that yielding the smallest loss.

These results can be better understood by looking at the dynamic adjustment of the main model variables to productivity and capital inflow shocks under the optimized monetary policy rules, discussed in the next section.

4.2 Impulse responses

The impulse-response functions to productivity and capital inflow shocks obtained under the optimized monetary policy rules are presented in Figures 8, 9 and 10. The purpose of this analysis is twofold. First, it allows to gain clearer insights on the results presented in the previous section, by inspecting the macroeconomic adjustment under different optimized monetary policy rules. Secondly, it allows to shed light on the interplay between financial frictions in both the financing of capital and real estate investment, and analyze the transmission of shocks across sectors, which is at the heart of this study. In particular, I focus on technology (Figure 8) and capital inflow shocks. The latter are defined as positive shocks to the perception of foreign lenders about domestic borrowers’ productivity, and can affect either entrepreneurs (Figure 9) or homeowners (Figure 10).

4.2.1 Domestic technology shock

A positive, one standard deviation technology shock (depicted in Figure 8) implies an unexpected improvement in domestic firms’ productivity and an abatement of marginal costs. On one side, this leads to a decreased demand for labor which drives down employment and wages. The resulting decrease in labor income depresses households’ consumption of both goods and housing services. On the other hand, the reduction in marginal costs leads firms to revise prices downwards, lowering home goods inflation. The decline in the price of domestic goods has two consequences on external balance. As domestic goods are cheaper, on one side export demand rises, and on the other hand a substitution effect kicks in, which shifts domestic purchases towards home produced goods, causing a decrease in imports. As a result, the trade balance shifts to surplus, and the resulting net inflow of currency puts appreciation pressures on the exchange rate. The reaction of the central bank depends on the chosen monetary policy strategy. If the central bank follows a fixed exchange rate regime, it keeps the nominal interest rate unaltered; if it follows a taylor rule, it lowers the nominal interest rate in response to the decrease in inflation.

The overall macroeconomic adjustment and the behavior of financial variables in both the entrepreneurial and the homeowners’ sector crucially depend on the monetary policy regime, mainly through its effects on aggregate demand and on borrowers’ balance sheets. Concerning aggregate demand, the domestic technology shock exerts opposite effects on the demand of consumption goods and housing. While improved productivity leads to a decrease in firms’ demand of labor and hence a drop in wages, dampening domestic consumption (including housing services), external demand offsets the decline in consumption and boosts production, increasing firms’ demand for capital. Hence, while demand for capital investment rises, demand for real estate investment contracts. Furthermore, it is important to notice that under a fixed exchange rate regime, the increase in export demand is much more pronounced, leading to a sharper expansion in production leading firms to limit their cutback in labor demand, which counteracts the fall in domestic consumption through a more muted decline in wage income. In any case, as capital demand surges, entrepreneurs engage in more projects, demand more credit and more unfinished capital goods, pushing up their price. While the raise in credit demand puts upwards pressures on entrepreneurial leverage, the increase in the price of capital partially offsets the worsening of entrepreneurs’ balance sheet.

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However, the monetary policy regime directly affects credit markets through balance sheet effects arising from exchange rate fluctuations. The exchange rate appreciation occurring when the central bank follows a Taylor rule decreases the effective debt burden of entrepreneurs and homeowners. In the entrepreneurial case, this counteracts the increased demand for loans by lowering the (foreign currency) value of debt, pushing leverage below steady state values and lowering the external finance premium. In the fixed exchange rate case, the favorable exchange rate effect does not occur, hence the loan burden increases, and with it entrepreneurial leverage, dampening the overall acceleration and leading to a smoother increment in capital investment. Hence, the interaction between monetary policy regime and credit frictions affects entrepreneurs through two effects acting in opposite
directions. A fixed exchange rate regime boosts exports but dampens the financial accelerator mechanism. An inflation targeting regime limits the effect of external demand, but strengthens the financial accelerator.

The balance sheet effect on homeowners operates in a similar fashion. Under flexible exchange rate, the loan burden decreases, thereby leading to lower leverage and external finance premium for homeowners and encouraging new real estate projects that ultimately reduce the rental price of houses. Hence, the positive balance sheet effect is able to offset the initial decrease in housing demand caused by the drop in overall consumption, leading to increased housing investment. In the case of a fixed exchange rate regime, a similar positive effect on real estate investment occurs, but for different reasons. Here, the increase in export demand leads domestic firms to reduce employment by less, implying a more muted effect on households’ wage income and hence a smoother drop in consumption. Hence, demand for housing services decreases less markedly. Hence, loan demand from homeowners declines by less, as well as leverage and the external finance premium, leading to an increase in housing investment that is even higher than in the case of flexible exchange rate. Hence, also in the case of homeowners, the interaction between monetary policy regime and credit frictions exerts two effects acting in opposite directions. A fixed exchange rate regime impacts less on housing demand but dampens the financial accelerator mechanism. An inflation targeting regime implies a greater fall in rental housing demand, but strengthens the financial accelerator.

Hence, in the event of a technology shock, credit frictions at the entrepreneurial and homeowners’ level can lead to different scenarios concerning the co-movement of financial variables in the two sectors depending on the monetary policy regime. Under a fixed exchange rate regime, albeit a positive co-movement of investment and asset prices in the two sectors, financial variables exhibit a negative correlation. While external finance premia and leverage increase in the entrepreneurial sector, they decrease in the real estate sector. On the contrary, under a flexible exchange rate regime as in the three taylor rules considered, financial variables co-move following a technology shock. While in both cases the shock exerts an opposite effect on the demand for goods (positive) and housing services (negative), in the flexible exchange rate regime, the balance sheet effect is able to offset the increase in leverage caused by the increased loan demand, while this effect does not operate in the case of pegged currency.

Finally, it is worth noting how the three optimized policy rules do not imply large differences in the dynamic adjustment of real and financial variables. This is a direct consequence of the pattern of co-movement between financial variables in the entrepreneurial and homeowners’ sectors. Following the shock, while entrepreneurs demand more credit, homeowners
do not and, as a result, aggregate credit does not increase so much to warrant a stronger reaction of the central bank.

4.2.2 Capital inflow shock: entrepreneurs

Figure 9 depicts the responses to a positive increase in foreign lenders’ perception of entrepreneurs’ productivity under the three optimal policy rules and fixed exchange rate scenarios. As foreign lenders become more optimistic concerning the profitability of entrepreneurs, implying a smaller perceived probability of default, they loosen credit conditions. Hence, on impact, the external finance premium charged on domestic entrepreneurs decreases. As borrowing conditions improve and entrepreneur net worth rises, leverage declines. Therefore, the abatement of the cost of external finance prompts entrepreneurs to engage in new investment projects, and to demand more credit. As capital investment increases and with it the supply of capital, production surges, and so does domestic price inflation. Furthermore, the positive inflow of capital exerts appreciating pressures on the domestic currency (the exchange rate decreases). After this impact effect, the macroeconomic adjustment crucially depends on the monetary policy rule followed by the domestic central bank. Under any optimized Taylor rule, implying a flexible currency, the exchange rate appreciation leads to a decrease in the price of imports, which offsets the increase in domestic price inflation and leads to a decline in CPI inflation. Under an inflation targeting regime, as output rises above steady state values more than inflation contracts, the central bank raises the policy rate, thereby accommodating the exchange rate appreciation after the initial impact. While this doesn’t improve the country’s export performance, it has positive consequences on borrowers’ balance sheet and it is the key channel of transmission of the shock to the real estate investment sector. In fact, as the exchange rate appreciates, borrowing conditions of homeowners improve. As the debt burden decreases, and with it the external finance premium and leverage, investment in the real estate sector grows. Hence, the positive effect of the initial shock to entrepreneurs’ borrowing conditions positively spills over to homeowners through balance sheet effects, leading to a positive co-movement of financial variables across sectors. If the central bank engages in exchange rate targeting, it tries to offset the initial exchange rate appreciation and increases the nominal interest rate by a smaller amount. While this reduces the negative effect on exports and boosts aggregate demand, it somewhat dampens the positive balance sheet effect. However, even in this case the shock positively spills over to homeowners leading to an increase in housing investment and prices.
The case of inflation and credit growth targeting is the most interesting one. As illustrated in Table 2, the optimal policy in this case prescribes a strong reaction to overall credit growth. As the capital inflow shock boosts entrepreneurial demand for credit and the balance sheet effect on homeowners' debt is not strong enough, overall credit growth rises. Hence the central bank reacts by increasing the policy rate by a larger amount than under an inflation targeting and an exchange rate targeting regime. However, in case of foreign currency borrowing, the relevant opportunity cost of investment is the foreign interest rate, which is taken as given by the small open economy and stays constant. As the monetary policy tightening results in a stronger exchange rate appreciation, it strengthens borrowers’ balance sheets, leading to a sharper decrease in leverage and a stronger improvement in balance sheet conditions. Furthermore, the stronger currency appreciation harms competitiveness, leading to a more pronounced fall in export demand which offsets the increase in output driven by
the rise in domestic demand. As a result, in pursuing such a monetary policy strategy, the domestic central bank obtains results that conflict with its objectives. First, it does not succeed in smoothing credit developments as the economy is dollarized. Second, it offsets the positive effect of export demand on output, counteracting the expansionary effect of the capital inflow shock. Hence, it can achieve a better result in terms of macroeconomic and financial stabilization if it includes an exchange rate term in the taylor rule, as this policy limits the effect on the exchange rate appreciation and, at the same time, it encourages export demand. This is also evident by inspecting column 10 of Table 2. While the standard deviations of output and inflation are slightly higher under an inflation targeting regime with exchange rate smoothing, credit volatility is smaller. In particular, the volatility of aggregate loans is 8% than in the case of inflation and credit growth targeting.

4.2.3 Capital inflow shock: homeowners

Figure 10 reveals that the effect of a capital inflow to the homeowners’ sector implies a similar macroeconomic dynamics as a shock to credit to entrepreneurs. On impact, the shock reduces the external finance premium paid on real estate mortgages, thereby increasing the net worth of homeowners and reducing their leverage. As homeowners find it more convenient to invest in real estate projects, investment in the housing sector increases and house prices rise. The shock has a positive effect on overall consumption through three effects. On one side, the rise in housing prices boosts households’ wealth, encouraging consumption. On the other hand, the increased supply of finished housing lowers their rental price, boosting demand. Finally, as the domestic currency appreciates, imports become cheaper and overall CPI inflation declines, stimulating purchases. The increased demand for domestic goods for consumption and housing investment purchases stimulates production, which leads to a raise in demand for capital goods. Hence, as returns to capital increase, the entrepreneurial sector is stimulated to undertake new investment projects. While this increases leverage and the external finance premium under a fixed exchange rate regime, if the central bank follows a taylor rule, the appreciation of the domestic currency exerts a favorable balance sheet effect on entrepreneurial leverage. However, once again, when the central bank reacts to credit growth, the increase in the domestic interest rate is stronger, leading to a sharper decrease in exports which depress demand. In this case, production remains below steady state for quite some time after the shock. As a result, in spite of the decrease in entrepreneurial leverage, capital investment is negatively affected by the decline in capital demand by firms. On the contrary, when the central bank engages in exchange rate smoothing, it prevents a sharp fall in exports while still allowing for a positive balance sheet effect arising from the currency appreciation. Hence, the growth in domestic demand is able to compensate the fall
in foreign demand, leading to an expansion of production.

Figure 10: Responses to a 0.01 standard deviation capital inflow (homeowners) shock, Model 2, optimal monetary policies under financial stability objective

Under the fixed exchange rate regime, the value of the currency is kept constant. This implies that the consumer price index does not benefit from the effect of the exchange rate appreciation, as the price of foreign goods is not affected. This, combined with the increase in domestic prices due to demand pressures implies that overall consumer price inflation rises. Furthermore, as the exchange rate does not appreciate, exports are only affected to the extent that the price of domestic goods rises. However, the fall in exports is negligible, and does not significantly counteract the increase in domestic demand, leading to higher production and hence higher capital investment. Furthermore, as balance sheet effects of exchange rate fluctuations are absent in this setting, entrepreneurial borrowing conditions
are worse than in the case of flexible currency. Therefore, after an initial decrease in leverage and premium due to the increase in asset prices, as capital investment peaks leverage rises, albeit to a small extent. Hence, once again the exchange rate regime determines the extent of co-movement between sectorial borrowing conditions in the small dollarized economy. While a monetary regime implying a flexible exchange rate leads to positive co-movement, when the currency is pegged, the correlation weakens and slightly reverts direction.

5 Conclusion

The recent financial crisis swiftly reverted the path of economic growth experienced by the emerging European countries in the years preceding the turmoil. In particular, the credit crunch interrupted the flow of foreign capital that, since the beginning of the new century, encouraged credit growth and fueled consumption and investment in both the production and real estate sector. However, the pre-crisis path of economic expansion was characterized by the build-up of vulnerabilities, mainly related to increased leverage, asset prices and large shares of private sector debt denominated in foreign currency, which exacerbated the impact of the downturn. Hence, it became clear that imbalances built-up in good times magnify the intensity of a slowdown. Furthermore, policymakers became increasingly aware of the fact that a monetary policy committed to guaranteeing an environment of stable inflation and economic growth is not necessarily conducive of financial stability. Therefore, academics and policymakers began to reconsider the potential role of financial stability in monetary policy making. In particular, the current debate centers on the issue as to whether central banks should amend their objectives to include financial, in addition to price, stability and, if so, monetary policy should be conducted by explicitly reacting to indicators of financial overheating.

In this paper, I analyze the interplay between financial frictions at the household and firm level, liability dollarization and monetary policy in a small open economy subject to capital inflow shocks. In particular, I focus on the interaction of firm and household leverage in the transmission of shocks to domestic technology and capital inflows, under optimized monetary policy rules. In particular, I consider three specifications of the monetary policy rule that have been widely considered in the literature for emerging economies (i.e. inflation targeting, exchange rate targeting and fixed exchange rate) and a Taylor rule reacting to credit growth. Furthermore, I analyze the optimized monetary policy rules under two specifications of the central banks’ objectives, namely macroeconomic stability and macroeconomic plus financial stability.

As a first step, I compare the optimized monetary policy rules under the two central
bank’s objectives. I find that, adding financial stability to the central bank’s objectives results in more inertial rules. Given that the relevant risk free rate for foreign lenders is the foreign one, this result might be puzzling. However, in a small open economy, changes in the nominal interest rate are mirrored by exchange rate fluctuations, which impact the balance sheet of borrowers with foreign currency debt, leading to more volatility in financial variables, including credit growth. Therefore, a central bank concerned with financial stability finds it optimal to smooth movements in the monetary policy rate. Furthermore, compared to the case where the central bank only cares about macroeconomic stability, the optimized Taylor rules under the financial stability objective achieve a lower volatility of inflation and of credit growth at the same time. However, this comes at the expense of a higher standard deviation of production. Finally, regardless of the central bank’s objective, a fixed exchange rate delivers the highest value of the loss function. In particular, while it implies a much reduced variability of inflation (mainly through the effect of exchange rate movements on the price of imported goods), output and credit are much more volatile. This is mainly due to the fact that pegging the currency avoids the dampening of exports that occurs when the currency is allowed to appreciate, with a larger effect on domestic production and the demand for credit to finance capital and real estate investments.

Concerning the optimized rules more specifically, some interesting results emerge. When macroeconomic stability is the only central bank’s concern, my findings largely mirror those of the literature. In this context, the optimized coefficients on credit growth and exchange rate are very close to zero the monetary policy authority’s best strategy is to engage in traditional inflation targeting. On the contrary, when financial stability is included among the central bank’s objectives, the optimized coefficients on credit growth and exchange rate are positive and large. However, engaging in exchange rate smoothing delivers the smallest value of the central bank’s loss function, mainly arising through a much reduced volatility of the credit aggregate. The reason why reacting to the credit aggregate is suboptimal in a small dollarized economy relates to the effect of the stronger interest rate increase that further appreciates the currency and reinforces, instead of dampening, the financial accelerator. Hence, contrary to the case where the central bank pursues price and output stability objectives, some degree of exchange rate smoothing is beneficial in this context.

Finally, this framework allows to draw interesting insights on the interaction of firm and household leverage in an open economy setting, on the transmission of shocks, and on the role of the monetary policy regime in shaping it. In the case of both technology and capital inflow shocks, the extent of co-movement of financial variables pertaining to entrepreneurs and homeowners crucially depends on whether the exchange rate is flexible or pegged. Specifically, under a fixed exchange rate regime, a negative correlation arises, i.e. stronger
balance sheet conditions of entrepreneurs lead to weakened or virtually unchanged balance sheet conditions for homeowners. Under a fixed exchange rate regime, a positive correlation of financial variables of the two types of borrowers arises, mainly operating through the balance sheet effect of exchange rate fluctuations. More specifically, a positive domestic productivity shock exerts opposite effects on capital and housing investment: while housing demand decreases (through a general decline in consumption demand due to lower wage income), capital demand increases because of increased production and external demand. Ceteris paribus, this leads to a fall in homeowners’ leverage and a surge in entrepreneurial leverage. While this happens in the case of fixed exchange rate, under a taylor rule the shock leads to a domestic currency appreciation, which strengthens the balance sheet of borrowers and offsets the opposite effect on investment demand in the two sectors.

In case of capital inflow shocks, similar conclusions can be drawn concerning the interaction between the monetary policy regime and the dynamics of financial variables across sectors. Furthermore, the analysis reveals that sectorial capital inflow shocks spill over to the other sector mainly through their effect on domestic production through increased demand of domestic goods used for investment purposes, and through balance sheet effects of currency appreciation.

The presented analysis can be extended in numerous directions, which will be explored in future research. First, in the presented model, I assumed that all debt is denominated in foreign currency, which is of course an extreme case. Hence, future work will focus on allowing for a less then full degree of foreign currency borrowing, and to examining the implications for monetary policy under different degrees of liability dollarization and economic openness. Second, asset price movements (including real estate prices) have significant consequences on banks’ balance sheets, which played a key role in its international transmission. As U.S. homeowners weren’t able to repay their mortgages, banks seized the collateral (houses) and tried to sell them, pushing real estate prices sharply downwards. This decreased, on one side, the wealth of households owning a house and, on the other side, the value of assets on banks’ balance sheets, leading them to cut lending both domestically and internationally, in the best case scenario. In the worst case scenario they declared default. In this light, extending the model to a two country setting featuring banks engaging in cross-border activities would allow to study issues related to the international transmission of real estate price shocks as well as the effect of policies aimed at regulating the banking sector. Furthermore, this setting would allow to study the interplay between monetary and prudential policies both within a country and from an international cooperation perspective.
References


6 Appendix

6.1 Derivation of the optimal contract between borrowers and lenders

Here I solve for the optimal credit contract. As entrepreneurs face the same problem as homeowner, in what follows I solve the optimization problem faced by the latter. Analogous first order conditions apply to the entrepreneurial sector. The first order conditions of the optimal contract are obtained by maximizing the expected payoff of homeowners subject to the lenders’ participation constraint:

$$\max_{\omega_{t+1}, H_{t+1}} E_t \left[ Q_{h,t} H_{t+1} R_{t+1}^h A^H (\bar{\omega}^H_{t+1}) \right]$$

Subject to:

$$Q_{h,t} H_{t+1} R_{t+1}^h B^H (\bar{\omega}^H_{t+1}, v^H_t) = R_t (Q_{h,t} H_{t+1} - NW^H_t)$$

Lagrangian:

$$L = E_t \left\{ Q_{h,t} H_{t+1} R_{t+1}^h A^H (\bar{\omega}^H_{t+1}) + \vartheta_{t+1} \left[ Q_{h,t} H_{t+1} R_{t+1}^h B^H (\bar{\omega}^H_{t+1}, v^H_t) - R_t (Q_{h,t} H_{t+1} - NW^H_t) \right] \right\}$$

Where $\vartheta_{t+1}$ is the Lagrange multiplier on the participation constraint. The first order conditions are:
\[ \frac{\partial L}{\partial \omega_{H+1}^t} = E_t \left\{ Q_{h,t} H_{t+1} R_{t+1}^h A^H(\omega_{t+1}^H) + \partial_{t+1} Q_{h,t} H_{t+1} R_{t+1}^h B^H(\omega_{t+1}^H, v_t^H) \right\} = 0 \]

\[ \Rightarrow E_t \left\{ Q_{h,t} H_{t+1} R_{t+1}^h A^H(\omega_{t+1}^H) \right\} = -E_t \left\{ \partial_{t+1} Q_{h,t} H_{t+1} R_{t+1}^h B^H(\omega_{t+1}^H, v_t^H) \right\} \]

\[ \Rightarrow \partial_{t+1} = E_t \left[ -\frac{A^H(\omega_{t+1}^H)}{B^H(\omega_{t+1}^H, v_t^H)} \right] \]

\[ \frac{\partial L}{\partial H_{t+1}} = E_t \left\{ Q_{h,t} R_{t+1}^h A^H(\omega_{t+1}^H) + \partial_{t+1} Q_{h,t} R_{t+1}^h B^H(\omega_{t+1}^H, v_t^H) - \partial_{t+1} R_t Q_{h,t} \right\} = 0 \]

\[ \Rightarrow E_t \left\{ R_{t+1}^h \left[ B^H(\omega_{t+1}^H, v_t^H) - \frac{B^H(\omega_{t+1}^H, v_t^H)}{A^H(\omega_{t+1}^H)} A^H(\omega_{t+1}^H) \right] \right\} = R_t \]

\[ \Rightarrow E_t \left\{ R_{t+1}^h \left[ \frac{B^H(\omega_{t+1}^H, v_t^H) A^H(\omega_{t+1}^H) - B^H(\omega_{t+1}^H, v_t^H) A^H(\omega_{t+1}^H)}{A^H(\omega_{t+1}^H)} \right] \right\} = R_t \]

\[ \Rightarrow E_t \left\{ R_{t+1}^h \right\} = R_t \left( \frac{A^H(\omega_{t+1}^H)}{B^H(\omega_{t+1}^H, v_t^H) A^H(\omega_{t+1}^H) - B^H(\omega_{t+1}^H, v_t^H) A^H(\omega_{t+1}^H)} \right) \]

\[ \frac{\partial L}{\partial \theta_{t+1}} = 0 \Rightarrow Q_{h,t} H_{t+1} R_{t+1}^h B^H(\omega_{t+1}^H, v_t^H) = R_t (Q_{h,t} H_{t+1}^H - NW_t^H) \]

### 6.2 Steady State

In the steady state, I set \( S = 1 \) and \( A = 1 \). Furthermore, all relative prices are set to 1, so as all inflation rates: \( \frac{p^d}{p^c} = 1 \), \( \frac{p^f}{p^c} = 1 \), \( \frac{p^h}{p^c} = 1 \), \( \pi = \pi^H = \pi^F = 1 \). Also the parameters representing lenders’ misperception of borrowers’ productivity are set to 1 in steady state: \( v_t^H = v_t^E = 1 \). The Euler equation implies, together wit the assumption that domestic and foreign interest rates are equal in steady state:

\[ R = R_* = \frac{1}{\beta} \]

From which it follows that

\[ \Psi = 1 \]

Equation (25) and its counterpart for housing investment imply that in steady state:

\[ Q_h = Q_h = 1 \]
The steady state of the credit market is computed assuming target values for three quantities: (1) The risk premium \((R^j - R)\), (2) The leverage ratio of borrowers and (3) the annualized default probability of borrowers \(F(\bar{\omega}^j)\) for both entrepreneurs and homeowners. I choose the value of parameters related to monitoring costs in the contract between financial intermediaries and entrepreneurs \((\mu^E)\) and between financial intermediaries and homeowners \((\mu^H)\), volatility of the idiosyncratic shocks \((\sigma^E, \sigma^H)\), steady state threshold productivity levels \(\bar{\omega}^E, \bar{\omega}^H\) and the survival rate of entrepreneurs \((\gamma^E)\) to match the aforementioned steady state quantities.

Given values of \(\sigma^E, \sigma^H\) and a target value for the default probability in each sector \(F(\bar{\omega}^E; \sigma^E), F(\bar{\omega}^H; \sigma^H)\) I can calculate the threshold productivity levels:

\[
\bar{\omega}^j = Ncdf^{-1}\left(\frac{\log \bar{\omega}^j + 0.5\sigma_j^2}{\sigma_j}\right)
\]

Which I can now use to calculate the quantities \(A(\bar{\omega}^j), A'(\bar{\omega}^j), B(\bar{\omega}^j)\) and \(B'(\bar{\omega}^j)\). I can then calculate the steady state external finance premium in both sectors:

\[
EFP^j = \frac{A^j(\bar{\omega}^j)}{B^j(\bar{\omega}^j) A^j(\bar{\omega}^j) - B^j(\bar{\omega}^j) A^E(\bar{\omega}^j)}
\]

From which it follows that

\[
R^E = EFP^E \cdot R
\]
\[
R^h = EFP^H \cdot R
\]

Using (22) and (31) and denoting \(lev^H = \frac{Q^H}{NW^H}\) and \(lev^E = \frac{Q^E}{NW^E}\):

\[
lev^H = \frac{1}{1 - B(\bar{\omega}^H) EFP^H}
\]
\[
lev^E = \frac{1}{1 - B(\bar{\omega}^E) EFP^E}
\]

I can now compute the rental rate of capital and the steady state share of housing services, using (28) and (16):

\[
r^K = R^E - (1 - \delta_k)
\]
\[
s = R^h - (1 - \delta_h)
\]
I now turn to the production side of the economy. From (45) (47) and (46) it results that:

\[ MC = \frac{\varepsilon - 1}{\varepsilon} \]

Then, using the equation defining the return to capital:

\[ \frac{K}{Y} = \frac{MC \cdot \alpha}{r^K} \]

And from the production function:

\[ \frac{K}{N} = \left( \frac{K}{Y} \right)^{\frac{1}{1-\alpha}} \]

Then using the labor demand equation:

\[ W = MC \cdot (1 - \alpha) \left( \frac{K}{N} \right)^{\alpha} \]

Fixing total labor supply at \( \frac{1}{3} \) of available time, \( N = 0.33 \) allows to compute\(^{45}\):

\[ K = \frac{K}{N} \cdot N \]
\[ Y = K \cdot \left( \frac{K}{Y} \right)^{-1} \]
\[ F_H = \frac{MC \cdot Y}{(1 - \beta \theta)} \]
\[ D_H = \frac{Y}{(1 - \beta \theta)} \]
\[ \tilde{p}^H = \frac{\varepsilon}{\varepsilon - 1} \frac{F_H}{D_H} \]
\[ W^E = MC (1 - \alpha) \Omega_E Y \]
\[ W^H = MC (1 - \alpha) \Omega_F Y \]

Equipped with \( K \), I can solve for investment and the expressions related to entrepreneurial net worth and consumption:

\(^{45}\)Recall that homeowners and entrepreneurs supply one unit of labor inelastically, hence \( N^H = N^E = 1 \).
\[ \begin{align*}
NW^E &= \frac{K}{l_{eV}E} \\
V^E &= R^E Q_k K A(\bar{\omega}^E) \\
\bar{\pi} &= \frac{NW^E - (1 - \alpha) \Omega^E MC \cdot Y}{V^E} \\
L^E &= Q_k K - NW^E \\
R^E_L &= \frac{\bar{\omega}^E R^E}{1 - \frac{NW^E}{Q_k K}} \\
C^E &= (1 - \bar{\pi}) R^E Q_k K A(\bar{\omega}^E)
\end{align*} \]

Now, fixing the steady state government expenditure at 20% of GDP: \( G = 0.2 \cdot Y \), I can use the national accounting identity, and back out the steady state value of the housing stock:

\[
H = \frac{(Y - G - \mu^E F \left( \frac{\ln \bar{\omega}^E - 0.5 \sigma_h^2}{\sigma_h} \right) \cdot R^E Q_k K - \delta_h K - C^E)}{\frac{\gamma_c}{1 - \gamma_c} s P^h + \delta_h + \mu^H F \left( \frac{\ln \bar{\omega}^H - 0.5 \sigma_h^2}{\sigma_h} \right) \cdot R^h Q_h}
\]

Which I can now use to calculate:

\[
\begin{align*}
NW^H &= \frac{H}{l_{eV}^H} \\
V^H &= R^h Q_h H A(\bar{\omega}^H) \\
D &= V^H + (1 - \alpha) \Omega^H MC \cdot Y - NW^H \\
\chi_D &= D \cdot l_{eV}^H \\
L^H &= Q_h H - NW^H \\
R^H_L &= \frac{\bar{\omega}^H R^h}{1 - \frac{NW^H}{Q_h H}} \\
h &= s H \\
c &= \frac{\gamma_c}{1 - \gamma_c} h \left( P^h \right)^\zeta \\
I_h &= \delta_h H
\end{align*}
\]

I set consumption of Ricardian and non-Ricardian consumers equal in steady state:

\[ C^R = C^{NR} = C \]

Then:
$$T^{NR} = D + W \cdot N - C^{NR}$$
$$T^R = G - T^{NR}$$

Now I can solve for the steady state of the labor market:

$$\chi_N = \frac{1}{C} W \cdot N^{-\varphi} \frac{\varepsilon_w - 1}{\varepsilon_w}$$
$$K^w = \frac{\varepsilon_w - 1}{\varepsilon_w} W \frac{N}{C (1 - \beta \theta_w)}$$
$$F^w = \frac{\chi_N N (1 + \varphi)}{(1 - \beta \theta_w)}$$

Finally, I set the net foreign asset position in steady state equal to zero, meaning a balanced current account in steady state:

$$B^* = 0$$
$$X = c^F + C^E + I^F_k + I^F_h$$
$$Y^* = \frac{X}{\gamma^*}$$

### 6.3 Derivations of $A(\bar{\omega})$, $B(\bar{\omega}, \nu)$, $A'(\bar{\omega})$, $B'(\bar{\omega}, \nu)$

In order to compute the external finance premia (cfr equations (21) and (22) in the text), I have to compute exact expressions for the quantities $A^j(\omega_{i+1}^j)$ and $B^j(\omega_{i+1}^j, \nu_i^j)$, where $j = H, E$. For notational simplicity, in what follows I will omit the time subscript and the superscript $j$. Recall:

$$A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega)d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega)d\omega$$  \hspace{1cm} (59)$$

$$B(\bar{\omega}, \nu) = \bar{\omega} \int_{\bar{\omega}}^{\infty} f^*(\omega)d\omega^* + (1 - \mu) \int_{0}^{\bar{\omega}} \omega^* f^*(\omega)d\omega^*$$  \hspace{1cm} (60)$$

The idiosyncratic shock $\omega$ is i.i.d., and is assumed to have log-normal distribution with parameters $\mu$ and $\sigma_\omega$: $\omega \sim \log N(\mu, \sigma^2_\omega)$. The density function at $\bar{\omega}$ is:
\[ f_\omega(\bar{\omega}) = \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} \exp \left\{ -\frac{(\ln \bar{\omega} - \mu)^2}{2\sigma_\omega^2} \right\} = \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} - \mu}{\sigma_\omega} \right) \] (61)

Where \( \phi(\cdot) \) is the standard normal pdf. The corresponding cumulative distribution function is:

\[ F_\omega(\bar{\omega}; \mu, \sigma_\omega) = \int_{-\infty}^{\ln \bar{\omega}} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp \left\{ -\frac{(y - \mu)^2}{2\sigma_\omega^2} \right\} \, dy = \int_{-\infty}^{\frac{\ln \bar{\omega} - \mu}{\sigma_\omega}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \, dt = \Phi \left( \frac{\ln \bar{\omega} - \mu}{\sigma_\omega} \right) \] (62)

Where \( \Phi \) is the standard normal cdf. The partial expectation is computed as:

\[
\int_{0}^{\infty} \omega f(\omega) d\omega = \int_{0}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(\ln \omega - \mu)^2} d\omega = \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y - \mu)^2} e^y \, dy = \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y^2 + \mu^2 + 2\mu y - 2\sigma_\omega^2 y)} \, dy = \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y^2 + \mu^2 - 2\mu y + \sigma_\omega^2)} \, dy = \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y^2 + \mu^2 - 2\mu y + \sigma_\omega^2)} \, dy = \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y^2 + \mu^2 - 2\mu y + \sigma_\omega^2)} \, dy = \exp \left( \mu + \frac{1}{2} \sigma_\omega^2 \right) \int_{\ln \omega}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2\sigma_\omega^2}(y^2 + \mu^2 - 2\mu y + \sigma_\omega^2)} \, dy = \exp \left( \mu + \frac{1}{2} \sigma_\omega^2 \right) \Phi \left( \frac{-\ln \omega + \mu + \sigma_\omega^2}{\sigma_\omega} \right) \] (63)

And

\[
\int_{0}^{\infty} \omega f(\omega) d\omega = \mathbb{E}(\omega | \omega \leq \bar{\omega}) \Pr(\omega \leq \bar{\omega}) = \exp \left( \mu + \frac{1}{2} \sigma_\omega^2 \right) \Phi \left( \frac{\ln \bar{\omega} - \mu - \sigma_\omega^2}{\sigma_\omega} \right) \] (64)

In order to derive the necessary expressions for (60), recall that lenders have a distorted perception of homeowners’ productivity, defined as \( \omega^* = \omega \nu \) where \( \nu \) is the misperception.
factor and $f^*(\omega)$ and $F^*(\omega)$ denote respectively the pdf and cdf of the productivity parameter as perceived by lenders. Hence,

$$F_{\omega^*}(\tilde{\omega}; \mu, \sigma_{\omega}) = \Pr (\omega^* \leq \tilde{\omega}) = \Pr (\omega \leq \tilde{\omega}) = \Pr \left( \omega \leq \frac{\tilde{\omega}}{v} \right) =$$

$$= \int_{0}^{\frac{\tilde{\omega}}{v}} \frac{1}{\omega \sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \omega - \mu}{\sigma_{\omega}} \right)^2} d\omega =$$

$$= \int_{-\infty}^{\frac{\ln \tilde{\omega}}{\sigma_{\omega}}} \frac{1}{\sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{y - \mu}{\sigma_{\omega}} \right)^2} dy = \Phi \left( \frac{\ln \tilde{\omega}/v - \mu}{\sigma_{\omega}} \right)$$

The partial expectation is:

$$\int_{0}^{\tilde{\omega}} \omega^* f(\omega^*) d\omega^* = E (\omega^*|\omega^* \leq \tilde{\omega}) \Pr (\omega^* \leq \tilde{\omega}) =$$

$$= E \left( \omega | \omega \leq \frac{\tilde{\omega}}{v} \right) \Pr \left( \omega \leq \frac{\tilde{\omega}}{v} \right) =$$

$$= \exp \left( \mu + \ln v + \frac{1}{2} \sigma_{\omega}^2 \right) \Phi \left( \frac{\ln \tilde{\omega}/v - \mu - \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right)$$

And, plugging (62) and (63) into (59):

$$A(\tilde{\omega}) = \exp \left( \mu + \frac{1}{2} \sigma_{\omega}^2 \right) \Phi \left( -\ln \tilde{\omega} + \mu + \frac{\sigma_{\omega}^2}{2} \right) - \tilde{\omega} \left( 1 - \Phi \left( \frac{\ln \tilde{\omega} - \mu}{\sigma_{\omega}} \right) \right)$$

Likewise, plugging (65) and (68) into (60):

$$B(\tilde{\omega}, v) = \tilde{\omega} \left( 1 - \Phi \left( \frac{\ln \tilde{\omega}/v - \mu}{\sigma_{\omega}} \right) \right) + (1 - \mu) \left[ \exp \left( \mu + \ln v + \frac{1}{2} \sigma_{\omega}^2 \right) \Phi \left( \frac{\ln \tilde{\omega}/v - \mu - \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \right]$$

In the particular case in which $E(\omega) = 1$ so that $\mu = -\frac{\sigma_{\omega}^2}{2}$ so that $\log \omega \sim N(-\frac{\sigma_{\omega}^2}{2}, \sigma_{\omega}^2)$ I obtain:

$$A(\tilde{\omega}) = \exp \left( -\frac{\sigma_{\omega}^2}{2} + \frac{1}{2} \sigma_{\omega}^2 \right) \Phi \left( -\ln \tilde{\omega} + \frac{\sigma_{\omega}^2}{2} + \frac{\sigma_{\omega}^2}{2} \right) - \tilde{\omega} \left( 1 - \Phi \left( \frac{\ln \tilde{\omega} + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \right) =$$

$$= 1 - \Phi \left( \frac{\ln \tilde{\omega} + \frac{\sigma_{\omega}^2}{2} - \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) - \tilde{\omega} \left( 1 - \Phi \left( \frac{\ln \tilde{\omega} + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \right)$$

(69)
\[ B(\bar{\omega}, v) = \bar{\omega} \left( 1 - \Phi \left( \frac{\ln \left( \frac{\bar{\omega}}{v} \right) + \frac{\sigma^2}{2}}{\sigma_\omega} \right) \right) + (1 - \mu) \left[ \exp \left( -\frac{\sigma^2}{2} + \ln v + \frac{1}{2} \sigma^2 \right) \Phi \left( \frac{\ln \left( \frac{\bar{\omega}}{v} \right) + \frac{\sigma^2}{2} - \frac{\sigma^2}{\sigma_\omega}}{\sigma_\omega} \right) \right] \]

\[ B(\bar{\omega}, v) = \bar{\omega} \left( 1 - \Phi \left( \frac{\ln \left( \frac{\bar{\omega}}{v} \right) + \frac{\sigma^2}{2}}{\sigma_\omega} \right) \right) + (1 - \mu) \cdot v \cdot \Phi \left( \frac{\ln \left( \frac{\bar{\omega}}{v} \right) + \frac{\sigma^2}{2} - \frac{\sigma^2}{\sigma_\omega}}{\sigma_\omega} \right) \]  

(70)

Now I have to compute \( A'(\bar{\omega}) = \frac{\partial}{\partial \bar{\omega}} A(\bar{\omega}) \) and \( B'(\bar{\omega}, v) = \frac{\partial}{\partial \bar{\omega}} B(\bar{\omega}, v) \).

Recall that in general:

\[ G(x) = \int_{x_0}^{f(x)} g(t) \, dt \Rightarrow G'(x) = g(f(x)) \cdot f'(x) \]

Furthermore, denoting as \( \phi(\cdot) \) the standard normal pdf, note that:

\[ \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2} - \frac{\sigma^2}{\sigma_\omega}}{\sigma_\omega} \right) = \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2} - \frac{\sigma^2}{\sigma_\omega}}{\sigma_\omega} \right)^2} = \]

\[ = \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{(\ln \bar{\omega})^2 + \left( \frac{\sigma^2}{2} \right)^2 + 2(\ln \bar{\omega}) \cdot \frac{\sigma^2}{2}}{\sigma_\omega^2} - 2(\ln \bar{\omega}) \cdot \frac{\sigma^2}{\sigma_\omega^2} - 2 \cdot \frac{\sigma^2}{2} \cdot \frac{\sigma^2}{\sigma_\omega^2} \right)} = \]

\[ = \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{(\ln \bar{\omega} + \frac{\sigma^2}{2})^2}{2\sigma_\omega^2} + \ln \bar{\omega} \right)} = \]

\[ = \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) = \]

\[ = \bar{\omega} \cdot \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) = \]

So that:
\[ A'(\bar{\omega}) = - \left[ \frac{d}{d\bar{\omega}} \int_0^{\bar{\omega}} \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \omega + \frac{\sigma^2}{2} - \sigma^2_\omega}{\sigma_\omega} \right)^2} d\omega \right] - 1 + \Phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) + \\
+ \bar{\omega} \cdot \frac{d}{d\bar{\omega}} \int_0^{\bar{\omega}} \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \omega + \frac{\sigma^2}{2} - \sigma^2_\omega}{\sigma_\omega} \right)^2} d\omega \\
A'(\bar{\omega}) = - \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} \exp \left[ - \frac{1}{2} \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2} - \sigma^2_\omega}{\sigma_\omega} \right)^2 \right] - 1 + \Phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) + \\
+ \bar{\omega} \cdot \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} \exp \left[ - \frac{1}{2} \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right)^2 \right] \\
A'(\bar{\omega}) = - \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2} - \sigma^2_\omega}{\sigma_\omega} \right) - 1 + \Phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) + \\
+ \bar{\omega} \left[ \frac{1}{\bar{\omega} \sigma_\omega} \phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) \right] \\
A'(\bar{\omega}) = -1 + \Phi \left( \frac{\ln \bar{\omega} + \frac{\sigma^2}{2}}{\sigma_\omega} \right) \quad (71) \]
And

\[ B'(\tilde{\omega}, v) = 1 - \Phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) - \tilde{\omega} \cdot \frac{1}{\tilde{\omega} v \sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] \cdot \frac{1}{v} + \]

\[ + (1 - \mu) \cdot v \cdot \frac{1}{\tilde{\omega} v \sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2} - \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] \cdot \frac{1}{v} \]

\[ B'(\tilde{\omega}, v) = 1 - \Phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) - \tilde{\omega} \cdot \frac{1}{\tilde{\omega} v \sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] + \]

\[ + (1 - \mu) \cdot v \cdot \tilde{\omega} \cdot \frac{1}{\tilde{\omega} v \sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] \cdot \frac{1}{v} \]

\[ B'(\tilde{\omega}, v) = 1 - \Phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) - \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] + \]

\[ + (1 - \mu) \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] \]

\[ B'(\tilde{\omega}, v) = 1 - \Phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) - \mu \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right)^2 \right] \]

\[ B'(\tilde{\omega}, v) = 1 - \Phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) - \frac{\mu}{\sigma_\omega} \phi \left( \frac{\ln (\tilde{\omega} v) + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) \]

Now I calculate the second derivatives \( A''(\tilde{\omega}) = \frac{\partial}{\partial \tilde{\omega}} A'(\tilde{\omega}) \) and \( B''(\tilde{\omega}, v) = \frac{\partial}{\partial \tilde{\omega}} B'(\tilde{\omega}, v) \):

\[ A''(\tilde{\omega}) = \frac{d}{d\tilde{\omega}} \int_0^{\tilde{\omega}} \frac{1}{\omega \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln (\omega + \frac{\sigma^2_2}{2})}{\sigma_\omega} \right)^2} d\omega = \frac{1}{\tilde{\omega} \sigma_\omega} \phi \left( \frac{\ln \tilde{\omega} + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) \]

\[ \int_0^{\tilde{\omega}} \frac{1}{\omega \sigma_\omega \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln (\omega + \frac{\sigma^2_2}{2})}{\sigma_\omega} \right)^2} d\omega = \frac{1}{\tilde{\omega} \sigma_\omega} \phi \left( \frac{\ln \tilde{\omega} + \frac{\sigma^2_2}{2}}{\sigma_\omega} \right) \]

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For the log-linearized model, I need the following derivatives: \( \frac{d}{d\tilde{\omega}} B(\tilde{\omega}, v) \) and \( \frac{d}{d\tilde{\omega}} B'(\tilde{\omega}, v) \)

\[
\begin{align*}
B''(\tilde{\omega}, v) &= -\frac{d}{d\tilde{\omega}} \int_{0}^{\tilde{\omega}} \frac{1}{\omega \sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \omega + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} d\omega - \frac{\mu}{\sigma_{\omega}} \cdot \frac{d}{d\tilde{\omega}} \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \tilde{\omega} + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} \right) \\
B''(\tilde{\omega}, v) &= -\left[ \frac{1}{\tilde{\omega} \sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \tilde{\omega} + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} \right] \cdot \frac{1}{v} - \frac{\mu}{\sigma_{\omega}} \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \tilde{\omega} + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} \cdot \left( \frac{\ln \frac{\tilde{\omega}}{v} + \frac{\sigma_{\omega}^{2}}{2}}{\sigma_{\omega}} \right) \right) \\
B''(\tilde{\omega}, v) &= -\left[ \frac{1}{\tilde{\omega} \sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \tilde{\omega} + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} \right] + \frac{\mu}{\sigma_{\omega}} \left( \frac{1}{\tilde{\omega} \sigma_{\omega} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln \tilde{\omega} + \sigma_{\omega}^{2}}{\sigma_{\omega}} \right)^{2}} \cdot \left( \frac{\ln \frac{\tilde{\omega}}{v} + \frac{\sigma_{\omega}^{2}}{2}}{\sigma_{\omega}} \right) \right) \\
B''(\tilde{\omega}, v) &= -\frac{1}{\tilde{\omega} \sigma_{\omega}} \phi \left( \frac{\ln \frac{\tilde{\omega}}{v} + \frac{\sigma_{\omega}^{2}}{2}}{\sigma_{\omega}} \right) \left[ 1 - \mu \cdot \left( \frac{\ln \frac{\tilde{\omega}}{v} + \frac{\sigma_{\omega}^{2}}{2}}{\sigma_{\omega}^{2}} \right) \right]
\end{align*}
\]

For the log-linearized model, I need the following derivatives: \( \frac{d}{d\tilde{\omega}} B(\tilde{\omega}, v) \) and \( \frac{d}{d\tilde{\omega}} B'(\tilde{\omega}, v) \)
\[
\frac{d}{dv} B' (\tilde{\omega}, v) = - \frac{d}{dv} \left[ \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} t^2} dt \right] - \frac{\mu}{\sigma_{\omega}} \cdot \frac{d}{dv} \left[ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \ln\left( \frac{\tilde{\omega}}{\sigma_{\omega}} \right) + \frac{\sigma_{\omega}^2}{2} \right)^2} \right] = \\
\frac{d}{dv} B' (\tilde{\omega}, v) = \left[ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \ln\left( \frac{\tilde{\omega}}{\sigma_{\omega}} \right) + \frac{\sigma_{\omega}^2}{2} \right)^2} \cdot \left( \frac{1}{\sigma_{\omega} \tilde{\omega} v^2} \right) \right] \\
\quad - \frac{\mu}{\sigma_{\omega}} \left[ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \ln\left( \frac{\tilde{\omega}}{\sigma_{\omega}} \right) + \frac{\sigma_{\omega}^2}{2} \right)^2} \cdot \left( \frac{\ln\left( \frac{\tilde{\omega}}{\sigma_{\omega}} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \cdot \left( \frac{1}{\sigma_{\omega} \tilde{\omega} v^2} \right) \right] \\
\frac{d}{dv} B' (\tilde{\omega}, v) = \frac{1}{v \sigma_{\omega}} \cdot \phi \left( \frac{\ln\left( \frac{\tilde{\omega}}{v} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) - \frac{\mu}{v \sigma_{\omega}} \left[ \phi \left( \frac{\ln\left( \frac{\tilde{\omega}}{v} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \cdot \left( \frac{\ln\left( \frac{\tilde{\omega}}{v} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \cdot \left( \frac{1}{\sigma_{\omega}} \right) \right] \\
\frac{d}{dv} B' (\tilde{\omega}, v) = \frac{1}{v \sigma_{\omega}} \cdot \phi \left[ \frac{\ln\left( \frac{\tilde{\omega}}{v} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right] \cdot \left[ 1 - \frac{\mu}{\sigma_{\omega}} \left( \frac{\ln\left( \frac{\tilde{\omega}}{v} \right) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}} \right) \right] \quad (72)
\]