Professional team sports generate indivisible joint-products. Neale (1964, p.2) captures this interrelatedness in the following sentence: ‘pure monopoly is disaster’. Or in short: teams need each other to produce games. Rottenberg (1956, p. 242) mentions that “The nature of the industry is such that competitors must be of approximate equal ‘size’ if any are to be successful”. The notion of competitive balance is founded upon this idea: for an attractive championship teams should not excel excessively in playing strength.

In the media and the professional sports sector this idea of competitive balance receives significant attention and underlies many sports policy decisions. In Sports Economics as well competitive balance is at least relevant. Over the last four decades several events occurred that can be considered to have had an impact on competitive balance. More specifically they affect the dominance of ‘large market’ teams. We define such teams as those located in a large city combined with a large fan base. Such imbalance is what Kesenne (2004) calls a ‘bad’ imbalance.

We theoretically research the impact of some major changes that caused important shifts in the revenue functions. We construct a two league –four teams model and include the market, local and national, as major determinants of the revenue functions of teams. We determine the choice of talent under both the win maximizing as the profit maximizing objective. The
difference in win percentages is used to measure the competitive imbalance. We calculate the total demand function for talent and intersect it with an elastic supply formalized into a simple linear supply function. The resulting equilibrium wage is discussed where possible.

We discuss three scenarios based on three successive periods in European football leagues. All periods are introduced by important changes generally discussed to influence dominance. We start from a benchmark scenario with a closed labour market in which ticket sales, based on the local market are considered to constitute the main income source. The sale of broadcast rights combined with shirt sponsoring introduces the second period. In most countries this new era began at the end of the 70ies and early 80ies. Both increased revenues substantially and are highly interrelated. Live matches persuaded sponsors to invest in the teams as well as in commercial blocks on tv. Sports fans were now able to enjoy live games of teams located in another part of the country so that the market of supporters increased with a part of the national market.

The third period is marked by a combination of three events. Jean-Marc Bosman, a professional Belgian football player, changed the labour market in professional team sports at the end of the nineties. He went to court to oppose against the transfer ruling. The European Court of Justice ruled that the transfer system concerning the European international football players violated the free movement of workers constituted in Article 48 of the ECC treaty. Following the verdict of 15 december 1995, the European Commission abandoned the policy of transfer restrictions and abolished the rule to limit the number of foreign players fielded as well, giving rise to a new chapter in the European football history: a more open and competitive labour market. The abolishment was in the middle of the season and the real
impact on the acquirement of talent can be assumed to begin at the earliest in season 1996-1997.

In 1997 the Champions League (CL) changed its selection criteria and its revenue distribution. The ‘market pool’ came into use, designating the revenue that is divided based on the national tv-market. Teams from countries with a larger market receive a larger share. This makes it possible that a CL champion receives a lower income than teams that ended lower in the ranking. Porto for example received in 2003-2004 € 19 million while runners-up AS Monaco FC received € 26.4 million. (Uefa.com, 8 June 2004) Even though the CL, named the Champions Cup before 1992, has an extensive history of adaptations, we consider this change as the most important one to substantially influence the dominance of teams in the national competition.

With the deregulation of the television market, the competition for broadcast rights intensified. Among other things, this boosted the broadcast revenues considerably. The introduction of digital television at the end of the nineties can be assumed to be the start of a continuing boom of broadcast rights and so influences the third period as well.

We provide a first empirical verification by constructing a measure that incorporates the identity of teams to focus on large market teams. Two European football leagues, England and Belgium are briefly discussed. In our conclusion we provide an overview table and discuss some future research topics.

1. Theoretic Model
Following Quirk & Fort (1995) the revenue function of professional sports teams with $n$ teams in the league should satisfy the following assumptions:

$$R_j = R(m_j, w_j) \quad \text{with} \quad \frac{\partial R_j}{\partial m_j} > 0 ; \quad \frac{\partial R_j}{\partial w_j} > 0 \quad \text{and} \quad \frac{\partial^2 R_j}{\partial w_j^2} < 0 \quad \text{for} \quad j = 1 \text{ to } n \quad (1)$$

The revenue function $R_j$ of a team $j$ depends on its local market size $m_j$ and its winning percentage $w_j$. The revenues increase in the market size. A team located in a large city and following having a large local market is assumed to be able to attract more spectators ceteris paribus. The difference in local drawing power will be shown to be substantial. The revenues also increase in the winning percentage but in a decreasing marginal matter so that the revenue function is concave in the win percentage. (El Hodiri & Quirk; 1974) Fans prefer that their team wins but appreciate it that the game is exciting and the difference in fielded talent should not be too big. A game where the home team wins with 2 to 1 can be assumed to be more attractive than when a team wins with 7 to 0. Following this preference, a team that has already a very high win percentage will not increase their revenues by as much or even at all when an extra win percent is added compared to when it has a low win percentage.

A team can not directly affect the win percentage but we follow the main literature that the win percentage of a team is determined by the talents acquired by team $j$, $x_j$, compared to the total talent in the league, $x$. (Quirk & Fort, 1995) Following Borghans & Groot (2005) the latter ratio forms the probability $p_{jk}$ that team $j$ wins against team $k$:

$$p_{jk} = \frac{x_j}{x_j + x_k} \quad (2)$$
The expected number of absolute wins for team \( j \), \( E(\text{wins}_j) \) is the sum of these probabilities for all games. In European football competitions the teams play each other twice.

\[
E(\text{wins}_j) = 2 \sum_{j \neq k} p_{jk} = 2x_j \sum_{j \neq k} \frac{1}{x_j + x_k}
\]  

(3)

We assume that there is no home team advantage for simplicity. We can now define the win percentage of team \( j \), \( w_j \), as the ratio of the total number of wins divided by the total number of games:

\[
w_j = \frac{E(\text{wins}_j)}{2(n-1)} = \frac{x_j}{n-1} \sum_{j \neq k} \frac{1}{x_j + x_k}
\]  

(4)

We construct a basic model in which we simplify these general formulations. The first subchapter describes our benchmark scenario. A subdivision is made based on the objective function: win maximization versus profit maximization. Next we introduce broadcasting and sponsor revenues by adding a fraction of the national market. The last scenario includes an open labour market and extra revenue from the market pool for the large team.

2. Benchmark: No Broadcasting/Sponsorship, Closed Labour and Product Market

A league consists of two countries (A and B) with each two teams (1 and 2). Country A is a large market country, while country B has a smaller national market. Within each country we define team 1 as the large market team and team 2 as the small market team. We assume that the local markets of the teams sum up to the total national market. So formally:
\[ m_A > m_B \; \; \; m_{i1} > m_{i2} \; \; \text{and} \; \; m_{i1} + m_{i2} = m_i \quad \text{for} \; i = A, B \]  

We combine two basic models of Kesenne (2006, 2007a) to specify a revenue function for each club in each country that fulfils (1). We begin with the local market as the main source of revenues and combine this with a parameter \( \beta > 0 \) in such that the concavity is fulfilled.

\[
R_{ij} = m_{ij} w_{ij} - \beta w_{ij}^2 \quad \text{for} \; i = A, B \; \text{and} \; j = 1, 2
\]  

To ensure positive win percentages and positive amounts of talent, Kesenne (2007b) shows that the following inequalities need to be satisfied. We adapt the formula to apply for our model.

\[
m_{i2} - m_{i1} < \beta < \frac{m_i}{2}
\]  

With only two teams, the winning percentage (4) amounts to:

\[
w_{ij} = \frac{x_{ij}}{x_{i1} + x_{i2}} = \frac{x_{ij}}{x_i}
\]  

The number of playing talents of team \( j \) in country \( i \) is presented by \( x_{ij} \) and \( x_i \) is the sum of talents in country \( i \).

On the cost side, we consider the player labour cost \( c_i \) as the sole cost of production, which is not an unreasonable assumption if a strong positive correlation between the capital cost and
the number of playing talents in a club exists (see Szymanski and Smith, 1997). We assume that the teams are wage takers. With only two teams we admit that this assumption is less realistic. The following total cost function is used:

\[ TC_{ij} = c_i x_{ij} \]  \hspace{1cm} (9)

The wage cost is determined endogenously. The aggregate demand function for talent is found by adding the individual demand functions horizontally. Equilibrium is found where demand intersects with the supply of talent.

We assume that the supply of talent is flexible. For computational simplicity it is restricted to be linearly increasing in wage. This can be justified based on the following reasonable assumptions. We assume that the highest league acquires substantial training so that people are not able to have a full time job elsewhere. Hence people are only willing to train their innate talents to reach the necessary level if they know they will receive considerable payment. The higher this payment, the more people are willing to train and so increase total number of supplied talents. This increase comes from the number of people entering the market as by the increase of talents within a player. Formalised we use:

\[ x^s_i = ac_i \]  \hspace{1cm} (10)

Theoretic modelling uses predominantly the objective of profit maximization. Some authors as Sloane (1969) and Cairns, Jennett & Sloane (1985) mention, without algebraic formalization, that utility maximizing, where wins give utility, can be more important than maximizing profits. Following their discussion, it is often stated that American sports leagues
are more profit maximizing while European leagues maximize winning. However, even for European team sports the profit objective remains predominant in theoretic research. We present results for both the profit maximization and the win maximization objective and start with the latter.

Professional football clubs in Europe are often considered to be mainly interested in winning as much games as possible. To achieve this they will buy as much talent as their budget constraint allows them to. Reaching a certain profit can be taken up in the budget constraint because win maximization does not exclude this. (Kesenne, 2006) Since calculations quickly become more complex when adding a positive, or a negative profit, we assume that teams hire as much talent as allowed to break even. The following method to find equilibrium is similar for the other two periods and is only discussed here in detail.

The maximisation problem for team j in league i is formulated using (6) and (9):

Maximize $x_j$

Subject to: $m_j w_j - \beta w_j^2 - c_i x_{ij} = 0$  \hspace{1cm} (11)

We use (8) and find that the break even constraint holds if:

$m_j \frac{1}{x_{i1} + x_{i2}} - \beta \left( \frac{1}{x_{i1} + x_{i2}} \right)^2 x_j = c_i$ \hspace{1cm} (12)

So the demand for talent is given by its average revenue function:
When the labour market is closed, the equilibrium cost of talent $c_j$ will be determined within each country so we added a subscript $i$.

The optimal number of talents ($x_{1i}$) of team 1 depends on the number of talents acquired by the other team ($x_{2i}$) and vice versa so a game theoretic approach applies. We assume that in each country the following simultaneous one shot game with complete information takes place. We have two players: team 1 and team 2. Each team chooses an amount of talent $x_j$ which lies between zero and infinity in order to maximize their win percentage given the break even constraint.

To find the Cournot-Nash equilibrium we intersect the reaction curves (12) of the two teams in a country:

$$\frac{m_{1i}}{x_j} - \beta \frac{x_{1i}}{x_j} = \frac{m_{2i}}{x_j} - \beta \frac{x_{2i}}{x_j}$$

(14)

We focus in this paper on dominance of large market teams within countries. Quirk & Fort (1995) mention that the difference in winning percentages measures competitive balance. As far as we know the present theoretic models all discuss the ratio of talents or the win percentages by itself. We introduce the measure of dominance ($D_i$) by subtracting the win percentage of team 2 from the win percentage of team 1. A positive $D_i$ reveals dominance of the large market team. After multiplying both sides of (14) by $x_j$, we can derive that:
\[ D_i = w_{i1} - w_{i2} = \frac{m_{i1} - m_{i2}}{\beta} \tag{15} \]

which is clearly positive, so that the large-market team dominates. The larger the difference in local markets is, the larger the dominance is.

Comparison between countries needs an assumption on the differences between the local markets. Large countries have generally the same number of teams, or just a few more, in the highest league. So the dispersion over the country is much larger and we can expect that most teams will be located in a larger local market area of their own. We believe that the differences in local markets will consequently be larger in small countries than in large countries. This will result in a more uneven balance in those smaller countries. However more research is necessary to support or contradict this.

The demand function of talent is found by summing the individual demand functions.

\[ x_i = \frac{m_i - \beta}{2c_i} \tag{16} \]

The equilibrium wage is found by intersecting the demand function (16) with the supply function of talents (10).

\[ c_i = \sqrt{\frac{m_i - \beta}{2a}} \tag{17} \]

Comparing the labour costs in both countries using (5) enables us to derive the following inequality:
meaning that the equilibrium wage is higher in the large-market country.

When we replace $c_i$ in (16) by (17) we find the total amount of talents purchased for each country. Because (5) applies we find the following inequality

$$x_d = \sqrt{a(m_d - \beta)/2} > x_b = \sqrt{a(m_b - \beta)/2}$$

so that the large market-country is more talented than the small-market country.

We can conclude from this analysis that, if teams, that only differ in local market size, maximize their win percentage, the large-market team in each country acquires more talents than the small-market team. With a linear supply curve, the large-market country has a higher equilibrium wage than the small-market country. The large-market country also acquires more talents than the small-market country.

The method of profit maximization, outlined here in detail, is similar for the next periods and is not repeated. The maximization problem of the profit function for team $j$ in league $i$, using (6) and (9) is presented as follows:

$$\max_{x^p} \Pi^p_{ij} = m_{ij} w^p_{ij} - \beta w^p_{ij} x^p_{ij} - c_i x^p_{ij}$$
We add a superscript P to indicate that we consider leagues with profit maximization as predominant objective.

The first order condition is:

\[ M R_{ij} = \frac{\partial R_{ij}}{\partial x_{ij}} = c_i \]  

With profit maximization the demand for talent is given by its marginal revenue function. A similar one shot game as with win maximization arises but now the profit function needs to be maximized. So in this case the first order conditions are the reaction functions. Intersecting these functions leads us to the following competitive balance:

\[ D_i^P = \frac{m_{i1} - m_{i2}}{m_i} \]  

which yields the well-known result that the large-market team dominates the small-market team.

The discussion concerning the differences in local markets applies here as well and if our intuition is correct that the differences are smaller for large market countries it follows that there is less large–market team domination in country A.

The demand function of talent can be derived as (see Appendix 1):
\[ x_i^p = \frac{(m_i - 2\beta_i)m_{i,2}}{c_i^p m_i^k} \]  

(23)

The equilibrium wage is now simply found by intersecting this demand function with the supply of talents (9),

\[ c_i^p = \frac{1}{m_i} \sqrt{\frac{(m_i - 2\beta_i)m_{i,2}}{a}} \]  

(24)

We were not able to compare the wage levels in both countries. If we calculate some examples incorporating the necessary conditions, we find a higher wage cost in country A than in country B. We have not found a contradicting example but more research is appropriate.

The total amount of talent in country \( i \) is found by substituting (24) into (23):

\[ x_i^p = \frac{\sqrt{\alpha(m_i - 2\beta_i)m_{i,2}}}{m_i} \]  

(25)

When comparing profit maximization to win maximization, and referring to (7), the competitive balance measure shows that the large market teams are more dominant in win maximizing leagues then they would be in profit maximizing leagues:

\[ D_i = \frac{m_{i,1} - m_{i,2}}{\beta} > D_i^p = \frac{m_{i,1} - m_{i,2}}{m_i} \]  

(26)
Comparing the wages we can derive the following result (see appendix 2):

\[ c_i > c_i^p \]  \hspace{1cm} (27)

We can conclude that, if teams that only differ in local market size, maximize profits, the large-market team acquires more talents and will hence dominate the small-market team. If teams are wage takers, the large-market teams are more dominant in win maximizing leagues then they would be in profit maximizing leagues. With a linear supply of talent, the market clearing wage will be higher in win maximizing leagues then it would be when profits are maximized.

3. Introduction of Live Broadcasting and Shirt-Sponsorship

Broadcasting of live matches and shirt sponsoring generate extra revenue. These revenue sources are rather determined by the size of the national market. To include such a change we add a fraction of the entire country to the local market of teams. We assume that large market clubs are able to attract bigger sponsors, resulting in higher revenue. The subscript $T_v$ is added to indicate the new period of sponsoring and broadcasting.

Since each team has a different potential to attract sponsors we multiply the total market of a country by a parameter $\alpha_{ij}$. We assume that the large market team attracts more money and hence receives a larger fraction. The national market is divided over the two teams.

\[ 0 < \alpha_{i2} < \alpha_{i1} < 1 \ ; \ \alpha_{i2} + \alpha_{i1} = 1 \]  \hspace{1cm} (28)
With these extensions to the model, all calculations are comparable to the ones from the previous section, so that can be derived that:

\[ D_{Tv,i} = D_i + \frac{m_i (\alpha_{i1} - \alpha_{i2})}{\beta} \]  \hspace{1cm} (29)

The dominance of the large-market club goes up if it receives a large fraction of the total market. This result follows from our assumption that the big team will be able to attract more revenue. If teams have an equal share of the total market so that \( \alpha_{d1} = \alpha_{d2} \) and \( \alpha_{b1} = \alpha_{b2} \) the competitive balance does not change. If the share of the total market would be used as an instrument by the league to promote competitive balance, the league could pool all the revenues of sponsoring and broadcasting. When they give a larger fraction to the small teams to compensate for the own local market a more equal talent distribution would be the result.

The demand of talent now equals:

\[ x_{Tv,i} = \frac{2 m_i - \beta}{2 c_{Tv,i}} \]  \hspace{1cm} (30)

so that the wage is larger in both countries due to the extra revenue.

\[ c_{Tv,i} = \sqrt{\frac{2 m_i - \beta}{2 a}} > c_i \]  \hspace{1cm} (31)

Under profit maximisation dominance becomes:
The broadcast and sponsor revenues can change the dominance of large market teams. If in a profit maximization league the small team receives a larger broadcast share than its local market share the difference in winning percentages decreases and so the national competitive balance improves. So the impact of the extra revenue depends on the dispersion of the revenues compared to the spread of the market shares. This is shown in Appendix 3.

\[
D_{Tv,d}^p = \frac{D_{Tv}^p}{2} + \frac{(\alpha_{i1} - \alpha_{i2})}{2}
\]

(32)

It is obvious that the total demand for talent in each country is now higher given the extra revenue, so that, with the same supply curve, also the equilibrium wage will be substantially higher. See Appendix 4 for algebraic notation.

When comparing profit and win maximization we find equal numerators of the dominance measure. Using (7) it is obvious that:

\[
D_{Tv,d}^p > D_{Tv,d}^p \iff \alpha_{e1} - \alpha_{e2} > \frac{m_{e1}}{m_e} - \frac{m_{e2}}{m_e}
\]

(33)

so that the concentration of talents remains higher in win maximization leagues then it would be with profit maximization, independent of the difference in shares.

We can conclude for this section that the broadcast and sponsor revenue increases the dominance of large market teams when these teams receive a larger fraction of the total
market. This result applies for a league that maximizes wins. With profit maximization the
dominance can already be lowered if the small team receives a higher percentage than its
local market share. That percentage can still be lower than what the large team acquires. For
both objectives the wages go up in both countries, and the win maximisation league remains
most dominated.

4. Introduction of the Champions League with an Open Labour Market and Large
Increase in Broadcast Rights

The Champions league joins the best national teams and gives them the opportunity to play
each other. Broadcasting and sponsoring, merchandizing and licensing revenues increase
because the market enlarges to the European market. We assume that the big market team of a
country reaches the Champions league, which is representative for the European football
leagues.

We consider the changes in 1997/1998 as the most influential on the dominance of large
market teams. Teams besides the champions of the national leagues were taken up in the
league and extra revenue is rewarded, called the ‘market pool’, based on the national tv
market. The larger the national tv-audience is, the more revenue they receive. We focus on
the market pool alone and add a parameter d to the national market. So we assume that this
enlargement can be presented as an increase in the fraction of the national market and that it
is equal for all teams and countries. Teams that enter the Champions league also receive a
lump sum independent of their market or number of games played. Playing in the Champions
League introduces however extra costs such as investments in the playing grounds to receive
international teams, in extra security, in transportation costs for away games, in extra medical
costs,… We assume that the lump sum covers these extra costs.

We assume that the teams do not take the talent of other countries into account when deciding
on their own talent. The fact that a team like Real Madrid bought star players, is assumed not
to have an effect on the talent of other teams playing in the Champions League if they do not
play in the Spanish Primera División.

At the end of the nineties the deregulation of the tv market boosted broadcast revenues. The
increases have not yet stabilized but the impact is comparable to the second scenario. We do
not explicitly include this in our model, but it is possible to multiply $\alpha_j$ by a factor larger than
one. If the increases in broadcast revenues are equally divided over all teams, no changes in
dominance occur. If the biggest increase is noted for the biggest teams, dominance increases
under win maximizing. Under profit maximizing the new broadcast shares need to be
compared to the local market shares.

The opening of the labour market after the Bosman arrest introduces a European labour cost.
We hence no longer use the subscript $i$. We assume that the European supply function can be
represented by the horizontal summation of the supply functions of each individual country.
Football players are indeed travelling all over Europe so this is a plausible simplification.

If only the large-market teams enter the CL, and teams are win maximizers, the maximization
problem is formulated including the parameter $d$ and adding the subscript $\text{Cl}$ as indicator of
this last period:
Maximise $x_{Cl,i}$

Subject to $(m_i + (\alpha_i + d)m_i)w_{Cl,i} - \beta w_{Cl,i}^2 - c_{Cl,i}x_{Cl,i} = 0$  \( (35) \)

The smaller team does not receive any extra income since it does not enter the Champions League.

Maximize $x_{Cl,2}$

Subject to $(m_2 + \alpha_2 m_i)w_{Cl,2} - \beta w_{Cl,2}^2 - c_{Cl,i}x_{Cl,2} = 0$  \( (36) \)

For both teams the budget constraints need to be fulfilled and hence act as reaction functions. Intersecting these shows that the increase of the revenue results in the following dominance measure:

$$D_{Cl,i} = D_{Tv,i} + \frac{dm_i}{\beta}$$  \( (37) \)

which yields the obvious result that the dominance of large market teams increases under win maximization when these teams alone receive extra income.

Because of the opening of the labour market the wage is now determined on the world market. The world demand for talent is derived by adding $x_{Cl,A}$ to $x_{Cl,B}$, so that:

$$x^D_{Cl} = \frac{m_A(2+d) + m_B(2+d) - 2\beta}{c}$$  \( (38) \)
We find the market wage cost by intersecting the demand and supply function. We need to adapt the supply function since now all talents can play in other countries. The aggregate world supply function is found by adding the two individual country supply functions so that:

\[ X_{ci}^s = 2ac \]  

(39)

Equilibrium is found by the intersection of (38) and (39) and the wage is now:

\[ c = \sqrt{\frac{m_A + m_B + \frac{d}{2} (m_A + m_B) - \beta}{a}} \]  

(40)

This shows again an increase compared to the second period. The proof is given in Appendix 5.

If the large teams in both countries maximize their profits, including their share of the market pool represented by the parameter d, and the small teams have the same maximization problem as in the second period, we find the new competitive balance:

\[ D_{ci, i}^p = D_{iv, i}^p + \frac{d}{2} \]  

(41)

so that the market pool increases the dominance of the large teams.

As in both previous periods, the win maximizing leagues are more dominated by their large market teams: (See Appendix 6):
We can conclude for this third period that the dominance of the large-market teams further increases in both the profit and the win maximisation case. It is also obvious that the extra revenue further increases demand and the equilibrium wage level in both countries for the win maximizing league. When teams maximize profits we are not able to derive the latter result algebraically. Again the dominance of the large-market teams is less extreme in the profit maximization leagues than it would be in win maximization leagues.

We summarize the results for all periods in a table in the last section. In the following section we provide a first empirical verification.

5. A first empirical verification

We add a first empirical analysis to verify some results. We did not find a measure in the literature that focuses on the identity of a team so we introduce a new dominance measure. This dominance measure focuses on large market teams and ranges between zero and 1. We choose England as our large market country with 49 million inhabitants and Belgium as the small market country with 10 million inhabitants. We consider a period of 30 seasons beginning with 1975 up to 2004.

Only three teams remained in the highest league in Belgium during our thirty seasons: RSC Anderlecht, Club Brugge en R. Standard de Liège. All three are located in a large local
market and enjoy a large fan base so are defined to be a large market team. In England four teams stayed in the highest league: Arsenal, Everton, Liverpool and Manchester United. To have as many teams as in Belgium we need three teams. The time frame was chosen to include Manchester United and we exclude Everton.

We focus on the sum of the win percentages of the chosen three large market teams. To range between 0 and 1 we compare the actual sum of win percentages to its minimum and its maximum. It is theoretically possible that the three large teams finish at the bottom of the ranking. The minimal sum of win percentages is attained when the last team has won no games, the second to last won the two games against the last and the third won against the other two, so four games. Combined they won 6 games out of 42 when there are 22 teams and out of 34 when there are 18 teams. Formulised for n teams and K large market teams you have the following minimal sum:

\[
\text{Minsum} = \frac{K(K-1)}{2(n-1)}
\]  

When the K teams are dominant they fill the top K. If they are perfectly dominant the first team wins all of its games, the second all except against the first, the third all except against the first two,… The maximal sum of win percentages is then reached (See Appendix 7):

\[
\text{Maxsum} = \frac{K[2n-K-1]}{2(n-1)}
\]  

\[\text{(43)}\]

\[\text{(44)}\]
The Dominance of K Large Teams measure for country i (DoKLaT_{i}-measure) is the ratio of the actual range of the sum of win percentages to the maximal range. It is modelled as follows:

\[
DoKLaT_i = \frac{\sum_{i=1}^{K} w_i \text{Min Sum}}{\text{Max Sum-Min Sum}} = \frac{2(n-1) \sum_{i=1}^{K} w_i - K(K-1)}{2K[n-K]}
\] (45)

So when the measure equals 1 the teams are perfectly dominating the competition. If the measure equals 0 a good imbalance (Kesenne, 2004) occurs and the large market teams end up at the bottom of the ranking, dominated by the small market teams.

Figure 5-1 shows our calculated time series for our two countries.

Descriptive analysis shows that in the beginning of the period, Belgium is more dominated by its three large market teams than England is. With the introduction of extra revenue for all teams at the beginning of the eighties the dominance of England’s large teams increases. This follows our theoretic model: when at the benchmark model the large country is less dominated than the small, in the next period both suffer from an increase in dominance but the large market teams of the large country increase their win percentage with a larger fraction. Consequently the imbalance measures come closer to each other. For Belgium no real increase is present. From season 1996 onwards, both countries show an increase in dominance. The large country, again England increases with a larger fraction than Belgium,
with now an almost equal evolution as a consequence. For these two countries the descriptive analysis supports our theoretic model very well.

6. Conclusions

Table 2.-1 summarizes our theoretical results: 2-1-a discusses the win maximizing leagues and 2-1-b the profit maximizing ones. The three periods are placed in the columns and each period is split up into two sub columns. The first shows the derived algebraic expression. The second column of the first period compares the begin situation of the two countries. For the other two periods we show when an increase is present, compared to the previous period. A simple arrow without further specification shows an unconditional increase. In the first row we place our dominance measure, in the second the wage cost.

Table 2-1

Caption Summary of the theoretical model

<table>
<thead>
<tr>
<th></th>
<th>1: Benchmark</th>
<th>2 = 1 + National market</th>
<th>3 = 2 + open + market pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$\frac{m_{b_1} - m_{b_2}}{\beta}$</td>
<td>$D_i + \frac{m_i(\alpha_{b_1} - \alpha_{b_2})}{\beta}$</td>
<td>$D_{TV,d} + \frac{d m_i}{\beta}$</td>
</tr>
<tr>
<td>Imb.</td>
<td>$\alpha_{b_1} &gt; \alpha_{b_2}$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$c$</td>
<td>$\sqrt{\frac{\beta - m_i}{2a}}$</td>
<td>$\sqrt{\frac{\beta - 2m_i - \beta}{2a}}$</td>
<td>$\sqrt{\frac{m_A + m_B + \frac{d(m_A + m_B)}{2} - \beta}{a}}$</td>
</tr>
<tr>
<td>$c\alpha &gt; c_B$</td>
<td></td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
</tbody>
</table>

2-1-a: Win Maximization
Large market teams are more dominant in a win maximizing league than in a profit maximizing one. Important increases in revenues, as the football leagues have encountered the last decades, increase demand of talent and consequently increase the wage cost. The distribution of extra revenue can improve competitive balance in a country. Our alfa-parameter is a possible revenue sharing instrument. Revenue sharing models mostly include shares of total revenue of other teams. Our model may be a more realistic alternative and invites further research. The extension of the model to n teams is not expected to alter our results significantly but deserves attention.

A balanced competition is shown only to be present when local markets are either equal or are compensated for. Extra revenue that is limited to only one, or a few teams, such as the market pool, is shown to increase the demand of talent of these teams. When the large teams enjoy these extra revenues their dominance increases even more. Concerns expressed in the media with the introduction of the market pool seem justified. A first empirical verification shows that although our theoretical model is a simplification it is not contradicted by the
actual evolution of 3 large teams in Belgium and England. An adaptation of the market pool seems appropriate when the national balance is of major concern. Researching the consequences on the international competition levels is an interesting extension of this research.
Appendices

Appendix 1: Total demand for talent in Profit maximizing benchmark scenario

Intersection of the first order conditions:

\[ m_{i1} \left( \frac{1}{x_{i1}^p} - \frac{x_{i1}^p}{(x_{i1}^p)^2} \right) - \beta \left( \frac{2x_{i1}^p}{(x_{i1}^p)^2} - \frac{2(x_{i1}^p)^2}{(x_{i1}^p)^3} \right) = m_{i2} \left( \frac{1}{x_{i2}^p} - \frac{x_{i2}^p}{(x_{i2}^p)^2} \right) - \beta \left( \frac{2x_{i2}^p}{(x_{i2}^p)^2} - \frac{2(x_{i2}^p)^2}{(x_{i2}^p)^3} \right) \]

Reduce to a common denominator and simplify:

\[ x_{i1}^p = x_{i2}^p \frac{m_{i1}}{m_{i2}} \] (*)

Use the latter expression to replace \( x_{i1}^p \) in the following rewritten FOC of team 2:

\[ m_{i2}x_{i1}^p(x_{i1}^p + x_{i2}^p) - 2\beta x_{i1}^p x_{i2}^p = c_i^p (x_{i1}^p + x_{i2}^p)^3 \]

Reduce to a common denominator, simplify and single out \( x_{i2}^p \):

\[ x_{i2}^p = \frac{m_{i2}^2 m_{i1}(m_i - 2\beta)}{c_i^p m_i^3} \]

We can use the latter expression in (*) to find \( x_{i1}^p \):
Total demand for talent is the sum of the individual demands:

\[ x_{i1}^P = \frac{m_{i1}^2 m_i^2 (m_i - 2\beta)}{c_i^P m_i^3} \]

Appendix 2: Comparison of wages in a profit maximizing league versus a win maximizing league

\[ c_A^P = \sqrt{\left(m_A - 2\beta\right) m_{A1}^2 m_{A2}^2} < c_A = \sqrt{\left(m_A - \beta\right) \frac{1}{2a}} \iff \]

\[ (m_A - \beta) \left(\frac{m_{A1}^2 m_{A2}^2}{m_A^2} - \beta \frac{m_{A1}^2 m_{A2}^2}{m_A^2}\right) < (m_A - \beta) \frac{1}{2} \iff \]

\[ (m_A - \beta) (m_{A1}^2 m_{A2}^2 - m_A^2) < 2\beta m_{A1}^2 m_{A2} \iff \]

This inequality is always fulfilled because \( m_A - \beta > 0 \) (7) and \( m_{A1}^2 m_{A2}^2 - m_A^2 < 0 \). So the wage cost in a win maximizing country is higher than in a profit maximizing league. The same applies to country B.

Appendix 3: Comparison of competitive balance under profit maximisation between the first two periods.
\[ D_{T,v,i}^p = \frac{(m_{i1} - m_{i2}) + m_i(\alpha_{i1} - \alpha_{i2})}{2m_i} > D_i^p = \frac{m_{i1} - m_{i2}}{m_i} \iff \]

\[ m_i(\alpha_{i1} - \alpha_{i2}) > \frac{1}{2} \left( \frac{m_{i1} - m_{i2}}{m_i} \right) \iff \]

\[ \alpha_{i1} - \alpha_{i2} > \frac{m_{i1} - m_{i2}}{m_i} \]

Appendix 4: Algebraic representation demand function and wage cost.

\[ x_{T,v,i}^p = \frac{2 \left( m_i - \beta \right)(m_{i1} + \alpha_{i1}m_i)(m_{i2} + \alpha_{i2}m_i)}{c_{T,v,i}^p m_i^2} \]

\[ c_{T,v,i}^p = \sqrt{\frac{2(m_i - \beta)(m_{i1} + \alpha_{i1}m_i)(m_{i2} + \alpha_{i2}m_i)}{am_i^2}} \]

Appendix 5: Comparison of the wage cost for win maximizing leagues when transition to third period.

\[ c = \sqrt{\frac{m_A + m_B + \frac{d}{2}(m_A + m_B) - \beta}{a}} > c_{T,v,A} = \sqrt{\frac{2m_A - \beta}{2a}} \iff \]

\[ m_A + m_B + \frac{d}{2}(m_A + m_B) - \beta > \frac{2m_A - \beta}{2} \iff \]
\[2m_B + d(m_A + m_B) > \beta \Leftrightarrow\]

Since \(2m_B > \beta\) (see (7)) the wage cost in the third period is necessarily larger than the one from the second period. This applies to country B as well because \(2m_A > \beta\) (see (7)).

Appendix 6: Comparison of dominance in win versus profit maximizing leagues when the market pool is introduced.

\[D_{ci,j}^P = \frac{m_{i1} - m_{i2} + m_i(\alpha_{i1} + d - \alpha_{i2})}{2m_i} < D_{ci,j} = \frac{m_{i1} - m_{i2} + m_i(\alpha_{i1} + d - \alpha_{i2})}{\beta} \Leftrightarrow\]

\[2m_i > \beta\]

This inequality is always fulfilled given (7).

Appendix 7: Maximal sum of win percentages:

When we start from K=3 the sum of the three win percentages is:

\[
\text{MaxSum} = \frac{2(n-1)}{2(n-1)} + \frac{2(n-1)-2}{2(n-1)} + \frac{2(n-1)-4}{2(n-1)}
\]

So

\[
\text{MaxSum} = \frac{3*2(n-1) - 2(\sum_{i=1}^{2} i)}{2(n-1)}
\]
For K teams:

\[
\text{MaxSum} = \frac{k \times 2(n-1) - 2\left(\sum_{i=1}^{k-1} i\right)}{2(n-1)}
\]

Solving the arithmetic series and further simplifying shows:

\[
\text{MaxSum} = \frac{k[2n-1-k]}{2(n-1)}
\]
Figure 5.1

Caption Do3LaT$_B$ and Do3LaT$_E$


