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ESTIMATION OF LOCALIZED DYNAMIC LOADS BY MEANS OF SPARSE OPTIMIZATION

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ABSTRACT
The identification of system parameters using forward approaches is not always practical due to the rising complexity of modern structures, leaving no chance for direct parameter measurements. In contrast to forward methods, inverse techniques have been gaining popularity, since the advent of high performing computers. This approach consists of the computation of input parameters of a system, with known output data and the system model. When the number of equations (sensors) becomes lower than the amount of unknowns (input parameters), or when the condition number of the system is high, the problem does not have a unique solution. The system becomes under-determined and highly sensitive to input perturbations. The discrete force identification problem in mechanics consists of estimating the applied force locations and their corresponding time history based on measured structural responses. In this article, the applicability of a recently proposed force identification technique (G-FISTA) will be tested using a real-life measurement on a footbridge. This iterative algorithm promotes structured sparsity in the force vector. This algorithm creates a new mathematical setting for the inverse problem, and then solves it using a mixed cost function of group-penalized least squares. This study shows that the location and time history of discrete forces applied on a footbridge can be correctly estimated using the proposed technique.

Keywords: Load identification, Operational modal analysis, Structured sparsity, Model updating

1. INTRODUCTION

Measuring the dynamic loads on structures constitutes a challenging topic in engineering or almost non-practical in some cases. The inverse load identification using system responses is especially of interest for structural health monitoring (SHM) in structural mechanics and civil engineering. For structures such as wind turbines or aircraft wings, the knowledge of the external loads is essential. Since the direct measurement of the applied forces on structures is impossible, many attempts were made to solve this problem by using indirect measurement techniques (inverse problems). This approach consists of computing the input parameters of a system (excitation loads), with known output (vibration) data and the model of the structure.

The great challenge with inverse problems is that they are mathematically ill-posed in almost all cases, meaning that there exists no unique solution to the problem, which continuously depends on the measured
data. This lack of stability is related to the fact that the system model is rank deficient and the condition number is high. In other words, a wide range of non-realistic (but mathematically correct) solutions may exist, all resulting in the same structural response (output) [1-3].

In the literature, the load identification problems are treated differently depending on the choice of working domain. In some studies such as in [4-8], time domain techniques are used to solve the problem. Recently, there have been great advances in the development of new time domain techniques, especially in the joint input and state identification [9]. The joint input-state estimation algorithm is introduced in structural dynamics as an extension of the joint input-state estimation algorithm proposed in [10], including the correlation between process noise and measurement noise. This correlation is inherently present for civil engineering applications when accelerations are measured in presence of ambient excitation [11]. The conditions for system inversion on structural dynamics were recently derived in [12]. The algorithm is used for the identification of hammer and artificial muscle forces applied to the bridge deck. Both harmonic and swept sine muscle excitation are considered. The verification is performed by comparing the estimated forces to the actual forces, that have been measured.

In the frequency domain, a classical solution for this problem is using the pseudo-inverse of the system model, [13, 15-17]. Although the pseudo-inverse approach can be computed easily, the outcome of this method is not satisfying in most cases. In optimization based techniques, the cost function can be penalized or not (regularization). In the study of Bond and Adams [14], the force identification is done after an external study on the signal entropy. Among the non-penalized methods, Parloo et al. [16] has suggested a technique that is based on $\ell_p$-norm cost functions for localized forces. The value of $p$ (norm) tends to zero in an iterative manner until convergence is achieved (non-convex problem). On the other hand, the cost function can contain a penalization term to stabilise the solution. The most common way of regularization is the addition of the $\ell_2$-norm to the cost function, such as in the study of Tikhonov and Arsenin [18]. The problem appears when the excitations are applied only on a few locations. There is a clear need for more adequate cost functions that can ensure both localization and amplitude reconstruction for point forces.

In this paper, we propose the application of the Grouped Fast Iterative Shrinkage Threshold algorithm (G-FISTA) for point force identification. This method includes the use of a new mixed cost function that combines the advantages of both $\ell_1$ and $\ell_2$-norms, taking into account the physics of the phenomenon. The variables (force locations) are linked together in separate groups, producing structured sparsity in the force vector. Finally, a modified implementation of an iterative algorithm will be used to solve the optimization problem, following [21].

Verification of force identification techniques proposed in literature so far is often limited to numerical simulations, where measurements errors are introduced by adding white noise to the simulated response signals, or to laboratory experiments. This paper presents a full-scale verification of an iterative sparse optimization algorithm, using data obtained from an in-situ experiment on a footbridge.

Figure 1: A wide view of the studied bridge in Ninove.
2. STATEMENT OF THE PROBLEM

The structure under consideration in this paper is a footbridge instrumented with accelerometers. By means of inverse techniques, we aim at localization and reconstruction of the applied forces. The structure is considered to follow a linear behavior in the range of applied excitations, and its dynamics are represented by a set of linear equations in the frequency domain as follows:

\[ X(f) = H(f)F(f) \], \quad f \in \{f_1, \ldots, f_{N_f}\} \tag{1} \]

where \( X(f) \) is the \( n \times 1 \) Fourier transform of the acceleration vector with \( n \) the number of the measured outputs, \( F(f) \) is the \( k \times 1 \) Fourier transform of the force vector with \( k \) the number of possible applied force locations, and \( H(f) \) is the \( n \times k \) frequency response function (FRF) model matrix. The \( H(f) \) matrix, which describes the behavior of the system to external excitations, is obtained by updating a Finite Element Model (FEM) using several sets of measured force/acceleration data. Equation (1) consists of a set of systems of linear equations, each one corresponding to a frequency \( f \), where \( N_f \) is the total number of frequency lines in the study range. Like most of the mechanical inverse problems, this system of equations is ill-conditioned and needs special consideration during data processing at the inversion step. This system is solved by the optimization of a regularized cost function that promotes sparsity in the solution, i.e. the force vector. The algorithm used in this study localizes and reconstructs the time history of the applied force vector. Not all of the available sensors are used in the load identification step.

3. EXPERIMENTAL SETUP

The structure under consideration in this paper is a footbridge, located in Ninove (Belgium). It is a two-span cable-stayed steel bridge with a main and secondary span of 36 m and 22.5 m, respectively (see figure 1). The vibration measurements have been performed using 12 wireless GeoSIG GMS-18 units and a National Instruments (NI) data acquisition system. The GMS-18 units include a high-sensitivity three-axial accelerometer for vibration monitoring. The NI system consists of a PXI-1050 chassis with four PXI-4472B modules and has been used to record data from four PCB 393B04 uniaxial accelerometers, two PCB 086D50 instrumented hammers, a PCB 222B load cell and a BD 5 load cell to measure the tension forces applied by the artificial muscles, and two AWLG 008M optical displacement sensors to measure the vertical displacements of the footbridge.

The data set considered in this study corresponds to the case of multi-sine excitation through a pneumatic actuator, attached to the bridge at location 7 (see Figure 2). The actuator acts like an artificial muscle and converts the pneumatic energy to a tensile force. The actuator is connected to a solid base on the ground beneath the bridge. The load cell installed between the muscle and the bridge records the actual forces induced to the structure.

4. STRUCTURAL MODEL

As described in previous sections, the system model is obtained by updating a finite element (FE) model based on the in-situ identified modal characteristics. This section shows a more detailed view of the calibration procedure of a FE model of the Ninove footbridge. A finite element model of the footbridge has been developed using the FE program ANSYS. The model has a total of 2421 nodes and 2508 elements.

The calibration consists of adapting the model parameters (mode shapes, natural frequencies and modal damping) such that an optimal agreement is found between the identified and calculated modal characteristics of the footbridge. This results in a better representation of the dynamic behavior of the structure and, therefore, increases the accuracy of the numerical predictions using the model. The purpose of the model calibration is to find the optimal parameters that minimize the least squares difference between the FE model output and experimental observations.
The bridge is instrumented with 12 tri-axial wireless accelerometers. The blue dots correspond to accelerometer locations (12 DOF), and other coloured dots represent force cells and auxiliary sensors.

The calibration parameters of the FE model considered in this analysis are:

- the stiffnesses of the neoprene bearings
- the Young’s modulus of the bridge deck
- the Young’s modulus of the pylons
- the effective Young's modulus of the cables.

The natural frequencies and mode shapes are used as the observed quantities in the calibration procedure. The initial values of the calibration parameters are based on prior knowledge on the materials used and the present support conditions. The model calibration is done for all the identified modes, in the frequency range up to 20 Hz.

The Frequency Response Function (FRF) matrix describing the input/output relations is deduced from the updated modal parameters resulting from the calibration process. This FRF matrix will be used in next sections in order to reconstruct the loads acting on the bridge.

5. FORWARD VERIFICATION

Verifying the quality of the system model (H) is needed for solving every inverse problem. The forward check constitutes a necessary but not sufficient condition for this purpose. This verification consists of comparing the measured accelerations with the estimated accelerations that are computed using the updated FEM and the measured forces. The results of the forward check show that in general, a very good
overall agreement between the measured and simulated response signals is obtained. As this condition is satisfying, the updated FEM can be used in the inverse calculation for force reconstruction, as explained in the next sections.

Figure 4: In the forward validation process, the estimated accelerations are calculated based on the FEM and the measured forces, then compared with acceleration measurements.

Figure 5: The good agreement of the estimated and measured accelerations validates the forward verification.

6. INVERSE CALCULATION

The first challenge in solving the problem in equation [1] is to reformulate the 3D system of equations ($[N_0 \times N_i \times N_f]$) into a 2D matrix form. To obtain the new settings, matrices $X(f)$, $H(f)$ and $F(f)$ are rearranged. Matrices $\bar{F}$ and $\bar{X}$ are created by concatenating respectively the matrices $F$ and $X$ of each frequency $f$ into a column vector. The $\bar{H}$ matrix is a block diagonal matrix obtained by placing all the matrices $H(f)$ on the diagonal. Another issue concerns the size of the mathematical system to solve. Only $N_s$ out of $N_f$ frequency lines are selected (around resonance peaks) in order to reduce the computation time in the localization step. The obtained FRF model is smaller in size, but still able to describe the main system dynamics. After the end of localization process, the force time history will be
calculated with all of the $N_f$ frequency lines. After the application of the settings, the new problem will become the following:

$$\bar{X}(f) = \bar{H}(f) \bar{F}(f), \ f \in \{f_1, \ldots, f_{N_s}\}$$  \hspace{1cm} (2)

The solution of this problem can be estimated using least squares as follows:

$$\hat{F} = \text{argmin}_F \left\{ \frac{1}{2} \| \bar{H} \bar{F} - \bar{X} \|_2^2 \right\}$$  \hspace{1cm} (3)

Since the number of rows in $\bar{H}$ is smaller than the number of columns ($k > n$), the problem in (3) is ill-posed. One promising method to overcome the ill-posedness problem is to penalize the least squares problem in equation (3) as follows:

$$\hat{F} = \text{argmin}_F \left\{ \frac{1}{2} \| \bar{H} \bar{F} - \bar{X} \|_2^2 + \lambda g(\bar{F}) \right\}, \ \lambda \geq 0$$  \hspace{1cm} (4)

where $g(\bar{F})$ is the penalty function and $\lambda$ is the regularization parameter. An appropriate choice of $g$ is crucial to obtain good results [1,21,22]. Considering the physics of the problem, we suggest that an adequate penalty function should possess at least the following three properties:

1. **Sparsity constraint:**
   Since the external forces are usually applied only at a few unknown (discrete) locations on the structure, the force vector is assumed to be zero on most of the candidate force locations. In this sense, the force vector is expected to resemble a high contrast pattern. In other words, along the force locations, a sparse solution is desirable for each frequency. Therefore, using an $\ell_1$-penalized model (over the candidate locations) seems to be more suitable (sparsity).

2. **Continuity constraint:**
   As the structure is excited with localized external forces (such as hammer or shaker), the force vector should retain its continuity over the frequency axis. In case of an impulse excitation (ex. hammer), the force will appear in almost every frequency due to the broadband frequency content of the impact force. To ensure the continuity of the force vector in the frequency axis, we suggest the use of the $\ell_2$-norm penalty. This means that at a given location on the structure, the probability for an element in the frequency axis to be non-zero depends on other elements of that particular location in the frequency axis. In other words, for each force location, the estimated force vector should be zero or non-zero along the frequencies (non-sparsity). Therefore, the use of $\ell_2$-norm is promoted along the frequencies.

3. **Convex cost function:**
   The convexity of the problem in equation (4) will guarantee a unique solution. For $\lambda \geq 1$, the cost function $g(\lambda)$ is convex.

Following the idea of group Lasso in [19,23], we propose to induce a group structure to the problem in equation (4) using an appropriate penalty function that includes all the required properties (sparsity, continuity and convexity). The groups are defined as follows:

$$G_i = \{F_i(f_1), \ldots, F_i(f_{N_s})\}, \ i \in \{1, \ldots, k\}$$  \hspace{1cm} (5)

In accordance with the previous section, the continuity constraint ($\ell_2$-norm penalty) is applied within each group elements, and the sparsity constraint ($\ell_1$-norm penalty) is applied between the groups. Therefore, the new penalty function becomes a hybrid $\ell_{1&2}$-norm, as described in equation (6). As this penalty
function is convex, it contains all the desirable properties mentioned in previous sections. The new penalized least squares problem is defined in equation (6).

\[ \| \tilde{F} \|_{1\&2} = \sum_{i=1}^{k} \| \tilde{F}_{Gi} \|_2 \] (6)

\[ \hat{F} = \text{argmin}_{F} \left\{ \frac{1}{2} \| \tilde{H} \hat{F} - \tilde{X} \|_2^2 + \lambda \| \hat{F} \|_{1\&2} \right\}. \] (7)

This problem is convex and consists of a smooth part (least squares) and non-smooth part (\( \ell_{1\&2} \)-norm). The proposed iterative algorithm in [24] (FISTA) is customized to be able to solve this problem. FISTA solves an \( \ell_1 \)-norm penalized least squares problem for real parameters. Therefore two major modifications are needed to adapt this algorithm for the problem in equation (7): the algorithm has to solve \( \ell_{1\&2} \)-norm penalized least squares for grouped variables, and it has to work with complex entries. In summary, the algorithm can be interpreted as a special case of Majorize-Minimization algorithm (MM) that tries to find the minimum of a surrogate cost function instead of the actual cost function [25].

The detailed algorithm is presented by Rezayat et al. in [21]. The convergence is achieved when the difference between the iteration steps becomes smaller than \( 1 \times 10^{-5} \). The G-FISTA method has the advantage of being simple to implement. The mathematical manipulations consist of a few matrix multiplications and the computation of the largest eigenvalue of a sparse matrix (only calculated once).

In a realistic case, there is no prior information on the exact location or amplitude of the applied forces. Model selection tools such as the Bayesian Information Criterion (BIC) are generally used in these cases to find the best trade-off between the system complexity and the fitting error [20]. The modified BIC used in this study as defined in [26] is the following:

\[ \text{BIC}(\lambda) = \log \left( 1 + \sum_{i=1}^{p} \| (\tilde{H} \hat{F}_{\lambda} - \tilde{X})_i \|_2^2 \right) + \frac{\log(p)}{2p} n_r \] (8)

where \( n_r \) is the number of zero elements in the residual vector \( R_{\lambda} = \| \tilde{H} \hat{F}_{\lambda} - \tilde{X} \| \), and \( p \) is the total number of unknowns \( N_s \ast k \). Some simulation and experimental results are illustrated in the next section. For practical reasons, we use the force index parameter \( \beta \) as a representative value of the force vectors [16]. This parameter is normalized over all the force locations (groups) and frequencies.

7. RESULTS AND DISCUSSIONS

The result of the force localization using G-FISTA algorithm is presented in figure 7. The estimated force signals in time domain are obtained by inverse Fourier transform. Figure 8 illustrates the fitting quality of the estimated force vectors. The agreement between the exact and estimated force signals emphasises the accuracy of the G-FISTA method. In this case, the force was applied at location 7, and the algorithm seems to be successful in finding the load position on the bridge.

While different choices of the penalty parameter \( \lambda \) produce different force estimates, using a larger \( \lambda \) would shrink the estimated force more toward zero, therefore, a larger \( \lambda \) in case G-FISTA would produce a more sparse estimation. Therefore, choosing an appropriate \( \lambda \) is the main challenge in all penalized methods. Like any other penalized methods, a model selection technique has to be applied in order to make an appropriate choice of \( \lambda \) parameter, that yields to the most physical force scenario. The tuning procedure of the regularization parameter needs an extensive study on its own. But in general, it seems that using a modified version of the BIC model is useful in case of structured sparsity. The \( \lambda \) with smallest BIC should be chosen.
Figure 6: The bridge is excited by means of pneumatic artificial muscles installed at location 7.

Figure 7: The bridge is excited by artificial muscles in the frequency range of [0.5 - 10] Hz. The normalized force index calculated based on G-FISTA method shows a good estimation of the force localization. A threshold of 5% is considered on the force index in this case.

As described in [21], one can observe that methods based on structured sparsity provide generally better results in comparison to the pseudo-inverse method. In the category of penalized algorithms, G-FISTA produces more accurate force estimations than Tikhonov regularization and FISTA ($\ell_1$ penalized approach). Algorithms purely based on $\ell_1$-norm penalization can result in good estimations when the applied point forces excite only a limited and discrete spectrum. The performance of latter methods is well pronounced when impact loads are considered in time domain [4].

The strength of G-FISTA in localizing point forces relies on the fact that this algorithm constitutes a general case with a mixed penalty function that can be simplified in either $\ell_1$ or $\ell_2$-norms as special cases. In other words, if only one big group (consisting of all components of $\bar{F}$ together) is defined, then the structured sparsity will act the same as usual $\ell_2$ regularization (ex. Tikhonov). However, if each frequency component of the force vector $\bar{F}$ is allocated to a separate group, (i.e. one element per group) the structured sparsity will become equivalent to the $\ell_1$ penalised case. Therefore, G-FISTA is a general algorithm that can deal with all cases of point force identification problems. This makes G-FISTA a strong toolbox to deal with different force configurations without the need for any prior knowledge on the type of the applied localized force.
8. CONCLUSIONS

In this contribution the load identification problem has been considered on the Ninove pedestrian bridge in presence of ambient noise. A dynamic model of the structure is composed from a detailed finite element model of the structure, that has been calibrated using a set of experimental modal characteristics. Numerical simulations show a good overall agreement between the true dynamic behavior of the footbridge and the one predicted by the model. The governing linear system of equations form an ill-conditioned problem that is sensitive to measurement noise.

The G-FISTA method proposed by [21, 24] was applied on the data set in frequency domain. The algorithm allows the localization of the applied force, and reconstructs the force vector. The mixed penalty function used in this study takes advantage of both the $\ell_1$ and $\ell_2$-norm properties simultaneously. The continuity constraint in the frequency axis is ensured by the $\ell_2$-norm, and the sparsity constraint is created using the $\ell_1$-norm over candidate force locations. Finally, the estimated solution is selected by minimizing the Bayesian Information Criterion (BIC). The obtained results illustrate the efficiency of G-FISTA approach in localizing point forces on the bridge. The experiments in this contribution show that the location and time history of discrete forces on real structures is correctly estimated using G-FISTA.

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