Revenue Sharing and Competitive Balance

Does the invariance proposition hold?

Prof. dr. Stefan Kesenne

Economics Department

University of Antwerp

and

Physical Education Department

Catholic University of Leuven (KUL)

University of Antwerp

Prinsstraat, 13, B-2000 Antwerp

Tel. 0032-3-2204105

Fax. 0032-3-2204799

e-mail: stefan.kesenne@ua.ac.be

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Abstract

This short paper, challenging the so-called invariance proposition, argues that, for a general n-team model with profit maximizing clubs and concave revenue functions, there are reasons to believe that revenue sharing can worsen the competitive balance among teams in a professional sports league. If clubs are win maximizers, revenue sharing improves the competitive balance.

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1. Introduction

Revenue sharing and its impact on the competitive balance in a sports league have been discussed by many sports economists during the last decades. The so-called 'Invariance proposition', claming that revenue sharing does not affect the distribution of playing talent among profit maximizing clubs, has been challenged by many economists since the seminal article of Rottenberg (1956) and the formal proof of the proposition by Quirk and El Hodiri (1974). Atkinson, Stanley and Tschirhart (1988), Marburger (1997) and Kesenne (2000) showed that, with a more general club revenue function, that also includes the absolute quality of a team, or the winning percentage of the visiting team, revenue sharing can improve the competitive balance. Fort and Quirk (1995) showed that, if clubs have other revenues that are not shared, revenue sharing can worsen the competitive balance. Kesenne (2001) showed that the effects of revenue sharing on competitive balance also depend on the specifics of the sharing arrangement, such as match gate sharing or pool sharing. In all these studies, one of the important implicit or explicit hypotheses of the model is that the supply of talent is constant, and moreover that clubs take this fact into consideration, meaning that team managers take into account that hiring an extra talent not only strengthen their team, but also weakens another team in the same league, which affects the marginal revenue of talent. This hypothesis can be realistic, at least in the short run, for the closed North

American Major Leagues, but it can hardly be a realistic assumption for the post-Bosman[1] European setting of many open national soccer leagues. In a recent paper, Szymanski and Kesenne (2003) have shown that revenue sharing worsens the competitive balance if the supply of talent is not constant, but flexible. Moreover, they argue that the constant supply approach in Quirk and El-Hodiri (1974), Fort and Quirk (1995) and Vrooman (1995) is not a Nash equilibrium, and show how the Nash concept implies a different solution. Also in the constant-supply approach, revenue sharing worsens the competitive balance. However, in their analysis, the authors start from a simplified two-team model, while it has been shown that not all results from a 2-club model, regarding the impact of revenue sharing, apply to an n-club model (Kesenne, 2000).

In this paper we try to find out how revenue sharing affects the distribution of playing talent among the clubs in a n-club league if the supply of talent is constant or flexible. In section 2, an appropriate n-club model is specified. Section 3 investigates the impact of a specific revenue sharing arrangement and section 4 concludes.

2. A simple n-club model with flexible talent supply

Let's start from the following concave revenue functions for the n clubs in the league:

$$R_i(m_i, w_i) \qquad \text{with } \frac{\partial R_i}{\partial m_i} > 0 \quad \frac{\partial R_i}{\partial w_i} > 0 \quad \frac{\partial^2 R_i}{\partial w_i^2} < 0 \quad \frac{\partial R_i^2}{\partial w_i \partial m_i} > 0 \quad \text{for } all\ i \qquad (1)$$

[1]The Bosman verdict (1995) by the European Court of Justice in Luxemburg established a free move of players between the countries of the European Union.

R is a club's total season revenue, w is the winning percentage and m is the size of the market or the drawing potential of the club. So, we assume that, besides the size of the market, only the own winning percentage, or the relative quality of the team, affects season revenue.

We further assume that the win percent of a team is equal to n/2 times its relative playing strength, so that we can write that:

$$w_i = \frac{n}{2} \frac{x_i}{\sum_k^n x_k} \qquad \textit{for all } i \tag{2}$$

where x_i is the number of playing talents (not the number of players) of a team. The factor n/2 is due to the fact of life that the sum of the winning percentages of n teams does not equal one but n/2, which is a constant, so that it can be deleted without affecting the analysis below. The sum of all talents in the denominator of (2) can be considered as the total supply of playing talent in the league. So far, in most analyses, this variable has been, implicitly or explicitly, assumed to be constant, and substituted into the objective function. In our approach, we assume the supply of talent to be flexible, given a fixed marginal cost of talent. It is clear that in the open national leagues of European soccer, where clubs can also hire talent from other national leagues, an extra talent for one club does not imply a loss of talent of another club in the same national league. It follows that the impact of an extra talent has a smaller effect on club revenue, if that talent is hired form another league, so that the marginal revenue of talent can be calculated as:

$$\frac{\partial R_i}{\partial x_i} = \frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial x_i} = \frac{\partial R_i}{\partial w_i}\frac{\sum_{k \neq i}^{n} x_k}{(\sum_{k}^{n} x_k)^2} \qquad \textit{for all } i \qquad (3)$$

The effect that a talent increase of one club has on the revenue of another club is then:

$$\frac{\partial R_j}{\partial x_i} = \frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial x_i} = \frac{\partial R_j}{\partial w_j}\frac{-x_j}{(\sum_{k}^{n} x_k)^2} \qquad \textit{for all } i \neq j \qquad (4)$$

But, even with a constant supply of talent that is not taken into account by team managers in determining their marginal revenue of talent, the marginal revenue of talent can be written as in (3) and (4). The difference with the "internalised" constant supply approach is clear: with a fixed talent supply $\sum_{k}^{n} x_k$, which can be rescaled to equal one, expressions (3) and (4) would have been:

$$\frac{\partial R_i}{\partial x_i} = \frac{\partial R_i}{\partial w_i}\frac{1}{\sum_{k}^{n} x_k} = \frac{\partial R_i}{\partial w_i} \qquad (5)$$

$$\frac{\partial R_j}{\partial x_i} = \frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial w_i}\frac{1}{\sum_{k}^{n} x_k} = \frac{\partial R_j}{\partial w_j}\frac{-1}{n-1} \qquad (6)$$

In this constant supply case, it is perfectly possible, and most likely, that the increase in win percent of one team only affects the win percent of one other team. However, for symmetry reasons, the last factor of (6) is based on the hypothesis that all n-1 teams bear an equal loss of win percent caused by the increase of the win percent of one team. This also implies that, in a large league with many teams, the right hand side of (6) is close to zero.

A simple 2-club model with quadratic revenue functions shows the impact of a flexible or a (non-internalized) constant supply of talent on the talent distribution among clubs. Let the revenue function of a large and a small market club be:

$$R_i = m_i w_i - 0.5 w_i^2 \qquad R_j = m_j w_j - 0.5 w_j^2 \qquad \textit{with } m_i > m_j \qquad \textit{(7)}$$

The supply of talent is put equal to unity, and is fully used in a competitive (free agency) player market. Because profit maximizing clubs hire talent until marginal revenue equals marginal cost, taking into account the constant supply of talent, the talent distribution is given by:

$$m_i - w_i = m_j - w_j \qquad \text{so that:} \qquad \frac{w_i}{w_j} = \frac{x_i}{x_j} = \frac{1 + m_i - m_j}{1 + m_j - m_i} \qquad \textit{(8)}$$

If the constant supply is not taken into consideration, the distribution is given by:

$$(m_i - w_i)x_j = (m_j - w_j)x_i \qquad \text{so that:} \qquad \frac{w_i}{w_j} = \frac{x_i}{x_j} = \frac{m_i}{m_j} \qquad (9)$$

This talent distribution of talent in (9) also emerges if the supply of talent is flexible, given a marginal cost of talent. Comparing the solutions (8) and (9), it is also clear that, for this simple model, the latter case results in a more balanced competition than the former. The hypothesis of a constant marginal product of talent leads to a competitive dominance bias.

3. The impact of a revenue sharing arrangement

The specific sharing arrangement we start from in this chapter is a pool sharing system, which consists of the contribution of a fixed percentage of a club's total season revenue to a league fund that is redistributed- equally among all clubs in the league. The pool sharing system is different from a typical gate sharing system where home and visiting team share the revenue of each match. The pool sharing system applies to the sharing of broadcasting rights, sponsoring and merchandizing money in some European countries and in the UEFA's Champions League.

If μ is the share parameter and R_i^* is the club's total season revenue after sharing, we can write this sharing arrangement[2] as:

$$R_i^* = \mu R_i + \frac{(1-\mu)}{n}\sum_{k}^{n} R_k = \mu R_i + (1-\mu)\overline{R} \qquad \textit{for all } i \qquad (10)$$

which can also be written as:

$$R_i^* = \frac{(n-1)\mu + 1}{n} R_i + \frac{(1-\mu)}{n}\sum_{k \neq i}^{n} R_k \qquad \textit{for all } i \qquad (11)$$

[2] Note that this pool sharing system and the typical gate sharing system are equivalent if there are only two clubs in the league.

To find out what effect is of this sharing system on the distribution of playing talents in the league, we will not use the appropriate comparative static analysis as in Szymanski and Kesenne (2003), which turned out to become very cumbersome, even for a simple 2-team model. We used the approach of Marburger (1997) and Kesenne (2000), investigating the shifts of the teams' demand curves for talent, caused by the sharing arrangement. Although this approach, based on partial derivatives, does not provide a full proof of the proposition, it does indicate that pool revenue sharing in an n-team model can worsen the competitive balance. The rationale behind this approach is the plausible assumption that, if, for any pair of clubs in the league, the downward shift of the demand curve of the higher talented club is smaller than the downward shift of the demand curve of lower talented club, measured at the initial player market equilibrium point, revenue sharing worsens the competitive balance.

If clubs are profit maximizers, the demand curve for playing talent is given by the marginal revenue product, i.e.:

$$\frac{\partial R_i^*}{\partial x_i} = \frac{(n-1)\mu + 1}{n} \frac{\partial R_i}{\partial w_i} \frac{\sum_{k \neq i}^{n} x_k}{(\sum_{k}^{n} x_k)^2} - \frac{1-\mu}{n} \sum_{k \neq i}^{n} \left(\frac{\partial R_k}{\partial w_k} \frac{x_k}{(\sum_{k}^{n} x_k)^2} \right) \quad \textit{for all } i \qquad (12)$$

In order to find the shifts of these demand curves, the first derivative of (12) with respect to the share parameter μ can be calculated as:

$$\partial \frac{\partial R_i^*}{\partial x_i} / \partial \mu = \frac{(n-1)}{n} \frac{\partial R_i}{\partial w_i} \frac{\sum_{k \neq i}^{n} x_k}{(\sum_{k}^{n} x_k)^2} + \frac{1}{n} \sum_{k \neq i}^{n} \left(\frac{\partial R_k}{\partial w_k} \frac{x_k}{(\sum_{k}^{n} x_k)^2} \right) > 0 \quad \textit{for all } i \qquad (13)$$

Because this expression is clearly positive, revenue sharing causes all clubs to reduce their demand for playing talent, for the obvious reasons that they all have to share their marginal revenue of hiring playing talent.

More important for the purpose of our paper is the size of these shifts of the demand curves at the initial equilibrium point, i.e. at the point where the club's marginal revenue equals the market clearing marginal cost of playing talent *c* before sharing, i.e. where:

$$\frac{\partial R_i}{\partial x_i} = \frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial x_i} = \frac{\partial R_i}{\partial w_i}\frac{\sum_{k\neq i}^{n} x_k}{(\sum_{k}^{n} x_k)^2} = c \qquad \textit{for all } i \qquad (14)$$

Substituting (14) into (13) for all clubs yields:

$$\partial\frac{\partial R_i^*}{\partial x_i}/\partial\mu = c[\frac{(n-1)}{n} + \frac{1}{n}\sum_{j\neq i}^{n}(\frac{x_j}{\sum_{k\neq j}^{n} x_k})] \qquad \textit{for all } i\neq j \qquad (15)$$

From this result it can be derived that the club with the higher number of playing talents before sharing (which is not necessarily the large-market club), will reduce its demand for talent less than the club with the lower number of playing talents, i.e.:

$$\text{If } x_i > x_j \quad \text{then} \quad \partial\frac{\partial R_i^*}{\partial x_i}/\partial\mu < \partial\frac{\partial R_j}{\partial x_j}/\partial\mu \qquad \textit{for all } i\neq j \qquad (16)$$

It follows that the revenue sharing arrangement worsens the competitive balance.

This can be illustrated with a 3-club model. For the 3 clubs the downward shifts of the demand curves, according to expression (15), are:

For club 1: $$c[\frac{2}{3}+\frac{1}{3}(\frac{x_2}{x_1+x_3}+\frac{x_3}{x_1+x_2})] \tag{17}$$

For club 2: $$c[\frac{2}{3}+\frac{1}{3}(\frac{x_1}{x_2+x_3}+\frac{x_3}{x_1+x_2})] \tag{18}$$

For club 3: $$c[\frac{2}{3}+\frac{1}{3}(\frac{x_1}{x_2+x_3}+\frac{x_2}{x_1+x_3})] \tag{19}$$

Comparing these shifts, it is clear that the expressions:

$$(17) < (18) < (19) \quad \text{if} \quad x_1 > x_2 > x_3. \tag{20}$$

which confirms the general result in (16)[3] .

A few remarks

- Expression (15) also shows that, if the number of teams in the league is very high, the term in brackets approaches one, so that the downward shift in demand for talent of all teams is more or less the same. It follows that in this case the invariance proposition still stands.
- A complication of the model above is that, in European soccer, clubs can hire players from both their own national league and from other foreign leagues. This mix of playing talents in a club can be introduced in the analysis by assuming that the

[3] It is also easy to check with this method that the Quirk and El Hodiri (1974) result was right that revenue sharing does not change the competitive balance in a closed league with a fixed talent supply.

playing talents in our model (x) can be decomposed in national league talents x_{1i} and foreign league talents x_{2i} so that:

$$x_i = x_{1i} + x_{2i} \qquad \textit{for all i} \qquad (21)$$

The marginal revenue of own-league talent and foreign-league talent are then given by (5) and (3) respectively, with (5) clearly larger than (3). It follows that, assuming the same marginal cost of own-league talent and foreign-league talent in the European competitive player market after Bosman, clubs will rather hire talent from their own national league than from foreign leagues, ceteris paribus. The reason is clear: when a club hires new playing talents from clubs in their own national league, it reduces also the playing strength of its competitors in the league, which enhances its own marginal productivity. The pool revenue sharing arrangement (10) in this mixed-talent model will still worsen the competitive balance, because of the unequal shift in the demand for foreign talent by high talented and low talented teams. Quirk and El Hodiri (1974) have shown that this revenue sharing arrangement causes an equal downward shift of the demand curves for own-league talent. And, as shown above, the club with the better team reduces its demand for foreign-league talent less than the club with the weaker team. The combination of these two results, in a mixed-talent model, indicates that this pool revenue sharing arrangement worsens the distribution of talent across teams in a league.

- Whereas the pool sharing system arrangement, as specified in (10), worsens the competitive balance in a league if all clubs are profit maximizers, it is obvious that

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Using (5) and (6), result (15) becomes: $\partial \dfrac{\partial R_i^*}{\partial x_i} / \partial \mu = c(\dfrac{n-1}{n} + \dfrac{1}{n-1})$ which is the same for all i.

revenue sharing improves the competitive balance if clubs are win maximizers given a fixed profit rate, even if the talent supply is flexible. In that scenario, the demand curve for talent is given by the net average revenue curve (i.e. after the subtraction of the fixed capital cost and the fixed profits, which can also be zero). Since the sharing arrangement in (10) reduces the total revenue of every club that has a higher revenue than the average revenue in the league, the result is an improved competitive balance.

- It is also possible that in one league, some clubs are profit maximizers and others are win maximizers. In the most likely case of the poorer clubs being win maximizers, and the richer and more talented clubs being profit maximizers, the impact of revenue sharing is clear. Because the poor clubs will increase their demand for talent and the rich clubs will reduce their demand for talent, the impact of revenue sharing is a better competitive balance.

4. Conclusion

The impact of revenue sharing on competitive balance is complicated, because it depends on many factors, such as the objectives of the clubs, the specification of the revenue functions, the specific sharing arrangement in operation and the supply of talent. Moreover, some results based on a simple two-team model do not apply to a general n-team model. This contribution only adds to our knowledge that a pool revenue sharing arrangement in a simple model with a flexible supply of talent, which is the case in the national soccer leagues in the European Union, or a non-internalized constant supply of talent, worsens the competitive balance, at least, if clubs are profit maximizers. This result does not imply that also gate revenue sharing worsens the competitive balance in a league, where also the win percent of the visiting team affects club revenue, or where some club revenue is not shared. For these complications, further research is needed.

References

- Atkinson S.E., Stanley L.R., Tschirhart J., (1988), Revenue sharing as an incentive in an agency problem: an example from the National Football League, RAND Journal of Economics, vol. 19, nr 1.
- Fort R., Quirk J. (1995), Cross-subsidization, Incentives and outcomes in Professional Team Sports Leagues, Journal of Economic Literature, XXXIII, pp. 1265-1299
- Kesenne S., (2000), Revenue Sharing and Competitive Balance in Professional Team Sports, Journal of Sports Economics, vol. 1, nr. 1, p.56-65
- Kesenne S., (2001), The different impact of different sharing systems on the competitive balance in professional team sports, European Sports Management Quarterly, vol. 1. Nr. 3, p. 210-218.
- Marburger D.R., (1997), Gate Revenue Sharing and Luxury Taxes in Professional Sports, Contemporary Economic Policy, vol. XV, april
- Quirk J., El Hodiri M., (1974), The Economic Theory of a Professional Sports League, in: Noll, R.G., ed. (1974), Government and the Sport Business, Brookings Institution, Washington DC.
- Rottenberg Simon (1956), The Baseball Players' Labor Market, Journal of Political Economy, 64
- Szymanski S. and Kesenne S. (2003), Competitive balance and gate revenue sharing in team sports, *Journal of Industrial Economics*, forthcoming.
- Vrooman J., (1995) A general theory of professional sports leagues, Southern Economic Journal, 61 / 4