Commuting, congestion tolls and noncompetitive labour markets: optimal congestion pricing in a wage bargaining model

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Abstract

The purpose of this paper is to study optimal congestion tolls in a bargaining model of the labour market. The model distinguishes commuting and non-commuting transport, and it allows some telecommuting. We first show that transport taxes as well as congestion levels raise negotiated wages and reduce employment levels; more efficient homework reduces wages. We then study optimal second-best labour and transport tax policies and find that they strongly depend on the impact of congestion on labour market outcomes. If transport taxes have to be uniform for commuting and other trip purposes, the optimal peak period transport tax positively depends on the impact of congestion on negotiated wages, and negatively on the wage effects of the congestion tax itself. Moreover, the tax rises to the extent that demand by people that are not active on the labour market represent a larger fraction of peak period transport flows. We further show that more efficient homework does not necessarily reduce congestion nor the optimal transport tax. If taxes can be differentiated according to trip purpose, ‘subsidizing’ commuting is justified as a way to shift the tax burden away from the employed. Finally, extending the model for multiple transport modes we find that the impact of congestion on negotiated wages provides an argument for subsidizing commuting by public transport but not by car, a proposal suggested in several European countries.

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Introduction

Congestion is a serious problem in many large urban areas throughout the world. In the US, for example, the Texas Transportation Institute reports very high increases in traffic congestion over the past two decades (Schrank and Lomax (2002)). In Europe, a comparative study of five urban areas, including London, Amsterdam and Brussels, shows a dramatic decline in average speed over the past decade; marginal external congestion costs have been estimated to exceed 1 Euro per kilometre in the most congested cities (De Borger and Proost (2001)). Economists have suggested a large variety of policy instruments to cope with congestion, and the use of some form of congestion pricing has taken a prominent place in this literature (see, among many others, Arnott, de Palma and Lindsay (1993), Verhoef, Nijkamp and Rietveld (1995), Mayeres and Proost (1997), and Small and Yan (2001))\(^1\). Congestion taxes are also high on the political agenda in several countries, and the first examples of actual implementation are either available (e.g., Singapore, Trondheim, Stockholm, Oslo, London) or under serious consideration (see, e.g., the Netherlands and several US states, including California, Florida and Texas).

The most severe congestion problems are typically associated with the journey-to-work, and the direct link between commuting and the labour market has raised some concern about the employment effects of congestion taxes. Several recent papers have studied this interaction between congestion taxes and outcomes on the labour market. For example, assuming competitive labour markets and perfect complementarity between commuting and labour supply, Parry and Bento (2001) analyze the implications of a revenue-neutral congestion tax, financed by a reduction in the tax on labour. They show that such a tax reform does not necessarily reduce labour supply due to the feedback effects of congestion: the congestion tax reduces congestion which indirectly raises labour supply. In fact, at relatively low levels of the transport tax, the employment effects of raising the tax are shown to be positive. Calthrop (2001) introduces multiple trip purposes, commuting and non-commuting, into the model and analyzes in detail the implications of the complementarity of commuting with labour supply. More recently, Van Dender (2003) studies the optimal

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\(^1\) Of, course, road pricing and congestion taxes are by no means the only instruments that have been suggested. For some recent contributions that focus on other instruments, see Arnott, Rave and Schöb (2005), Parry and Small (2005), and De Borger and Mayeres (2006).
tax structure, allowing for multiple trip purposes and multiple transport modes, and finds empirical support for taxing commuting at a lower rate than non-commuting transport. Moreover, the welfare cost of not differentiating the transport tax according to trip purpose is found to be substantial.

Although these seminal papers have yielded important new insights, the underlying assumptions are somewhat less than desirable, especially in a European context. Competitive labour markets are clearly unrealistic, since in most countries wages and employment levels are the result of an explicit bargaining process between unions and employer organisations. Moreover, perfect complementarity between labour supply and commuting obscures the distinction between taxes on labour and commuting taxes (Van Dender (2003)), and it has been challenged in view of recent advances in telecommuting possibilities (see, e.g., Safirova (2002)). The purpose of this paper is, therefore, to study optimal congestion tolls in a bargaining model of the labour market. The model allows for different trip purposes and it relaxes the perfect complementarity between work and commuting, incorporating telecommuting possibilities. Within this framework, we first analyze to what extent changes in congestion and congestion taxes are likely to affect the outcome of the bargaining process. We then study the optimal second-best tax problem faced by a budget-constrained benevolent government that cares about the wellbeing of all citizens (the employed as well as those not employed) and private sector profits. It uses taxes on transport and on labour as the main instruments. We consider both the case where transport taxes cannot be differentiated according to trip purpose and the case of optimal tax differentiation between commuting and other transport.

We obtain several interesting results. First, for various specifications of union preferences we show that, under plausible peak period conditions, both exogenous increases in congestion levels and in transport taxes directly raise negotiated wages and reduce employment levels. The total impact of higher transport taxes on employment, taking account of the indirect feedback effects through congestion changes, is not unambiguous; it depends on the relative importance of congestion and the transport tax at the negotiation table. Second, we show that if transport taxes cannot be differentiated according to trip purpose, the optimal transport tax positively depends on the impact of congestion on negotiated wages, and negatively on the wage effects of the congestion tax itself. Intuitively, if wages strongly rise due to congestion then it is optimal to raise the congestion tax: by doing so one reduces congestion and,
hence, lowers wages and stimulates employment. Similarly, the more sensitive negotiated wages are to higher congestion taxes, the lower the optimal tax will be, again to reduce employment effects. For specific union utility functions, we further find that transport taxes rise to the extent that demand by people not currently active on the labour market represent a larger fraction of the peak period transport flow. Moreover, it is found that higher efficiency of telecommuting does not necessarily reduce congestion and the optimal transport tax. Third, if taxes can be differentiated according to trip purpose we find that commuting subsidies are generally justified to the extent that peak period transport flows include demand by people that don’t work, allowing taxes to be shifted away from the employed. The commuting tax is shown to be below marginal external cost. Finally, when we consider multiple transport modes we find that, if congestion affects negotiated wages, this may provide an argument for subsidizing commuting by public transport only, a proposal suggested in several European countries (Potter et al. (2006)).

This paper is related to, and contributes to, several strands of the recent literature. First, like Parry and Bento (2001), Van Dender (2003) and Safirova (2002) we study the relation between congestion taxes, commuting and the labor market, but we do so in a model of wage bargaining between unions and firms. As far as we know, with the exception of a recent paper by Van Ommeren and Rietveld (2005), this is the first study using a bargaining framework to look at this interaction of congestion and the labour market\(^2\). Second, the paper contributes to the growing literature on the implications of externality taxation in bargaining models of the labour market (see, e.g., Koskela, Schob and Sinn (1998), Strand (1998), Schneider (1997), Holmlund and Kolm (2000), Bayundir-Uppmann and Raith (2003), and Schöb (2005)). These models typically focus on environmental externalities in the production sector and assume separable utility structures; this implies that the outcome of the bargaining process is independent of the level of environmental quality. However, congestion associated with commuting is the prototype example of a non-separable externality which generates feedback effects on demand (see, e.g., Mayeres and Proost (1997)). As a consequence, both the externality itself and the externality tax are likely to affect negotiated wages and employment levels. Third,

\(^2\) Van Ommeren and Rietveld (2005) develop a search model of the labor market to explain the fact that the average commuting time has hardly varied over time. However, they did not incorporate congestion effects, nor did they study the implications for optimal taxation.
indirectly the paper also contributes to the growing literature on the potential desirability of ‘subsidizing’ commuters, in the sense of allowing tax deductibility of commuting expenses. Different countries have very divergent policies on this issue (for a recent survey, see Potter et al. (2006)). For example, the US and Canada do not allow deductibility, but many European countries such as Germany, France and the Scandinavian countries do. Economic arguments in favor include the presence of distortionary or suboptimal labor taxes (Wrede (2000), Van Dender (2003)) and the mobility of firms and households in a spatial economy (Wrede (2001)). Recently, Borck and Wrede (2005) study the problem from a distributive angle. They show that desirability of commuting subsidies strongly depends on the distribution of landownership across income classes and the location of different income groups in an urban area. Note, however, that with the exception of Van Dender (2003) none of these models incorporates congestion; moreover, none takes account of bargaining on the labor market.

Structure of the paper is as follows. In Section 1 we introduce the most basic version of the model. Section 2 looks at the effects of congestion, transport taxes and labour taxes on negotiated wages and employment. In Section 3 we derive and interpret the optimal tax structure for the case of uniform transport taxes across trip motives. Section 4 deals with tax differentiation between commuting and non-commuting transport. The optimal treatment of different transport modes is looked at in Section 5. Finally, Section 6 summarizes the main conclusions.

1. Structure of the basic model

In this section we present the structure of the model. We make the following assumptions. First, we normalize the population at one. A fixed fraction \( m \) \((0 < m < 1)\) of the population participates on the labour market, a fraction \((1-m)\) does not participate at all (retired people, discouraged workers, etc.). Employment as a fraction of the population is given by \( L \) \((0 < L < m)\), unemployment amounts to \((m-L)\). To save on notation, we assume that all participants on the labour market are union members and that they are represented by a single union; in other words, union membership is given by \(m\). However, although we assume a single
union, we follow the literature in assuming that it behaves like a ‘small’ union (see, e.g., Schöb (2005)). This implies, among others, that it does not internalize the indirect effects of employment and wages on congestion when negotiating with firms. Second, to keep the bargaining process manageable, we assume fixed effective working times per worker, so that negotiations between union and firm are about wages and employment only, not about the labour input per worker (e.g., hours of work)\(^3\). Third, labour supply and commuting are treated as imperfect complements. Possibilities for telecommuting are incorporated by assuming exogenous productivity differences between working time on the job and work done at home. Fourth, the road network, used by commuters and non-commuters alike, is represented as a single road link. The model is assumed to be representative of peak period transport conditions; transport flows on the network consist of both commuting and non-commuting transport. There is no freight transport. Finally, in this section we assume uniform taxes across trip purposes; differentiation between commuting and other transport is considered in Section 4.

1.1. Household behaviour

We consider two types of households. The first type are the employed; the second type are people that, for various reasons, are not employed\(^4\). First, consider the employed. Their preferences are defined over consumption of a general consumption good, transport, and leisure; moreover, they care about working at home or working on the job. Effective labour supply per worker is fixed, but telecommuting allows some choice between working on the job and working at home. However, the effective labour input per day of home work is only a fraction \(\alpha (0 < \alpha < 1)\), exogenous to the worker, of a day worked on the job. Finally, the wage per effective

\(^3\) In principle, of course, bargaining might not only deal with wages and employment but it might also include hours of work. Unfortunately, models where wages, employment and hours are simultaneously negotiated in a Nash setting yield few general intuitive results, and they suffer from serious technical difficulties (see, e.g., Hart (2004, p. 66) for discussion). Moreover, various alternative simplifying models have been criticized because of their lack of realism. For example, assuming a right to manage setting where negotiation is about wages only and firms then unilaterally decide on employment as well as hours is likely to be unacceptable to the union. Similarly, negotiating about wages and hours and then letting the firm decide on employment is probably unacceptable to the firm (Hart (2004)).

\(^4\) Note that this group not only includes the unemployed but also people not participating on the labour market at all (retired, discouraged workers, etc.). What matters for the results is whether or not people are employed (and hence are subject to a labour tax), see below.
time unit is given by \( w \). In other words, \( w \) is the wage for a day at the job; it is the outcome of the negotiations between union and firm.

The model considers typical peak-period transport conditions over extended periods of time. It is assumed that the employed demand both commuting and non-commuting trips (leisure trips during days off, dropping kids at school, etc.). To save on notation, we assume that each day of work on the job requires one commuting (round) trip so that the demand for commuting transport (i.e., the number of commuting roundtrips) equals the number of days worked on the job. Non-commuting demand is expressed in the same units (i.e., in standardized commuting roundtrips). The producer price of transport is normalized at one. The employed face taxes imposed by the government on labour and on transport; the relevant unit tax rates are \( \tau_L, \tau_T \), respectively. The government also provides a lump-sum transfer, denoted by \( G \).

Given the above assumptions, the choice problem faced by the employed is to maximize:

\[
\max \ u(C^e, T_n^e, T_c^e, \ell^e, H^e)
\]

subject to the constraints

\[
C^e + (1 + \tau_T)T_n^e = [w - \tau_L - (1 + \tau_T)]T_c^e + (w - \tau_L)\alpha H^e + G
\]

\[
\ell^e + (T_n^e + T_c^e)\alpha + (T_c^e + H^e) = R
\]

\[
T_c^e + \alpha H^e = D
\]

In this formulation \( C^e \) is consumption, \( T_n^e \) and \( T_c^e \) are non-commuting and commuting transport demand (the latter equals, by our assumptions, time worked on the job), \( \ell^e \) is leisure, and \( H^e \) is time worked at home. Finally, \( a \) is the time needed to travel one commuting roundtrip, \( R \) is the exogenously given time available, and the effective labour time per worker is fixed at \( D \).

The first constraint is the budget restriction. The left hand side captures spending on consumption and non-commuting transport; the right hand side is net total income. It consists of the net labour income derived from working on the job and at home, plus the transfer from the government. The second constraint captures time

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\(^5\) Note that, from the viewpoint of the firm, work on the job and home labour are assumed to be perfect substitutes. The parameter \( \alpha \) may reflect a marginal productivity differences; an alternative interpretation is that it reflects some cost for the firms associated with telecommuting. Alternative ways to introduce the telecommuting option are available (Safirova (2002)); they would not affect the qualitative results of this paper.
allocation across leisure time, travel time, and total working time. The third equation divides fixed effective labour supply per worker across working on the job and at home.

Associating multipliers \( \lambda^e, \beta^e, \gamma^e \) with the constraints, we have the first-order conditions:

\[
\begin{align*}
  u_C &= \lambda^e \\
  u_t &= \beta^e \\
  u_{t_e} &= \lambda^e (1 + \tau_T) + \beta^e a \\
  u_{t_e} &= \lambda^e (1 + a + \tau_T) + \beta^e (1 + a + \gamma^e) \\
  u_{H} &= -\lambda^e [\alpha (w - \tau_L)] + \beta^e + \alpha \gamma^e
\end{align*}
\]

where subscripts of the utility function denote partial derivatives. Following Becker (1965) and Jara-Diaz (2000) we define the value of time \( v^e \) as the marginal utility of leisure relative to consumption. Manipulating the first-order conditions we then easily show:

\[ v^e = \frac{u_{t_e}}{u_C} = \frac{\lambda^e + [(w - \tau_L) - (1 + \tau_T)] - \frac{\gamma^e}{\lambda^e}}{1 + a} \]

The value of time depends on the wage net of labour tax and commuting costs, on the disutility of commuting, on congestion and on the disutility of work (also see De Borger and Van Dender (2003)).

The solution of the optimisation problem yields the indirect utility function of the employed \( V^e(w - \tau_L, \tau_T, a, \alpha, G) \). The envelope theorem implies:

\[
\begin{align*}
  \frac{\partial V^e}{\partial \tau_T} &= -\lambda^e (T_c^e + T_n^e) < 0 \\
  \frac{\partial V^e}{\partial \tau_L} &= -\lambda^e < 0 \\
  \frac{\partial V^e}{\partial a} &= -\beta^e (T_c^e + T_n^e) = -v^e \lambda^e (T_c^e + T_n^e) < 0 \\
  \frac{\partial V^e}{\partial \alpha} &= \left[ \lambda^e (w - \tau_L) - \gamma^e \right] H^b > 0
\end{align*}
\]
It follows that taxes and congestion both reduce utility, more effective home labour, in
the sense of a higher $\alpha$, raises utility. The last inequality follows as long as there is
disutility of work. Indeed, it is implied by $u_e < 0, u_H < 0$, see equations (3d, 3e).

Next we turn to people that have no employment. They maximise:

$$u(C^u, T_n^u, \ell^u, G)$$

subject to

$$C^u + (1 + \tau_T)T_n^u = G$$

$$l^u + T_n^u a = \bar{L}$$

where all variables have the same interpretation as before, and the subscript $u$ refers to
those not employed. The value of time is denoted similarly as $v^u = \frac{\beta^u}{\lambda^u}$, where the
multipliers have the same interpretation as before. Indirect utility is $V^u(\tau_T, a, G)$, where, by the envelope theorem:

$$\frac{\partial V^u}{\partial \tau_T} = -\lambda^u T_n^u < 0$$

$$\frac{\partial V^u}{\partial a} = -\beta^u T_n^u = -v^u \lambda^u T_n^u < 0$$

1.2. Employer behaviour

Competitive firms produce a single output using labour as the only variable
factor of production. The production function is given by:

$$f(L)$$

where $L$ is employment (defined as a fraction of total number of households, see
before). The output of the firm is sold at a fixed competitive world price, for
convenience normalised to 1, so that profits are given as:

$$\pi(w, L) = f(L) - wL$$

Here $w$ is the wage per effective time unit of labour input, and $f'(L) > 0$, $f''(L) < 0$.
Applying the implicit function theorem to (7) immediately implies that iso-profit
curves are downward sloping as long as wages exceed the marginal product of labour.

1.3. Union behaviour

The union is assumed to care about wages and employment. Moreover, at the
bargaining stage it treats taxes, congestion levels and efficiency of home work as
given. The union’s utility function is written in general:
\[ \Omega(w, L; \tau_L, \tau_T, a, \alpha) \]  

We assume positive marginal utilities of wages and employment, so that indifference curves are negatively sloped. The role of taxes, congestion and the efficiency of home work in union preferences will be described in Section 2 below.

1.4. The specification of congestion

The level of congestion depends on the aggregate demand for transport. Total transport demand by the employed and unemployed is given by:

\[ T = L(T^e + T^u) + (1 - L)T^w \]  

Note that \( T^e \) is the number of round trip commutes per employed person, the other transport demands are expressed in the same normalized units. Congestion is specified as:

\[ a = a(T), \text{ where } a' > 0, \ a'' > 0 \]  

1.5. Government behaviour

The government collects revenue from taxes on wages, transport and profits. Tax revenues are used to finance the lump sum transfer \( G \) provided to consumers. It is assumed for the purpose of this paper that \( G \) is fixed; this allows us to focus on the optimal tax structure for a government that is required to generate a fixed amount of revenues. The government budget constraint is thus given by:

\[ \tau_T T + \tau_L L + \tau_\pi \pi \geq G \]  

where \( \tau_\pi \) is the profit tax on the firm.

2. Congestion and the impact of taxes on labour market outcomes

In this section, we analyze the labour market effects of taxes and other exogenous parameters. We first look at the impact of changes in congestion, in (labour and transport) taxes and in the efficiency of home work on negotiated wages and employment levels. Then we consider the total or long-run impact of tax increases that take into account all feedback effects of congestion on the transport and labour markets.
2.1. The effect of taxes and congestion on negotiated wages and employment levels

We focus throughout this section on the right-to-manage bargaining model, in which the union and the firm bargain over wages only; the firm determines employment along its labour demand curve, given the negotiated wage. We denote the labour demand function as $L'(w)$; it is the solution of the firm’s first order condition for profit maximisation $w = f'(L)$. Hence, its slope is given by $\frac{\partial L'(w)}{\partial w} = \frac{1}{f''(L)} < 0$.

The outcome of the negotiation process can be modelled as the solution to the problem:

$$\max_w Z(w) = \mu \ln(\Omega) + (1 - \mu) \ln(\pi)$$

(12)

where $\mu$ reflects the relative power of the union. Moreover, profit and union utility were specified before, see (7) and (8), respectively. To obtain the impact of exogenous parameters on negotiated wages, we take the first derivative of $Z(w)$ with respect to $w$ and we use $w = f'(L)$ in the resulting first-order condition. This yields:

$$Z_w = \frac{\mu}{\Omega} \frac{\Omega_w}{\pi} - \frac{(1 - \mu)}{\pi} L = 0$$

(13)

where $\Omega_w = \frac{d\Omega}{dw} = \frac{\partial \Omega}{\partial w} + \frac{\partial \Omega}{\partial L} \frac{1}{f''}$. Applying the implicit function theorem to (13) then yields, after straightforward manipulations:

$$\frac{dw}{d\tau_L} = \frac{Z_w}{Z_{ww}} = N\left[\Omega_w \Omega_{\tau_L} - \Omega_{w\tau_L} \Omega\right]$$

(14a)

$$\frac{dw}{d\tau_T} = \frac{Z_w}{Z_{ww}} = N\left[\Omega_w \Omega_{\tau_T} - \Omega_{w\tau_T} \Omega\right]$$

(14b)

$$\frac{dw}{d\alpha} = \frac{Z_w}{Z_{ww}} = N\left[\Omega_w \Omega_\alpha - \Omega_{w\alpha} \Omega\right]$$

(14c)

$$\frac{dw}{d\alpha} = \frac{Z_w}{Z_{ww}} = N\left[\Omega_w \Omega_\alpha - \Omega_{w\alpha} \Omega\right]$$

(14d)

Here $N = \frac{\mu}{Z_{ww} \Omega^2}$, and $Z_{ww} < 0$ by the second order conditions of the optimisation problem. It follows that $N < 0$.

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*We also considered Nash bargaining over wages and employment; the results were qualitatively very similar. Details are available from the author.*
Of course, the sign and magnitude of many of the second cross derivatives of the union utility function appearing in (14a-14d) is not a priori obvious. To obtain further insight, we therefore proceed by focusing on a concrete specification for the union utility function. Specifically, assume that union preferences are given by a standard Cobb-Douglas expression of the expected utility of employed and unemployed members:

\[ \Omega = \left[ L V^e(.) \right]^\beta \left[ (m-L)V^u(.) \right]^{1-\beta} \]  

(15)

where \( V^e(.) = V^e(w-\tau_L, \tau_T, a, \alpha, G) \) and \( V^u(.) = V^u(\tau_T, a, G) \) are indirect utility of employed and unemployed members, respectively. Note that \( \beta \) reflects the relative importance the union gives to the wellbeing of the two groups of members. In Appendix 1 we show that this specification implies the following signs of the partial effects of exogenous changes in taxes, congestion, and efficiency of home work:

\[ \frac{\partial w}{\partial \tau_L} > 0, \quad \frac{\partial w}{\partial \tau_T} > 0, \quad \frac{\partial w}{\partial a} > 0, \quad \frac{\partial w}{\partial \alpha} < 0 \]  

(16)

Since wage-employment outcomes are on the labour demand curve, it immediately follows that \( \frac{\partial L}{\partial k} = \frac{1}{f^w} \frac{\partial w}{\partial k}, k \in (\tau_L, \tau_T, a, \alpha) \). Hence, employment effects have the opposite sign compared to the effects on wages.

Note that, consistent with the literature (see, e.g., Lockwood and Manning (1993), Pissarides (1998), Bayundir-Uppman and Raith (2003)), we find that labour taxes raise negotiated wages and reduce employment. Consistent with Van Ommeren and Rietveld (2005), the same holds for an exogenous increase in transport taxes. We further find that congestion itself positively affects the negotiated wage. Although earlier models have not included congestion, this seems plausible. It implies that increases in the monetary cost (transport tax increase) and in the time cost (congestion increase) have a similar impact on negotiated outcomes. Finally, increasing the

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7 Other specifications were used as well with qualitatively similar results, see below. The problem is that very little is known about the importance of congestion and transport taxes in union preferences. Direct information on the channels by which transport problems enter union preferences is unavailable, and the degree to which congestion issues are explicitly emphasized at the negotiation table is unobservable.
efficiency of home work reduces the negotiated wage and raises employment. Home work makes union members better off, reducing wage claims. We considered various other specifications of union utility that capture different attitudes towards the mobility problem of union members. Appendix 1 has all the details. The results summarized in (16) were found to be quite robust across specifications. All alternatives considered imply that labour taxes raise negotiated wages. Moreover, they all suggest that, assuming peak period conditions, transport taxes and congestion levels raise negotiated wages and reduce employment.

2.2. The total impact of labour and transport tax changes on labour market outcomes

In this subsection, we derive the total effect of labour and transport tax changes on wages and employment, taking into account all feedback effects on transport and labour markets through congestion. The total wage effect consists of a direct effect (at constant congestion) and an indirect effect through congestion changes:

\[
\frac{dw}{d\tau_L} = \frac{\partial w}{\partial \tau_L} + \frac{\partial w}{\partial a} \frac{dT}{d\tau_L}, \\
\frac{dw}{d\tau_T} = \frac{\partial w}{\partial \tau_T} + \frac{\partial w}{\partial a} \frac{dT}{d\tau_T}
\]

(17)

where the partial derivatives on the right hand side are the effects on negotiated wages studied in subsection 2.1. Similarly, we have for the total employment effects of tax changes:

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8 It is well known that in a right-to-manage bargaining setting the effect of higher union power is to raise wages. An interesting question is then whether or not a more powerful union will be more successful in transmitting higher congestion taxes and higher congestion into higher wages. In Appendix 1 we show that this depends on the production structure. For example, for the iso-elastic production function considered by Van Ommeren and Rietveld (2005), it is shown that a more powerful union will be satisfied with smaller wage increases when transport costs or congestion increase. However, this is not a general result. A more general production structure may yield the opposite, viz. that a more powerful union may use its power to obtain a larger wage increase to compensate workers for the higher commuting costs.

9 For several specifications, (16) holds unambiguously. For others, a transport tax increase raises wages as long as transport demand of the employed exceeds demand of people not employed, i.e., as long as \((T_e + T_u) - T_u > 0\). Similarly, for some specifications congestion raises negotiated wages as long as the employed ‘suffer’ more from congestion than people not employed; the condition is that \([v^e(T_e + T_u) - v^uT_u] > 0\), where the \(v^i\) (\(i = e, u\)) are the respective values of time of the employed and unemployed. Reflecting peak period conditions and reasonable assumptions about time values, both conditions seem plausible. For details, see Appendix 1.
To evaluate these expressions, we first consider the final term on the right hand side, the total impact of taxes on transport demand. To do so, let us rewrite the definition of transport demand given in (9) as follows:

\[ T = L(T^e) + (1 - L)T^u \]  

(19)

Where, to save on notation, \( T^e = T_c^e + T_a^e \) and \( T^u = T_a^u \). Consumer demand for transport depends on taxes and congestion, see section 1.1:

\[
T^e = T^c(w - \tau, \tau, a, \alpha, G) \\
T^u = T^u(\tau, a, G)
\]  

(20)

Moreover, wages and employment depend on taxes and congestion via the negotiations between unions and firms, as argued in subsection 2.1:

\[
L = L(\tau, \tau, a, \alpha) \\
w = w(\tau, \tau, a, \alpha)
\]  

(21)

Using (19)-(21), and noting that congestion \( a(T) \) depends on total transport demand (see (10)), simple manipulations yield the following results:

\[
\frac{dT}{d\tau_L} = \frac{1}{1 - \rho} \left[ L \frac{\partial T^e}{\partial \tau} (\frac{\partial w}{\partial \tau_L} - 1) + (T^e - T^u) \frac{\partial L}{\partial \tau_L} \right]
\]  

(22)

\[
\frac{dT}{d\tau_T} = \frac{1}{1 - \rho} \left[ L \frac{\partial T^e}{\partial \tau} + (1 - L) \frac{\partial T^u}{\partial \tau_T} + (T^e - T^u) \frac{\partial L}{\partial \tau_T} + L \frac{\partial T^e}{\partial \tau_T} \frac{\partial w}{\partial \tau_T} \right]
\]  

(23)

where

\[
\rho = L \frac{\partial T^e}{\partial a} a' + (1 - L) \frac{\partial T^u}{\partial a} a' + L \frac{\partial T^e}{\partial w} \frac{\partial w}{\partial a} a' + (T^e - T^u) \frac{\partial L}{\partial a} a'
\]

(24)

is the total feedback effect of congestion on transport demand. The feedback accounts for the fact that congestion reduces transport demand directly as well as indirectly (via changes in employment and wages). Unless conditional transport demand is extremely wage-sensitive, we have \( \rho < 0 \), and the feedback effect reduces the direct effect\(^{10}\).

Consider the total effect of the labour tax on transport demand given in (22). The labour tax affects negotiated wages as well as employment levels. Employment

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\(^{10}\) On the feedback effect see, among others, Sandmo (2000) and Mayeres and Proost (1997).
changes affect the demand for transport to the extent that the conditional demands for transport by the employed and by people that do not work differ (see the final term between square brackets). Moreover, increases in wages affect transport demand by the employed to the extent that it changes the after-tax wage and that transport demand is wage-sensitive (first term)\textsuperscript{11}. Similarly, the transport tax affects demand through various different channels, see (23). For given wage and employment levels, it reduces the demand of both the employed and those not employed. Moreover, there are additional impacts through negotiated wages and, to the extent transport demand by the unemployed and the employed differ, through changes in employment. Expression (23) shows that the overall effect is highly plausibly negative.

Finally, substitute (22)-(23) into (17)-(18) to find, after straightforward algebra:

\[
\frac{dL}{dT} = \frac{1}{1 - \rho} \left[ \left( 1 - L \frac{\partial T^e}{\partial a} \right) \frac{\partial L}{\partial T} - (L \frac{\partial T^e}{\partial w} \frac{\partial w}{\partial a}) \right] \tag{25}
\]

\[
\frac{dw}{dT} = \frac{1}{1 - \rho} \left[ \left( 1 - L \frac{\partial T^e}{\partial a} \right) \frac{\partial w}{\partial T} - (L \frac{\partial T^e}{\partial w} \frac{\partial L}{\partial a}) \right] \tag{26}
\]

\[
\frac{dL}{\tau} = \frac{1}{1 - \rho} \left[ \left( 1 - L \frac{\partial T^e}{\partial a} \right) \frac{\partial L}{\partial \tau} - \left[ L \frac{\partial T^e}{\partial \tau} + (1 - L) \frac{\partial T^u}{\partial \tau} \right] \frac{\partial L}{\partial a} \right] \tag{27}
\]

\[
\frac{dw}{\tau} = \frac{1}{1 - \rho} \left[ \left( 1 - L \frac{\partial T^e}{\partial a} \right) \frac{\partial w}{\partial \tau} + \left[ L \frac{\partial T^e}{\partial \tau} + (1 - L) \frac{\partial T^u}{\partial \tau} \right] \frac{\partial w}{\partial a} \right] \tag{28}
\]

Relations (25)-(26) suggest that higher labour taxes reduce employment and raise wages, unless the effects of congestion on the outcomes of the negotiations are very strong and wage effects of conditional transport demand by the employed are large. Interestingly, (27)-(28) show that transport taxes may raise or decrease employment depending on the relative impact of transport taxes and congestion at the negotiation table\textsuperscript{12}. Consider two extremes. Suppose a transport tax increase has a large effect on

\textsuperscript{11} In principle, a labour tax increase can be over-shifted, so that net of tax wages rise (see Lockwood and Manning (1993), Calthrop and De Borger (2004)).

\textsuperscript{12} Parry and Bento (2001) also find that a transport tax increase has ambiguous effects on employment, depending on the relative impact of tax and congestion changes. However, direct comparison with (27)-(28) is not obvious for several reasons. First, they consider revenue-neutral transport tax increases. The results given here concern exogenous increases in transport taxes; they hold independent of possible additional tax recycling effects on employment. Second, in their model wages are exogenously fixed; moreover, employment and transport demand are perfectly complementary.
the outcomes of the negotiations whereas congestion itself does not, then the first term between the brackets of (27)-(28) dominates: higher transport taxes raise wages and employment declines. Alternatively, however, if congestion is an important issue in the negotiations but transport taxes have little effect on negotiated outcomes, then the second term dominates and higher transport taxes raise employment.

3. Optimal transport and labour taxes in a bargaining model

In this section we turn to the optimal second-best tax problem faced by a benevolent government. We first set up the problem and derive the first-best solution; next we derive and interpret the general optimal second-best tax structure. We further discuss the role of transport demand by people not employed, and we briefly analyze the impact of more efficient home work on optimal congestion taxes. Throughout, technical details are delegated to Appendices.

3.1. General formulation of the problem and the first-best solution

The government maximises social welfare subject to financing constraints, taking into account the bargaining process. It has three tax instruments: the labour tax on the employed, a congestion tax on all transport, and a tax on domestic profits. The profit tax is mainly introduced to illustrate the first-best outcome, see below. Social welfare takes account of the well being of all households (the employed, the unemployed and those not participating on the labour market) as well as firm profits. Moreover, the government faces two constraints. First, a budget restriction requiring total tax revenues to cover exogenous expenditures to finance a fixed lump-sum transfer G. Second, for political or institutional reasons the tax on domestic profits may be restricted\(^1\).

Making abstraction of distributive issues, we formulate the problem just described in the simplest possible way. The government is assumed to face the following problem:

\[
\text{Maximize} \quad \frac{1}{\lambda_0} LV^e (w - \tau_L, \tau_T, a) + \frac{1}{\lambda_0} (1 - L) V^u (\tau_T, a) + (1 - \tau_T) \left[ f(L) - wL \right]
\]

\(^1\) For example, as noted by Schöb (2005), it may have difficulty distinguishing between pure profit and return to capital investments, or it may limit profit taxes in order to prevent firms from fleeing the country. For the role of profit taxes in optimal tax models see, e.g., Richter and Schneider (2001).
\[
\text{s.t. } \tau_T + \tau_L + \tau_x \left[ f(L) - wL \right] = G
\]
\[
\tau_x \leq \bar{\tau}_x
\]
where \( \bar{\tau}_x \) is the maximum profit tax rate politically or technically feasible. Note that the share of people not employed \((1 - L)\) includes the unemployed \((m - L)\) as well as those not participating \((1 - m)\). Moreover, we have divided indirect utility by the marginal utility in a reference situation (denoted by \( \lambda_n^c, \lambda_n^w \)) to transform utility into monetary units.

We associate multipliers \( \gamma \) and \( \eta \) with the budget constraint and the profit restriction, respectively. In Appendix 2 we show that the set of first-order conditions imply:

\[
\frac{dL}{d\tau_L} \left[ V^e - V^u + \gamma \tau_L \right] + L \frac{dw}{d\tau_L} \left[ \tau_x (1 - \gamma) \right] + \frac{dT}{d\tau_L} \left[ \gamma \tau_T - MEC \right] + L(\gamma - 1) = 0 \quad (29)
\]
\[
\frac{dL}{d\tau_T} \left[ V^e - V^u + \gamma \tau_L \right] + L \frac{dw}{d\tau_T} \left[ (1 - \gamma) \tau_x \right] + \frac{dT}{d\tau_T} \left[ \gamma \tau_T - MEC \right] + T(\gamma - 1) = 0 \quad (30)
\]
\[
\eta - \pi(\gamma - 1) \leq 0; \quad \tau_x \left[ \eta - \pi(\gamma - 1) \right] = 0 \quad (31)
\]
\[
\tau_x - \bar{\tau}_x \leq 0; \quad \eta \left[ \tau_x - \bar{\tau}_x \right] = 0 \quad (32)
\]

To facilitate the interpretation, in the above expressions we have assumed constant marginal utility of income, normalized for simplicity at one (see Appendix 2 for details). The \( MEC \) is the marginal external congestion cost; it reflects the welfare effect for road users of a small increase in transport demand. Further note that the total derivatives in (29)-(30) are given by expressions (22)-(23) and (25)-(28) of the previous section.

The first-best solution is easily described by assuming that an unrestricted profit tax is available. If the profit constraint is not binding we have \( \eta = 0 \) (see (32)). Expression (31) then implies \( \gamma = 1 \): the shadow cost of public funds equals one. Substituting \( \gamma = 1 \) in (29)-(30) then immediately reveals the first-best solution: tax transport at marginal external cost, and use the labour tax to exactly offset the distortion on the labour market that is due to the negotiation process. The optimal labour tax is therefore negative: the government subsidizes the monetary value of the
utility difference between employed and unemployed to make sure the gross social opportunity cost of labour to firms is on the contract curve, and hence efficient\textsuperscript{14}.

3.2. The optimal second-best tax structure

If the profit restriction does bind, the government has to raise any required revenues via distortionary taxes on labour and transport. It follows from (31)-(32)) that the shadow price of the government budget constraint exceeds one ($\gamma > 1$). We can then rewrite (29)-(30) in matrix notation:

\[
\begin{bmatrix}
\frac{dL}{d\tau_L} & \frac{dT}{d\tau_L} \\
\frac{dL}{d\tau_T} & \frac{dT}{d\tau_T}
\end{bmatrix}
\begin{bmatrix}
V^e - V^u + \gamma \tau_L \\
\gamma \tau_T - MEC
\end{bmatrix} =
\begin{bmatrix}
(1-\gamma) \left( T \frac{dL}{d\tau_L} - T \frac{dT}{d\tau_L} \right) \\
(1-\gamma) \left( T - \tau_s \frac{dL}{d\tau_T} - T \frac{dT}{d\tau_T} \right)
\end{bmatrix}
\]

Using Cramer’s rule, the solution satisfies:

\[
V^e - V^u + \gamma \tau_L = \frac{1-\gamma}{H} \left\{ L \frac{dT}{d\tau_T} - T \frac{dT}{d\tau_L} + \left[ L \tau_s \left( \frac{dL}{d\tau_L} - \frac{dT}{d\tau_T} \right) - \frac{dL}{d\tau_T} \right] \right\}
\]

\[
\gamma \tau_T - MEC = \frac{1-\gamma}{H} \left\{ T \frac{dL}{d\tau_L} - L \frac{dL}{d\tau_T} + \left[ L \tau_s \left( \frac{dL}{d\tau_T} - \frac{dT}{d\tau_L} \right) - \frac{dT}{d\tau_T} \right] \right\}
\]

where

\[
H = \frac{dL}{d\tau_L} \frac{dT}{d\tau_T} - \frac{dL}{d\tau_T} \frac{dT}{d\tau_L}
\]

is the determinant of the system.

The tax rules are elaborated in Appendix 2, using the various total derivatives derived above. The general rules are quite cumbersome, and interpretation is facilitated by assuming zero income effects of transport demand. The optimal tax structure is then given by the following expressions:

\[\text{\textsuperscript{14} See, among others, Schöb (2005), for discussion. Note that, with Nash bargaining over wages as well as employment, the wage-employment outcome is on the contract curve. One then easily shows that the first-best optimal labour tax is zero.}\]
\[
\tau_L = -\left(\frac{V^e-V^u}{\gamma}\right) + \left(\frac{1-\gamma}{\gamma}\right) \left\{ L[S]\left[1-\tau\frac{\partial w}{\partial \tau_L}\right] + (T^e-T^u) \left( L\frac{\partial L}{\partial \tau_T} - T\frac{\partial L}{\partial \tau_L} \right) \right\} \frac{\partial L}{\partial \tau_L} [S] \right) 
\]

\[
\tau_T = \frac{MEC}{\gamma} + \left(\frac{1-\gamma}{\gamma}\right) \left[ T - \left(\frac{1-\gamma}{\gamma}\right) S \right] \frac{\partial w}{\partial \tau_T} - \left(\frac{1-\gamma}{\gamma}\right) \left( \frac{\partial w}{\partial \tau_T} - \left(\frac{\partial w}{\partial \tau}\right) \right) \frac{\partial w}{\partial \tau_L} \right) 
\]

where

\[
S = L\frac{\partial T^e}{\partial \tau_T} + (1-L)\frac{\partial T^u}{\partial \tau_T} < 0 
\]

\[
\rho_l = \left(\frac{L\frac{\partial T^e}{\partial \tau} + (1-L)\frac{\partial T^u}{\partial \tau}}{\partial a} \right) a' < 0 
\]

The term \(\rho_l\) summarizes the direct feedback effects of congestion on transport demand. Higher congestion reduces demand, hence \(\rho_l < 0\). Note that expressions (33)-(34) are quite general: they are independent of the specification of union preferences. Union utility functions only determine the various partial effects of taxes and congestion on wages and employment, see sub-section 3.3 below.

Briefly consider the optimal labour tax in (33). If the budget restriction is not strong (in the sense that \(\gamma\) is close to one), the first term on the right hand side dominates and the optimal tax is negative: it corrects the distortion created by the wage negotiations. If the constraint is sufficiently stringent, the labour tax is plausibly positive. The profit tax is typically restricted to be substantially smaller than 100%. Moreover, transport demand by the employed typically exceeds demand from those not employed. Finally, the term \(L\frac{\partial L}{\partial \tau_T} - T\frac{\partial L}{\partial \tau_L}\) is plausibly positive\(^{15}\). Observe that the optimal labour tax declines with increases in the profit tax; it also declines to the extent that a labour tax increase strongly affects negotiated wages.

To interpret the transport tax rule first note that, if transport taxes and congestion do not affect negotiated wages, the rule simplifies to:

\(^{15}\) We were unable to determine its sign in general; however, it was found to be positive for all union utility specifications considered in Appendix 1.
\[ \tau_r = \frac{\text{MEC}}{\gamma} + \left( \frac{(1 - \gamma)(1 - \rho)}{\gamma} \right) [S] \]

This well-known rule states that the optimal tax consists of a Pigovian component and a revenue-raising Ramsey term (see, among many others, Bovenberg and Goulder (1996)). The Ramsey term is inversely proportional to the own price effect, captured by \([S]\). The weight of the Pigovian component declines when government funds are more highly valued.

Let us focus now on the final two terms on the right-hand-side of (34). It has been argued in the previous section that transport taxes and congestion are likely to affect negotiated wages and employment. If this is the case, (34) suggests that this may have important implications for congestion taxes. First, suppose that commuting taxes are partially shifted forward to employers in the form of higher negotiated wages. Then the third term on the right hand side of (34) suggests that this reduces the optimal transport tax. Intuitively, a lower commuting tax reduces wages and, at given congestion levels, raises employment. Second, however, to the extent that congestion itself positively affects negotiated wages the optimal transport tax rises, see the final term on the right hand side of (34). The intuition here is that, if congestion strongly raises wages and reduces employment, raising the transport tax reduces traffic levels and hence congestion. This reduction in congestion in turn reduces wages at the negotiation table and, therefore, partially counteracts the effect of the congestion tax itself on employment. Note that the optimal transport tax is higher the larger the (negative) effect of the labour tax on employment; this is due to the correspondingly lower labour tax in that case, so that the congestion tax has more of a revenue generating role.

Together, these findings imply that depending on the role of congestion and congestion taxes in wage negotiations, there may be a case for imposing transport taxes that are ‘structurally’ above or below the standard Ramsey tax. At one extreme, suppose that commuting taxes play an important role at the negotiation table but that congestion is not really an issue. This is an argument for relatively low optimal transport taxes. Alternatively, if congestion itself is a major issue during negotiations then this provides an argument for raising commuting taxes. This reduces congestion and hence dampens the negative effect of congestion on employment.
The role of the wage effects of transport taxes and congestion for the optimal tax structure can also be interpreted in a slightly different way. Reformulate (34) as follows:

\[
\tau_T = MEC + \left\{ \frac{1-\gamma}{\gamma} \left( 1 - \rho_T \right) \right\} \left( T - L \frac{\partial \tau_T}{\partial \tau_L} \right) + \left[ \frac{1-\gamma}{\gamma} \right] \left( MEC - L \frac{\partial \tau_T}{\partial \tau_L} \right) \right\} \quad (37)
\]

This expresses the optimal tax as the marginal external cost plus two ‘correction’ terms. The first one captures the difference in the trade-off between transport and labour taxes for consumers and for workers. To see this, note that, by Roy’s identity, transport demand gives the trade-off (i.e., the relative welfare effects on consumers) between a transport and a labour tax increase at given wages. The trade-off from the viewpoint of workers is given by the relative compensation they can obtain at the negotiation table for higher transport and labour taxes. Similarly, the second correction term captures the different trade-offs between congestion and money for consumers and workers. The marginal congestion cost is the welfare effect for the consumer, expressed in monetary terms, of an increase in congestion. The final term is the trade-off between congestion and money for the employed as it results from the wage negotiations.

Interpretation of (37) is then clear. If workers are ‘overcompensated’ in the wage negotiations for an increase in the transport tax, the optimal response is to reduce the tax. Similarly, if workers are overcompensated for congestion changes the optimal tax on transport rises. In other words, the tax rises if reducing congestion is valued more highly by the labour market than by consumers.

3.3. The role of the composition of transport demand

The results in the previous subsection were independent of the specification of union preferences. Some further relevant insights can be obtained by elaborating the results for specific union utility functions. To illustrate this point, we worked out the tax rules for the Cobb-Douglas specification of union preferences (15), used above. For this specification, we show in Appendix 2 that the transport tax rule can be simplified to:

\[
\tau_T = MEC + \left( \frac{(1-\gamma)}{\gamma} \right) \left( \frac{(1-\rho_T)}{(S)} \right) \left( T^u \right) \quad (38)
\]
where, as before, \( v' \) is the value of time of people not employed.

Expression (38) nicely illustrates the role of transport demand by people that are not employed. Since the shadow cost of funds exceeds one and the term between square brackets is necessarily negative, the tax rule states that the higher the demand by transport users that are not currently active on the labour market, the more the transport tax exceeds the marginal external cost. The reason for the higher transport tax is ‘tax shifting’ (see de Mooij and Bovenberg (1998) for an illustration of tax shifting in the context of capital taxation): by raising transport taxes, and correspondingly reducing labour taxes, the tax burden is implicitly shifted from employed people to people that are not working. The labour tax is more efficient in generating revenues on the employed, but this tax in unable to tax those not working. Note that the increase in transport tax will be larger the smaller the fraction of the employed in the population (yielding lower labour taxes, hence more revenues have to be generated by transport taxes), the larger the feedback effects of congestion on transport demand (large feedbacks reduce the revenues from given congestion taxes) and the smaller the own price sensitivity of transport demand.

Note that the above result (38) has a rather subtle implication. It is not the distinction between commuting and non-commuting that matters for the tax rule, it is whether demand comes from people that are employed or not. What matters for tax shifting is whether or not people are affected by the labour tax, and not whether the trips they make are commuting or non-commuting trips. To make this very concrete, suppose that everyone where employed, but that on an average day a certain fraction of peak period trips are non-commuting trips (some people having a day off, people dropping kids off at school, etc.). Then (38) suggests this is not a reason for raising the tax above marginal external cost; it does allow tax shifting. Only when demand comes from people not employed does the tax rise above MEC.

Finally, note that for this specification, the commuting tax equals marginal external cost if transport demand by those not employed is zero. However, this is not a general result. In Appendix 2 one easily shows that for some union utility specifications deviations between taxes and marginal external costs remain even if transport demand comes from the employed only. This happens whenever the valuation of congestion and transport taxes by the union differs from the value to
commuters. The government then ‘corrects’ the too high or too low valuation of the union by appropriately adjusting optimal transport taxes.

3.4. The impact of more efficient home work on optimal transport taxes

If technological improvements raise the efficiency of telecommuting, the question is how this will affect marginal congestion costs and optimal transport taxes. One easily shows that the effect of more efficient homework is generally ambiguous.

Although the efficiency of home work not only affects the marginal external cost of a given transport increase but also the trade-offs between transport variables and money, we focus on a particularly simple case to illustrate our point. Consider the Cobb-Douglas specification of union preferences and suppose $T^u = 0$, so that (38) implies the tax rule $\tau_T = MEC$. Note that, assuming normalized marginal utility of income equal to one,

$$MEC = -\left[L \frac{\partial V^e}{\partial a} + (1 - L) \frac{\partial V^u}{\partial a}\right] a'$

We then easily derive the effect of more efficient home work on marginal external cost as:

$$\frac{\partial \tau_T}{\partial a} = -a'L \frac{\partial^2 V^e}{\partial a \partial \alpha} - L \frac{\partial V^e}{\partial a} \frac{\partial L}{\partial a} + \frac{\partial L}{\partial \alpha} \frac{\partial V^e}{\partial \alpha} - L^2 a'' \frac{\partial V^e}{\partial a} \frac{\partial \tau_T}{\partial \alpha}$$

Given the signs derived before, we know $\frac{\partial^2 V^e}{\partial a \partial \alpha} < 0$, $\frac{\partial L}{\partial a} > 0$. Moreover, the appreciation of more efficient homework is likely to rise at high congestion, so we plausibly have $\frac{\partial^2 V^e}{\partial a \partial \alpha} > 0$. Hence the first term on the right hand side is negative, the second is positive; the third term is negative if more efficient homework reduces the demand for transport. If the decline in transport demand per employed worker is substantial and the congestion function is highly convex, then marginal cost and the optimal tax go down. However, if more efficient home work leads to substantially more employment, then this may actually raise marginal congestion costs because of higher total transport demand.\(^{16}\) In sum, the overall effect is ambiguous.

\(^{16}\) For more detailed analysis of home work efficiency on transport demand, see Safirova (2002). She shows that it reduces transport costs, that it reduces agglomeration effects, and that it raises the diversity of inputs used in urban production. Our model does not contain the last two effects, but it suggests that the first effect may in fact go in the opposite direction.
4. Tax differentiation: should commuting be subsidized?

In this section, we look at the problem of tax differentiation between commuting and other transport. This issue is relevant in view of the large recent literature dealing with the potential desirability of subsidizing commuters (Wrede (2000, 2001), Richter (2004), Borck and Wrede (2005)). Assuming competitive labor markets and perfect complimentarity between commuting and work, the numerical results of Van Dender (2003) make a clear case for commuting subsidies, based on the much higher negative labor supply effects of commuter taxes. In this section, we briefly reconsider the issue within the framework of the bargaining model developed in this paper. The setup is simple. As before, we distinguish three types of transport: commuting and other transport by the employed \((T_c, T_e,\) respectively) and non-commuting by the unemployed \((T_n)\). The government, however, is assumed to consider taxing commuting and non-commuting transport differently; the two tax rates are \(\tau_c, \tau_n\), respectively\(^{17}\).

The government’s problem is to

\[
\begin{align*}
\max_{\tau_c, \tau_n, \tau_e} & \quad LV_c(w-\tau_e, \tau_c, \tau_n, a, G) + (1-L)\nu_n(\tau_n, a, G) + (1-\tau_n)\left[f(L) - wL\right] \\
\text{s.t.} & \quad \tau_c \left[LT_c^e\right] + \tau_n \left[LT_n^e + (1-L)T_n^u\right] + \tau_e L + \tau_n \left[f(L) - wL\right] = G \\
& \quad \tau_n \leq \bar{\tau}_n
\end{align*}
\]

where we have normalized marginal utility of income at one. To gain some economic intuition, we focus in what follows on the simplified case with zero income effects of conditional transport demands, no feedbacks in conditional transport demand, and zero cross-price effects between commuting and non-commuting transport\(^{18}\). Moreover, we present results for one concrete specification of union preferences, viz. (15). In Appendix 3 we show that the optimal tax structure can be written as:

---

\(^{17}\) There is, of course, the issue of how to implement tax differentiation between commuting and non-commuting transport. In practice, the way to do this at the lowest implementation cost is probably via partial tax deductibility of commuting expenses.

\(^{18}\) The general case with income effects, feedbacks and cross price effects is worked out in an unpublished appendix. It complicates matters substantially and does not yield additional economic insight.
\[ \gamma T_L + V^e - V^u = \left[ \frac{L(1-\gamma)}{\partial \tau_L} \right] \left[ 1 - \tau_L \frac{\partial w}{\partial \tau_L} \right] - \frac{(1-\gamma) S}{\tilde{S}} \left( (T^e - T^u) T^u (1-L) \right) \] (39)

\[ \tau_c = \frac{MEC}{\gamma} \left( 1 - \gamma \right) \frac{L \partial w}{\partial a} \frac{a'}{\partial \tau_L} \] (40)

\[ \tau_u = \frac{MEC}{\gamma} \left( 1 - \gamma \right) \left[ L \frac{\partial w}{\partial a} \frac{a'}{\partial \tau_L} \left( \frac{T^u (1-L)}{\tilde{S}} \right) \right] \] (41)

where \( \tilde{S} = L \frac{\partial T^e}{\partial \tau_u} + (1-L) \frac{\partial T^u}{\partial \tau_u} \) is the own price effect of total non-commuting demand.

The results are easily interpreted. First, noting that \( \tilde{S} < 0 \), direct comparison of (41) with (40) shows that non-commuting is taxed at a higher rate than commuting; hence, in this ‘relative’ sense it is optimal to subsidize commuting. As suggested before, the subsidy argument is not limited to non-competitive labour markets, and it has been numerically supported before (Van Dender (2003)). However, the model of this section makes the argument much more transparent. We have:

\[ \tau_u - \tau_c = \left( \frac{1-\gamma}{\gamma} \right) \left[ \frac{(1-L) T^u}{\tilde{S}} \right] \geq 0 \] (42)

Unless all transport demand comes from the employed, commuting is subsidized. Note again that the crucial issue is whether demand comes from the employed or not. As explained in Section 3, the intuitive reason for this finding is the presence of an optimal labour tax. Suppose only the employed demand transport. Then, as long as labour can be taxed optimally, there is no reason to treat their commuting and non-commuting transport differently. However, if a part or all of non-commuting demand comes from people that are not employed and hence not subject to labour taxation, then commuting subsidies are justified because their behaviour cannot be corrected through labour taxes. Finally, note from (39) the corresponding role of demand by the unemployed for the optimal labour tax: to the extent that their demand rises, more tax revenues are generated on transport and less on labour.

Second, the tax rule for commuting implies that commuting should be taxed below marginal external cost: commuting is also subsidized in this ‘absolute’ sense.
To see this note that, using the definition of marginal external cost, it immediately follows that (40) can be rewritten as (see Appendix 3):

\[ \tau_c = MEC + \frac{(1-\gamma)}{\gamma} (1-L)v^nT_n a' \]

where \( v^n \) is the value of time of people not working. Hence, to the extent that congestion affects people not employed it is optimal to reduce the commuting tax below marginal external cost.

5. Should we subsidize commuting by public transport only?

A standard second-best argument in the literature is that public transport subsidies can be justified when private car transport is taxed below marginal external cost for technical or political reasons. In this subsection we extend the model for multiple transport modes to investigate whether public transport subsidies can be justified even if labour and car transport is optimally taxed. The policy relevance of doing so is clear. In many countries objections are raised against subsidizing car commuters, but many politicians do favour subsidizing public transport commuters (see Potter et al. (2006)).

To analyze the issue we make some simplifying assumptions. First, we focus on commuting transport only; hence non-commuting transport of the employed as well as transport demand by those not employed is ignored. The employed either use car transport or rail to commute to work. Rail is assumed not to create externalities. Denote the number of commutes by car and rail as \( T_{car} \) and \( T_{rail} \), respectively. Rail transport is assumed to be produced at constant marginal cost; the tax equals the rail fare minus marginal cost. The respective tax rates on car use and rail transport are denoted \( \tau_{car}, \tau_{rail} \). Fixed costs of rail transport are taken into account in the government budget restriction.

We derive the optimal tax structure in Appendix 4. To keep the results transparent, we present the optimal tax rules under the maintained assumption that marginal utilities equal one, that feedback effects of congestion on transport demand are zero, and that there are zero income effects of transport demand. Moreover, we use Cobb Douglas union preferences. We find the following optimal taxes for
commuting by car and by rail, respectively (the labour tax rule is given in Appendix 4 but not reproduced here):

\[
\tau_{\text{car}} = MEC + \left(\frac{1-\gamma}{\gamma}\right) \left\{ \frac{B}{K} \right\} \left\{ -(L + T_{\text{rail}}Z) \frac{\partial T_{\text{rail}}}{\partial \tau_{\text{car}}} + T_{\text{rail}} Z \frac{\partial T_{\text{car}}}{\partial \tau_{\text{car}}} \right\}
\]

(43)

\[
\tau_{\text{rail}} = \left(\frac{1-\gamma}{\gamma}\right) \left\{ \frac{B}{K} \right\} \left\{ L \frac{\partial T_{\text{car}}}{\partial \tau_{\text{car}}} \right\}
\]

(44)

where

\[
B = \tau \left(\frac{\partial w}{\partial a} a' Z (T_{\text{car}} - T_{\text{rail}}) \frac{\partial T_{\text{car}}}{\partial \tau_{\text{rail}}} \right)
\]

\[
K = (L + T_{\text{rail}}Z) \left\{ \frac{\partial T_{\text{car}}}{\partial \tau_{\text{car}}} \frac{\partial T_{\text{rail}}}{\partial \tau_{\text{rail}}} - \frac{\partial T_{\text{car}}}{\partial \tau_{\text{rail}}} \frac{\partial T_{\text{rail}}}{\partial \tau_{\text{car}}} \right\}
\]

and

\[
Z = \frac{L \frac{\partial L}{\partial a} a'}{1-T_{\text{car}} \frac{\partial L}{\partial a} a'} < 0
\]

To interpret these results, start with the rail tax in (44). The sign of the optimal rail tax depends on the signs of the terms \( B, K \). Assume the own price elasticity of car transport is negative. The sign of \( B \) then only depends on the relative demands for car and rail transport. Suppose, consistent with empirical observations in most countries, that the number of car commuting trips exceeds the number of rail trips. Then we have \( B < 0 \). To determine the sign of the term \( K \), note that

\[
L + T_{\text{rail}}Z = \frac{L \left[ 1-(T_{\text{car}} - T_{\text{rail}}) \frac{\partial L}{\partial a} a' \right]}{1-T_{\text{car}} \frac{\partial L}{\partial a} a'}
\]

If car demand exceeds rail demand, then we have \( L + T_{\text{rail}}Z > 0 \). Moreover, assuming own price effects dominate cross effects it follows that \( K > 0 \). Hence, if all assumptions made hold, then we find that rail transport should be subsidized, in the strong and literal sense of a negative tax: \( \tau_K < 0 \).

Importantly, this result crucially depends on congestion affecting wages and rail taxes having an impact on car demand. If this is not the case, then \( B = 0 \) and the
optimal rail tax is zero. If congestion raises wages and cross price effects are non-zero, however, then it is optimal to subsidize rail because this reduces car demand and hence congestion. This in turn reduces wages and has favourable employment effects.

Next consider the optimal car tax. In the one-mode case analyzed in the previous section, the optimal car tax would simply equal marginal external cost, because we assumed all transport demand comes from commuters. In the multi-mode setting of this section, the optimal car tax generally deviates from $MEC$, and it again depends crucially on the labour market effects of congestion. To see this, under the assumptions made ($B<0$, $K>0$) the relation of the tax and the marginal external cost depends on the sign of

$$
-\left(\frac{\partial T_{\text{rail}}}{\partial \tau_{\text{car}}} + T_{\text{rail}} Z \frac{\partial T_{\text{car}}}{\partial \tau_{\text{car}}} \right)
$$

This can be rewritten as:

$$
\left[-L \frac{\partial T_{\text{rail}}}{\partial \tau_{\text{car}}} + T_{\text{rail}} Z \left(\frac{\partial T_{\text{car}}}{\partial \tau_{\text{car}}} - \frac{\partial T_{\text{rail}}}{\partial \tau_{\text{car}}} \right)\right]
$$

(45)

If the labour market effects of congestion are trivial, then $Z$ is negligible and the first term in (45) dominates; since car and rail are substitutes the expression is then negative. This implies that, see (43):

$$
\tau_{\text{car}} < MEC
$$

However, if the negotiated wage and employment effects of congestion are substantial, then $Z$ is large in absolute value and may dominate so that (45) is positive. In that case, the optimal car tax exceeds marginal external cost:

$$
\tau_{\text{car}} > MEC
$$

The policy implications of these simple results are clear. If commuting demand by car far exceeds demand for rail trips, if cross elasticities are non-zero, and if congestion has substantial implications for wages and employment, then it is optimal to subsidize rail transport, but not car commuting trips. The latter are actually heavily taxed at more than marginal external cost. If the labour market effects of congestion are small, rail subsidies will be limited and are combined with car taxes that are below marginal external cost.

Finally, note that these findings may have to be amended when a substantial fraction of trips comes from people not active on the labour market, as argued in
Section 4. In that case commuting taxes will be adjusted to shift taxes away from the employed.

6. Summary and conclusion

The purpose of this paper was to reconsider optimal congestion pricing in a model that takes account of the intimate relation between commuting, congestion, transport taxation and the labour market. Wages and employment outcomes on the labour market were assumed to be the result of bargaining between firms and unions. The model allowed for different trip motives and incorporated the possibility of telecommuting options.

The main findings of this paper are easily summarized. First, for various specifications of union preferences we show that, under plausible peak period conditions, both exogenous increases in congestion levels and in congestion taxes raise negotiated wages and reduce employment levels. More efficient telecommuting reduces wages and raises employment. Second, we show that if transport taxes cannot be differentiated according to trip purpose, optimal transport taxes positively depend on the impact of congestion on negotiated wages, and negatively on the wage effects of the congestion tax itself. If negotiated wages strongly rise due to higher congestion then this provides an argument to raise the congestion tax. Similarly, the more sensitive negotiated wages are to higher congestion taxes, the lower the optimal tax will be. In both cases, the reason is that these are indirect ways to limit negative employment effects. For specific union utility functions, we further find that transport taxes rise to the extent that demand by people not currently active on the labour market represent a larger fraction of the peak period transport flow. Moreover, it is found that higher efficiency of telecommuting does not necessarily reduce congestion.

Third, if taxes can be differentiated according to trip purpose commuting subsidies are generally justified if this allows shifting the tax burden away from the employed; this will be the case if peak period transport flows include demand by people that are not active on the labour market. Finally, if multiple transport modes are considered, we show that the positive effect of congestion on negotiated wages implies an argument for providing absolute subsidies to commuters using public transport, despite car commuting transport being heavily taxed.
References


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APPENDIX 1 – The effect of taxes and congestion on bargaining outcomes

As shown in the main body of the paper, the effect of taxes, congestion and the efficiency of home work on negotiated wages are given by:

\[
\frac{d w}{d \tau_L} = -\frac{Z_{w \tau_L}}{Z_{w w}} N \left[ \Omega_w \Omega_{\tau_L} - \Omega_{w \tau_L} \right] \tag{A1.1}
\]

\[
\frac{d w}{d \tau_T} = -\frac{Z_{w \tau_T}}{Z_{w w}} N \left[ \Omega_w \Omega_{\tau_T} - \Omega_{w \tau_T} \Omega \right] \tag{A1.2}
\]

\[
\frac{d w}{d a} = -\frac{Z_{w a}}{Z_{w w}} N \left[ \Omega_w \Omega_a - \Omega_{w a} \Omega \right] \tag{A1.3}
\]

\[
\frac{d w}{d \alpha} = -\frac{Z_{w \alpha}}{Z_{w w}} N \left[ \Omega_w \Omega_{\alpha} - \Omega_{w \alpha} \Omega \right] \tag{A1.4}
\]

where \( N = \frac{\mu}{Z_{w w} \Omega} < 0. \)

We work out these effects for various specifications of union preferences. First, suppose union preferences are given by a standard Cobb-Douglas specification of the expected utility of employed and unemployed members:

\[
\Omega = \left[ L V^e(.) \right]^\beta \left[ (m - L) V^u(.) \right]^{1-\beta} \tag{A1.5}
\]

where \( V^e(.) = V^e(w - \tau_L, \tau_T, a, \alpha, G) \) and \( V^u(.) = V^u(\tau_T, a, G) \) are indirect utility of employed and unemployed, respectively. Note that \( \beta \) reflects the relative importance the union gives to the wellbeing of the two groups of members. Working out the first and second derivatives of the union utility function and substituting into (A1.1)-(A1.4) one easily derives the various partial effects of taxes, congestion and home work efficiency on negotiated wages. For simplicity, we report results under the maintained assumption of constant marginal utility (normalized at one) throughout\(^{19}.\) We find:

\[
\frac{\partial w}{\partial \tau_L} = N \frac{\beta(\Omega)^2}{(V^e)^2} \frac{\partial V^e}{\partial \tau_L} \tag{A1.6}
\]

\[
\frac{\partial w}{\partial \tau_T} = N \frac{\beta(\Omega)^2}{(V^e)^2} \frac{\partial V^e}{\partial \tau_T} \tag{A1.7}
\]

\[
\frac{\partial w}{\partial a} = N \frac{\beta(\Omega)^2}{(V^e)^2} \frac{\partial V^e}{\partial a} \tag{A1.8}
\]

\[
\frac{\partial w}{\partial \alpha} = N \frac{\beta(\Omega)^2}{(V^e)^2} \frac{\partial V^e}{\partial \alpha} \tag{A1.9}
\]

\(^{19}\) More general results are available from the author. However, they do not change the main insights derived from this analysis. Moreover, we will also systematically assume constant marginal utility of income in the analysis of optimal tax structures, see below.
Using expression (4) reported in the main body of the paper, it then immediately and unambiguously follows that:

\[ \frac{\partial w}{\partial \tau_L} > 0, \quad \frac{\partial w}{\partial \tau_T} > 0, \quad \frac{\partial w}{\partial a} > 0, \quad \frac{\partial w}{\partial \alpha} < 0. \]

To see how robust these findings are, we considered several alternative specifications of union utility. A second specification assumes that union preferences are given by the expected utility of its members (see, e.g., Pissarides (1998)):

\[ \Omega = LV^\epsilon(\cdot) + (m - L)V^\alpha(\cdot) \]

A third union utility function assumes the union is interested in maximizing the rent of employment relations (Hart (2004)):

\[ \Omega = L \left[ V^\epsilon(\cdot) - V^\alpha(\cdot) \right] \]

Finally, a fourth option we looked at is based on the observation that the three specifications considered so far implicitly assume that the union fully ‘accepts’ the preferences of its members in judging the implications of transport taxes and congestion when it negotiates with firms. Although this is interesting as a benchmark, it may not be fully realistic. For example, the union may judge that monetary and time costs of commuting are to some extent the result of earlier rational decisions by workers (e.g., the decision as to where to live), and it may, therefore, attach relatively low weights to these issues at the negotiation table. To reflect this possibility, we consider the following specification of union preferences:

\[ \Omega = L^\theta \tilde{w} \]

where

\[ \tilde{w} = w - \tau_L - \epsilon \tau_T - \delta a \]

This is a standard Cobb-Douglas specification of utility in terms of employment and the net of tax wage of commuters, ‘corrected’ for the union’s evaluation of commuting costs and congestion. The parameters \( \epsilon \) and \( \delta \) reflect the implicit weights the union places on exogenous increases in transport costs and congestion, relative to the importance given to a wage change. These weights are not observed and may strongly differ from the relative valuation of taxes and congestion relative to wages in consumer preferences.

The results of the derivations are summarized in Table A.1. All specifications imply that labour taxes raise negotiated wages. Second, for the first and the fourth specification it is unambiguously the case that both transport taxes and congestion levels will raise negotiated wages and hence reduce employment. For the second and the third specification it holds as long as we assume that the employed ‘suffer’ more from congestion and congestion taxes than the unemployed. More precisely, sufficient conditions are:

---

20 Note that the fallback wage was set arbitrarily to zero. This does not affect any of the implications we are dealing with in this paper.

21 Some other specifications were considered as well, with qualitatively similar results.
\[
\left[(T^e_e + T^e_u) - T^u_u\right] > 0 \\
\left[v^e(T^e_e + T^e_u) - v^u T^u_u\right] > 0
\]

where the \(v^i (i = e,u)\) are the respective values of time of the employed and unemployed. Assuming the employed demand more transport and have higher time values than the unemployed, then both conditions seem plausible.

Given the wage effects reported in Table A.1, one immediately derives the implications for employment, using the labour demand curve \(L = L'(w)\). Indeed, we have:

\[
\frac{\partial L}{\partial \tau_L} = \frac{1}{f'' \partial \tau_L}, \quad \frac{\partial L}{\partial \tau_T} = \frac{1}{f'' \partial \tau_T}, \quad \frac{\partial L}{\partial \alpha} = \frac{1}{f'' \partial \alpha}, \quad \frac{\partial L}{\partial \alpha} = \frac{1}{f'' \partial \alpha}
\]
Table A.1: Effects of exogenous parameters on wages and employment for the right-to-manage bargaining model (*).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
<th>Sign</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega = [LV^e(\cdot)]^\beta [(m - L)\nu^u(\cdot)]^{1-\beta}$</td>
<td>$\frac{\partial w}{\partial \tau_L} &gt; -N[A] &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega = LV^e(\cdot) + (m - L)\nu^u(\cdot)$</td>
<td>$\frac{\partial w}{\partial \tau_T} &gt; N[A] \frac{\partial V^e(\cdot)}{\partial \tau_T} &gt; 0$</td>
<td>Sign ambiguous</td>
<td></td>
</tr>
<tr>
<td>$\Omega = L[V^e(\cdot) - \nu^u(\cdot)]$</td>
<td>$\frac{\partial w}{\partial \alpha} &gt; N[A] \frac{\partial V^e(\cdot)}{\partial \alpha} &gt; 0$</td>
<td>Sign ambiguous</td>
<td></td>
</tr>
<tr>
<td>$\Omega = L^2 w$, where $w = w - \tau_L - \alpha \tau_T - \delta a$</td>
<td>$\frac{\partial w}{\partial \alpha} &lt; 0$</td>
<td>$NL^2 \frac{\partial V^e(\cdot)}{\partial \alpha} &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 1. $N = \frac{\mu}{(\Omega)^2 Z_{ww}} < 0$. The definition of $\Omega$ and, hence, of $Z_{ww}$ differs between columns.

2. $[A] = \frac{\beta(\Omega)^2}{(V^e)^2} > 0$

3. $[B] = \left[L^2 - \frac{mV^u}{f''} \right] > 0$

4. $[C] = L + \frac{1}{f''} (V^e - \nu^u)$. Note that it follows from the first-order condition $Z_w = 0$ that $[C] > 0$. 

(*)
Finally, note that the effect of union power on the wage implications of transport variables is not a priori clear. As an example, consider union utility specification (15) that is used in the main body of the paper. Now look at the wage effect of congestion when the union becomes more powerful. We easily show:

$$\frac{\partial^2 w}{\partial a \partial \mu} = \frac{f \Omega^2}{(V^*)^2} \frac{\partial V^*}{\partial a} \frac{\partial N}{\partial \mu}$$

Using the definition of $N$ it follows:

$$\frac{\partial N}{\partial \mu} = \frac{Z_{ww} - \mu \frac{\partial Z_{ww}}{\partial \mu}}{(Z_{ww})^2}$$

In this expression, $Z_{ww}$ is the second derivative of the objective function with respect to the wage. Differentiating $Z_w$, taking into account the definition of the labour demand curve as a function of wages, we find:

$$Z_{ww} = \mu \left\{ \frac{\Omega \Omega_{ww} - (\Omega_w)^2}{\Omega^2} \right\} - (1 - \mu) \left\{ \frac{f - wL}{f^n} + L^2 \right\} \left\{ \frac{f - wL}{f^n} + \frac{f''}{(f - wL)^2} \right\}$$

where we have used the condition $f' - w = 0$. It follows:

$$\frac{\partial Z_{ww}}{\partial \mu} = \left\{ \frac{\Omega \Omega_{ww} - (\Omega_w)^2}{\Omega^2} \right\} + \left\{ \frac{f - wL}{f^n} + \frac{f''}{(f - wL)^2} \right\}$$

Substitution then yields, after slight rearrangement:

$$\frac{\partial N}{\partial \mu} = - \frac{1}{(Z_{ww})^2} \left( \frac{f}{f''} \right) \left( \frac{f - wL}{L - f' +Lf''} \right)$$

The sign of the term between brackets depends on the specification of the production function. For example, for the simple iso-elastic function often used in theoretical work $f(L) = aL^b$, $0 < b < 1$ one easily shows that it is positive. Hence $\frac{\partial N}{\partial \mu} > 0$ and

$$\frac{\partial^2 w}{\partial a \partial \mu} = \frac{f \Omega^2}{(V^*)^2} \frac{\partial V^*}{\partial a} \frac{\partial N}{\partial \mu} < 0$$

One similarly shows under these conditions that $\frac{\partial^2 w}{\partial \tau \partial \mu} < 0$. This is consistent with findings of Van Ommeren and Rietveld (2005). Using an iso-elastic production function they find that more powerful unions have less extra wage demands in response to increasing commuting costs. However, note from the above that this is
not a general result. More flexible specifications of production imply \( \left\{ \frac{f}{L} - f' + Lf'' \right\} < 0 \) and, hence \( \frac{\partial^2 w}{\partial a \partial \mu} > 0 \), \( \frac{\partial^2 w}{\partial \tau_r \partial \mu} > 0 \). In that case, a more powerful union uses its power to obtain higher wage increases after an increase in commuting costs.

**APPENDIX 2 – Derivation of the optimal tax expressions**

We first derive the first-order conditions in the general case. Then we derive the optimal tax rules analyzed in the main body of the paper. Finally, we simplify these rules for different concrete specifications of the union utility function.

1. Derivation of the first-order conditions

Consider the problem:

\[
\begin{align*}
\text{Max}_{\tau^e, \tau^u} & \quad \frac{1}{\lambda^e_0} L V^e(w - \tau_L^e, \tau^e, a, \alpha, G) + \frac{1}{\lambda^u_0} (1 - L)V^u(\tau^u, a, G) + (1 - \tau_r^e)\left[f(L) - wL\right] \\
\text{s.t.} & \quad \tau^e_T + \tau^e_L + \tau^u_L \left[f(L) - wL\right] = G \quad (\gamma) \\
& \quad \tau^e \leq \overline{\tau}^e \quad (\eta)
\end{align*}
\]

Note that indirect utility is normalized by marginal utility of income in a reference situation in order to make utility and the firm’s after tax profit comparable. The first-order condition with respect to the labour tax is given by:

\[
\begin{align*}
L \frac{1}{\lambda^e_0} \frac{dV^e}{d\tau_L} + V^e \frac{1}{\lambda^e_0} \frac{dL}{d\tau_L} + (1 - L) \frac{1}{\lambda^u_0} \frac{dV^u}{d\tau_L} - V^u \frac{1}{\lambda^u_0} \frac{dL}{d\tau_L} + (1 - \tau_r^e) \left[f'(w) \frac{dL}{d\tau_L} - L \frac{dw}{d\tau_L}\right] \\
+ \gamma \left[\tau^e_L \frac{dL}{d\tau_L} + L + \tau^e_T \frac{dT}{d\tau_L} + \tau^u_L \left[f'(w) \frac{dL}{d\tau_L} - L \frac{dw}{d\tau_L}\right]\right] = 0
\end{align*}
\]

(A2.1)

In this expression, the full effect of the labour tax on utility of the employed and unemployed can be written as, respectively:

\[
\begin{align*}
\frac{1}{\lambda^e_0} \frac{dV^e}{d\tau_L} &= \frac{1}{\lambda^e_0} \left\{ -\lambda^e + \lambda^e \frac{dw}{dT} + \frac{\partial V^e}{\partial a} \frac{dT}{d\tau_L} \right\} \\
\frac{1}{\lambda^u_0} \frac{dV^u}{d\tau_L} &= \frac{1}{\lambda^u_0} \left\{ \frac{\partial V^u}{\partial a} \frac{dT}{d\tau_L} \right\}
\end{align*}
\]

where we have used \( \frac{\partial V^e}{\partial \tau_L} = -\lambda^e \) and the fact that congestion depends on total transport demand through the congestion function \( a(T) \). Substituting in (A2.1) and rearranging yields:
\[
\frac{dL}{d\tau_L} \left\{ \frac{1}{\lambda_0^e} V^e - \frac{1}{\lambda_0^u} V^u + [1 - \tau_x + \gamma \tau_x] (f' - w) + \gamma \tau_L \right\} + L \frac{d\omega}{d\tau_L} \left\{ \frac{1}{\lambda_0^e} \lambda^e - (1 - \tau_x) - \gamma \tau_x \right\} \\
+ \frac{dT}{d\tau_L} \left\{ \lambda^e \frac{1}{\lambda_0^e} \frac{\partial V^e}{\partial a} + (1 - L) \frac{1}{\lambda_0^u} \frac{\partial V^u}{\partial a} \right\} + \gamma \tau_T \right\} + L (\gamma - \frac{1}{\lambda_0^e} \lambda^e) = 0 \\
(A2.2)
\]

Now note that the term
\[
a' \left[ L \frac{\partial V^e}{\partial a} \frac{1}{\lambda_0^e} + (1 - L) \frac{\partial V^u}{\partial a} \frac{1}{\lambda_0^u} \right]
\]
is closely related to the marginal external cost of an increase in transport demand \(T\).

To see this, note that for an individual belonging to group \(i = \{e, u\}\) the willingness to pay for a unit reduction in the congestion level \(a\) is given by \(-\frac{1}{\lambda^i} \frac{\partial V^i}{\partial a}\). Hence the marginal cost of a transport increase that is suffered by the employed and unemployed can be defined as, respectively:

\[
MEC_e = -L \frac{1}{\lambda^e} \frac{\partial V^e}{\partial a} a' \\
MEC_u = -(1 - L) \frac{1}{\lambda^u} \frac{\partial V^u}{\partial a} a'.
\]

Consequently, we have:

\[
a' \left[ L \frac{\partial V^e}{\partial a} \frac{1}{\lambda_0^e} + (1 - L) \frac{\partial V^u}{\partial a} \frac{1}{\lambda_0^u} \right] = -a' \left( \lambda^e \frac{1}{\lambda_0^e} MEC_e + \lambda^u \frac{1}{\lambda_0^u} MEC_u \right) \quad (A2.3)
\]

Using (A2.3) in (A2.2) the latter can be written as:

\[
\frac{dL}{d\tau_L} \left\{ \frac{1}{\lambda_0^e} V^e - \frac{1}{\lambda_0^u} V^u + [1 - \tau_x + \gamma \tau_x] (f' - w) + \gamma \tau_L \right\} + L \frac{d\omega}{d\tau_L} \left\{ \frac{1}{\lambda_0^e} \lambda^e - (1 - \tau_x) - \gamma \tau_x \right\} \\
+ \frac{dT}{d\tau_L} \left\{ \gamma \tau_T - \lambda^e \frac{1}{\lambda_0^e} MEC_e - \lambda^u \frac{1}{\lambda_0^u} MEC_u \right\} + L (\gamma - \lambda^e \frac{1}{\lambda_0^e}) = 0 \\
(A2.4)
\]

Next consider the first order condition with respect to the congestion tax. It is given by, under similar assumptions:

\[
L \frac{1}{\lambda_0^e} \frac{dV^e}{d\tau_T} + \frac{1}{\lambda_0^u} \frac{dL}{d\tau_T} + (1 - L) \frac{1}{\lambda_0^u} \frac{dV^u}{d\tau_T} \frac{1}{\lambda_0^u} \frac{dL}{d\tau_T} - (1 - \tau_x) \left[ (f' - w) \frac{dL}{d\tau_T} - L \frac{d\omega}{d\tau_T} \right] \\
+ \gamma \left\{ \tau_L \frac{dL}{d\tau_T} + \tau_T \frac{dL}{d\tau_T} + \tau_T + \gamma \tau_T \left[ (f' - w) \frac{dL}{d\tau_T} - L \frac{d\omega}{d\tau_T} \right] \right\} = 0
\]
The full utility effects of a congestion tax are given by

\[
\frac{1}{\lambda^e_0} \frac{dV^e}{d\tau_T} = -\lambda^e \frac{1}{\lambda^e_0} T^e + \lambda^e \frac{1}{\lambda^e_0} \frac{dw}{d\tau_T} + \frac{1}{\lambda^e_0} \frac{\partial V^e}{\partial a} \left[ \frac{dL}{d\tau_T} (T^e - T^u) + L \frac{dT^e}{d\tau_T} + (1 - L) \frac{dT^u}{d\tau_T} \right]
\]

\[
\frac{1}{\lambda^u_0} \frac{dV^u}{d\tau_T} = -\lambda^u \frac{1}{\lambda^u_0} T^u + \lambda^u \frac{1}{\lambda^u_0} \frac{dw}{d\tau_T} + \frac{1}{\lambda^u_0} \frac{\partial V^u}{\partial a} \left[ \frac{dL}{d\tau_T} (T^e - T^u) + L \frac{dT^e}{d\tau_T} + (1 - L) \frac{dT^u}{d\tau_T} \right]
\]

where we have used Roy's identity \( \frac{\partial V^i}{\partial \tau_T} = -\lambda^i T^i \) \((i = e, u)\). Substituting, using the definitions of the marginal external costs given before, yields:

\[
\frac{dL}{d\tau_T} \left\{ \frac{1}{\lambda^e_0} V^e - \frac{1}{\lambda^u_0} V^u + \left[ (1 - \tau_x) + \gamma \tau_x \right] (f' - w) + \gamma \tau_L \right\} + L \frac{dw}{d\tau_T} \left[ \frac{1}{\lambda^e_0} \lambda^e - (1 - \gamma) \tau_x \right]
\]

\[
+ \frac{dT}{d\tau_T} \left\{ \gamma \tau_T - \frac{1}{\lambda^e_0} \lambda^e MEC_e - \frac{1}{\lambda^u_0} \lambda^u MEC_u \right\} + T \gamma - L \frac{1}{\lambda^e_0} \lambda^e T^e - (1 - L) \frac{1}{\lambda^u_0} \lambda^u T^u = 0
\]

(A2.5)

To simplify these expressions, first note that the right to manage model implies \( f' - w = 0 \). Moreover, we focus in the main body of the paper on the case of constant marginal utility of income throughout. Specifically, we assume \( \lambda^e = \lambda^u = \lambda_0 = 1 \). This is a strong assumption, but it will simplify the interpretation. With the described assumptions, equations (A2.4-A2.5) reduce to:

\[
\frac{dL}{d\tau_T} \left\{ V^e - V^u + \gamma \tau_L \right\} + L \frac{dw}{d\tau_T} \left[ (1 - \gamma) \tau_x \right] + \frac{dT}{d\tau_T} \left\{ \gamma \tau_T - MEC \right\} + L(\gamma - 1) = 0
\]

(A2.6)

\[
\frac{dL}{d\tau_T} \left\{ V^e - V^u + \gamma \tau_L \right\} + L \frac{dw}{d\tau_T} \left[ (1 - \gamma) \tau_x \right] + \frac{dT}{d\tau_T} \left\{ \gamma \tau_T - MEC \right\} + T(\gamma - 1) = 0
\]

(A2.7)

where overall marginal external cost is given by

\[
MEC = MEC_e + MEC_u
\]

Finally, the first-order condition with respect to the profit tax and the complementary slackness conditions due to the weak inequality constraint, result in:

\[
\eta - \pi(\gamma - 1) \leq 0; \quad \tau_x \left[ \eta - \pi(\gamma - 1) \right] = 0
\]

\[
\tau_x - \tau^e_x \leq 0; \quad \eta \left( \tau_x - \tau^e_x \right) = 0
\]

(A2.8)

System (A2.6)-(A2.7)-(A2.8) is the system considered in the main body of the paper.
2. Derivation of the tax rules: the general case

Assuming the profit restriction is binding and solving by Cramer’s rule then implies that the second-best optimal tax rules satisfy:

\[ V^* - V^u + \gamma_T = \frac{1 - \gamma}{H} \left\{ L \frac{dT}{d\tau_T} - T \frac{dT}{d\tau_L} + \left[ L_T \frac{dw}{d\tau_T} \frac{dT}{d\tau_L} - \frac{dw}{d\tau_T} \frac{dT}{d\tau_L} \right] \right\} \]

\[ \gamma_T - MEC = \frac{1 - \gamma}{H} \left\{ T \frac{dL}{d\tau_L} - L \frac{dL}{d\tau_T} + \left[ L_T \frac{dw}{d\tau_T} \frac{dL}{d\tau_L} - \frac{dw}{d\tau_T} \frac{dL}{d\tau_L} \right] \right\} \]

where

\[ H = \frac{dL}{d\tau_L} \frac{dT}{d\tau_T} - \frac{dL}{d\tau_T} \frac{dT}{d\tau_L} \]

(A2.11)

Using the total effects of taxes on transport demands, employment and wages (see (22)-(23) and (25)-(28)) and manipulating the results, we can write the individual terms appearing in (A2.9)-(A2.10)-(A2.11) as follows:

\[ H = \frac{dL}{d\tau_L} \frac{dT}{d\tau_T} - \frac{dL}{d\tau_T} \frac{dT}{d\tau_L} = \frac{1}{1 - \rho} \left\{ \frac{\partial L}{\partial \tau_L} [S] + L \frac{\partial L}{\partial \tau_T} \frac{\partial T^e}{\partial w} \right\} \]

(A2.12)

\[ L \frac{dT}{d\tau_T} - T \frac{dT}{d\tau_L} = \frac{1}{1 - \rho} \left\{ L \left[ (S) + T \frac{\partial T^e}{\partial w} \right] + (T^e - T^u)(Q_L) + L \frac{\partial T^e}{\partial w} (Q_w) \right\} \]

(A2.13)

\[ \frac{dw}{d\tau_T} \frac{dL}{d\tau_T} - \frac{dw}{d\tau_L} \frac{dT}{d\tau_T} = -\frac{1}{1 - \rho} \left\{ \frac{\partial w}{\partial \tau_L} [S] + L \frac{\partial T^e}{\partial w} \frac{\partial w}{\partial \tau_T} \right\} \]

(A2.14)

\[ T \frac{dL}{d\tau_l} - L \frac{dL}{d\tau_T} = -\frac{1}{1 - \rho} \left\{ (Q_L)(1 - \rho_a) + L \frac{\partial L}{\partial a} \frac{\partial a}{\partial \tau_T} \left[ (S) + T \frac{\partial T^e}{\partial w} \right] \right\} \]

(A2.15)

\[ \frac{dw}{d\tau_T} \frac{dL}{d\tau_T} - \frac{dw}{d\tau_L} \frac{dT}{d\tau_T} = 0 \]

(A2.16)

where

\[ S = L \frac{\partial T^e}{\partial \tau_T} + (1 - L) \frac{\partial T^u}{\partial \tau_T} \]

(A2.17)

\[ Q_L = L \frac{\partial L}{\partial \tau_T} \frac{\partial T^e}{\partial \tau_T} - T \frac{\partial L}{\partial \tau_L} \]

(A2.18)

\[ Q_w = L \frac{\partial w}{\partial \tau_T} \frac{\partial T^e}{\partial \tau_T} - T \frac{\partial w}{\partial \tau_L} \]

(A2.19)

\[ \rho_a = L \frac{\partial T^e}{\partial a} \frac{\partial a}{\partial \tau_T} + (1 - L) \frac{\partial T^u}{\partial a} \]

(A2.20)

The reason for (A2.16) being equal to zero is that congestion and labour taxes both move the wage-employment combination along the labour demand curve; relative effects of congestion and taxes on wages and employment must therefore be equal. It is easily shown using (A1.10).
Substituting (A2.12)-(A2.16) into the optimal tax expressions (A2.9)-(A2.11) yields, after straightforward algebra:

\[
\tau_L = -\left(\frac{V^e - V^u}{\gamma}\right) + \left(1 - \frac{\gamma}{1}\right) \frac{L[S]\left(1 - \frac{\partial w}{\partial \tau_L}\right) + L \frac{\partial T^e}{\partial w} [K] + (T^e - T^u)(Q_L)}{\frac{\partial L}{\partial \tau_L} [S] + L \frac{\partial T^e}{\partial w} \frac{\partial L}{\partial \tau_T}}
\]

\[
\tau_T = \left(\frac{MEC}{\gamma}\right) - \left(1 - \frac{\gamma}{1}\right) \frac{(1 - \rho_i)(Q_L) + L \frac{\partial L}{\partial a^*} \left[\frac{\partial T^e}{\partial w}\right]}{\frac{\partial L}{\partial \tau_L} [S] + L \frac{\partial T^e}{\partial T^e} \frac{\partial L}{\partial \tau_T}}
\]

where \([S]\) and \([Q_L]\) are as defined above, and:

\[
[K] = T(1 - \frac{\partial w}{\partial \tau_L}) + L \frac{\partial w}{\partial \tau_T}(1 - \tau_i)
\]

These expressions are difficult to interpret in general. To do so, it is instructive to assume zero income effects of transport demand\textsuperscript{22}. In that case we have, using the definition of \(Q_L\), noting from Appendix 1 that

\[
\frac{\partial L}{\partial \tau_L} = \frac{\partial w}{\partial \tau_L}, \quad \frac{\partial L}{\partial a} = \frac{\partial w}{\partial \tau_L},
\]

and rearranging to facilitate the interpretation:

\[
\tau_L = -\left(\frac{V^e - V^u}{\gamma}\right) + \left(1 - \frac{\gamma}{1}\right) \frac{L[S]\left(1 - \frac{\partial w}{\partial \tau_L}\right) + (T^e - T^u)(Q_L)}{\frac{\partial L}{\partial \tau_L} [S]}
\]

\[
\tau_T = \left(\frac{MEC}{\gamma}\right) + \left(1 - \frac{\gamma}{1}\right) \left(1 - \rho_i\right) \left(\frac{L \frac{\partial w}{\partial \tau_T}}{\frac{\partial L}{\partial \tau_T} [S]}\right) \left(\frac{\partial \tau_T}{\partial \tau_L}\right) - \left(1 - \frac{\gamma}{1}\right) \frac{\frac{\partial \tau_T}{\partial \tau_L}}{\frac{\partial L}{\partial \tau_T}} \frac{\partial w}{\partial \tau_T} a^*
\]

These are the expressions discussed further in the main body of the paper.

\textsuperscript{22} Note that the case of constant marginal utility of income and zero income effects boils down to the assumption of quasi-linear preferences in the general consumption good, see section 1.1.
3. Tax rules for specific union utility functions

We here work out the tax rules for various utility function specifications. First, for the Cobb-Douglas specification (15), it is easily shown, using (A1.6)-(A1.9), (A2.18) and \( \frac{\partial L}{\partial \tau_L} = \frac{\partial w}{\partial \tau_L} \), that:

\[
Q_L = L \frac{\partial L}{\partial \tau_L} - T \frac{\partial L}{\partial \tau_L} = -(1-L)T^u \frac{\partial L}{\partial \tau_L}
\]

\[
T - L \frac{\partial w}{\partial \tau_L} = (1-L)T^u
\]

Substituting in (A2.21)-(A2.22) yields:

\[
\tau_L = - \left( \frac{V^e - V^u}{\gamma} \right) + \left(1-\frac{1}{\gamma} \right) \left\{ L \frac{\partial w}{\partial \tau_L} \left[ 1 - \frac{\partial w}{\partial \tau_L} \right] - T^u (1-L) \right\} \frac{(T^e - T^u)}{(S)} \}
\]

(A2.23)

\[
\tau_T = \left( \frac{MEC}{\gamma} \right) - \left(1-\frac{1}{\gamma} \right) \left\{ L \frac{\partial w}{\partial \tau_L} a' \left[ 1 - \frac{\partial w}{\partial \tau_L} \right] - T^u (1-L) \right\} \frac{(1-\rho_a)}{(S)} \}
\]

(A2.24)

To further simplify, note that:

\[
MEC = -a \left[ L \frac{\partial V^e}{\partial a} + (1-L) \frac{\partial V^u}{\partial a} \right]
\]

Moreover, specification (15) implies, see Appendix 1, that:

\[
L \frac{\partial w}{\partial \tau_L} a' = -L \frac{\partial V^e}{\partial a} a'
\]

Using these results in (A2.24) and noting that \( \frac{\partial V^u}{\partial a} = -\gamma T^u \), it immediately follows that:

\[
\tau_T = MEC + \left(1-\frac{1}{\gamma} \right) \left[ \frac{(1-\rho_a)}{(S)} - (1-L)\gamma \right] T^u
\]

Entirely analogous procedures can be followed for the other specifications of union preferences. Specification two yields:
\[
\tau_L = -\left( \frac{V^e - V^u}{\gamma} \right) + \frac{1 - \gamma}{\gamma} \left\{ L \left( 1 - \tau_L \frac{\partial w}{\partial \tau_L} \right) - T^u \left[ 1 - \frac{Lm(C)}{(B)} \right] \frac{\left( T^e - T^u \right)}{(S)} \right\}
\]

\[
\tau_T = \left( \frac{MEC}{\gamma} \right) - \frac{1 - \gamma}{\gamma} \left\{ L \frac{\partial w}{\partial \tau_L} \frac{a'}{\partial \tau_L} - T^u \left[ 1 - \frac{Lm(C)}{(B)} \right] \frac{\left( 1 - \rho_i \right)}{(S)} \right\}
\]

Specification three yields:

\[
\tau_L = -\left( \frac{V^e - V^u}{\gamma} \right) + \frac{1 - \gamma}{\gamma} \left\{ L \left( 1 - \tau_L \frac{\partial w}{\partial \tau_L} \right) - T^u \left[ (1 - L) - (\varepsilon - T^e)L \right] \frac{\left( T^e - T^u \right)}{(S)} \right\}
\]

\[
\tau_T = \left( \frac{MEC}{\gamma} \right) - \frac{1 - \gamma}{\gamma} \left\{ L \frac{\partial w}{\partial \tau_L} \frac{a'}{\partial \tau_L} - T^u \left[ (1 - L) - (\varepsilon - T^e)L \right] \frac{\left( 1 - \rho_i \right)}{(S)} \right\}
\]

This is the same as specification one, except for the term (1-L).

Finally, specification four leads to:

\[
\tau_L = -\left( \frac{V^e - V^u}{\gamma} \right) + \frac{1 - \gamma}{\gamma} \left\{ L \left( 1 - \tau_L \frac{\partial w}{\partial \tau_L} \right) - T^u \left[ (1 - L) - (\varepsilon - T^e)L \right] \frac{\left( T^e - T^u \right)}{(S)} \right\}
\]

\[
\tau_T = \left( \frac{MEC}{\gamma} \right) - \frac{1 - \gamma}{\gamma} \left\{ L \frac{\partial w}{\partial \tau_L} \frac{a'}{\partial \tau_L} - T^u \left[ (1 - L) - (\varepsilon - T^e)L \right] \frac{\left( 1 - \rho_i \right)}{(S)} \right\}
\]

If zero transport for the unemployed holds, then:

\[
\tau_L = -\left( \frac{V^e - V^u}{\gamma} \right) + \frac{1 - \gamma}{\gamma} \left\{ L \left( 1 - \tau_L \frac{\partial w}{\partial \tau_L} \right) + [(\varepsilon - T^e)L \frac{T^e}{(S)} \right\}
\]

\[
\tau_T = MEC - \frac{1 - \gamma}{\gamma} \left[ L\delta a' - MEC \right] + [(\varepsilon - T^e)L \frac{(1 - \rho_i)}{(S)} \right]\]

Remember that \( \delta \) and \( \varepsilon \) reflected, see above, the implicit value the union placed on congestion and transport taxes, respectively. Note that this implies deviations between tax and marginal external cost even at zero demand by those not employed. A lower valuation of congestion by the union than by the government \([L\delta a' - MEC] < 0\) yields lower optimal transport taxes. The government corrects the ‘error’ of the union. A
lower valuation of transport taxes by the union than by the public \((\varepsilon-T^\pi)L<0\) yields higher optimal transport taxes. The government corrects the too low evaluation of transport taxes.

**APPENDIX 3: Optimal tax differentiation**

Consider the government’s problem:

\[
\begin{align*}
\text{Max}_{\tau_L, \tau_c, \tau_n, \tau_\pi} & \quad LV_c(w - \tau_L, \tau_c, \tau_n, a, \alpha, G) + (1 - L)V_u(\tau_n, a, G) + (1 - \tau_\pi)[f(L) - wL] \\
\text{s.t.} & \quad \tau_c[T_c] + \tau_n[T_n] + \tau_L L + \tau_\pi [f(L) - wL] = G \\
& \quad \tau_\pi \leq \bar{\tau}_\pi
\end{align*}
\]

In this formulation, marginal utilities have been normalized at one. Note that the transport demands for commuting and non-commuting are denoted by subscripts c and n, respectively. We have by definition:

\[
\begin{align*}
T_c &= LT^c_c \\
T_n &= LT^c_n + (1 - L)T^w_n
\end{align*}
\]

Moreover, employment and wages result from negotiations between union and firm; they can be written in general as \(L(\tau_L, \tau_c, \tau_n, a, \alpha), w(\tau_L, \tau_c, \tau_n, a, \alpha)\).

Assuming that the profit tax constraint binds, we can write the first-order conditions in matrix notation as follows:

\[
\begin{bmatrix}
\frac{dL}{d\tau_L} & \frac{dT_c}{d\tau_L} & \frac{dT_n}{d\tau_L} \\
\frac{dL}{d\tau_c} & \frac{dT_c}{d\tau_c} & \frac{dT_n}{d\tau_c} \\
\frac{dL}{d\tau_n} & \frac{dT_c}{d\tau_n} & \frac{dT_n}{d\tau_n}
\end{bmatrix}
\begin{bmatrix}
\gamma \tau_L + V^c - V^u \\
\gamma \tau_c - \text{MEC} \\
\gamma \tau_n - \text{MEC}
\end{bmatrix}
= 
\begin{bmatrix}
(1 - \gamma)L\left(1 - \tau_\pi \frac{dw}{d\tau_L}\right) \\
(1 - \gamma)\left[T_c - L\tau_\pi \frac{dw}{d\tau_c}\right] \\
(1 - \gamma)\left[T_n - L\tau_\pi \frac{dw}{d\tau_n}\right]
\end{bmatrix}
\]

We focus throughout on the case with zero income effects in transport demand, zero transport feedbacks and zero cross-price effects between commuting and non-commuting demands\(^{23}\). To solve (A3.2), we develop, using similar procedures as in Appendix 2, the various total derivatives, and then solve by Cramer’s rule.

First, consider the total impact of taxes on employment. Using \(L(\tau_L, \tau_c, \tau_n, a, \alpha)\) and the definition of congestion, they are found to be given by:

\[^{23}\text{The more general case is available on request in an unpublished appendix. It does not yield much additional insight.}\]
\[\frac{dL}{d\tau_L} = \frac{\partial L}{\partial \tau_L} \frac{1}{1 - \rho}\]
\[\frac{dL}{d\tau_c} = \frac{\partial L}{\partial \tau_c} + L \frac{\partial L}{\partial a} a' \left( \frac{\partial T^e}{\partial \tau_c} \right) \frac{1}{1 - \rho} \quad (A3.4)\]
\[\frac{dL}{d\tau_n} = \frac{\partial L}{\partial \tau_n} + \frac{\partial L}{\partial a} a' \left( \frac{L}{\partial \tau_n} \frac{\partial T^e}{\partial \tau_n} \right) + (1 - L) \frac{\partial T^u}{\partial \tau_n} \frac{1}{1 - \rho} \]

In these expressions,
\[\rho = (T^e_c + T^e_n - T^u_n) \frac{\partial L}{\partial a} a'\]
is the feedback of congestion on transport demand via changes in employment. The pure transport feedbacks are assumed to be zero for simplicity, see above. Analogously, the impact of taxes on wages is easily shown to be given by:
\[\frac{dw}{d\tau_L} = \frac{\partial w}{\partial \tau_L} \frac{1}{1 - \rho}\]
\[\frac{dw}{d\tau_c} = \frac{\partial w}{\partial \tau_c} + L \frac{\partial w}{\partial a} a' \left( \frac{\partial T^e}{\partial \tau_c} \right) \frac{1}{1 - \rho} \quad (A3.5)\]
\[\frac{dw}{d\tau_n} = \frac{\partial w}{\partial \tau_n} + \frac{\partial w}{\partial a} a' \left( \frac{L}{\partial \tau_n} \frac{\partial T^e}{\partial \tau_n} \right) + (1 - L) \frac{\partial T^u}{\partial \tau_n} \frac{1}{1 - \rho}\]

Next, using the definitions of transport demand given in (A3.1)-(A3.2), we can write the total effects of labour taxes on commuting and non-commuting transport as follows:
\[\frac{dT_c}{d\tau_L} = \frac{T^e_c}{1 - \rho} \frac{\partial L}{\partial \tau_L}\]
\[\frac{dT_n}{d\tau_L} = \frac{(T^e_n - T^u_n)}{1 - \rho} \frac{\partial L}{\partial \tau_L} \quad (A3.6)\]

Finally, the effects of transport taxes on transport demands are calculated in a similar way. We find:
\[
\frac{dT_c^e}{d\tau_c} = \frac{T_c^e \frac{\partial L}{\partial \tau_c} + L \frac{\partial T_c^e}{\partial \tau_c} \left[1 - (T_n^e - T_n^u) \frac{\partial L}{\partial a'}\right]}{1 - \rho}
\]

\[
\frac{dT_n^e}{d\tau_n} = \frac{(T_n^e - T_n^u) \left[\frac{\partial L}{\partial \tau_n} + L \frac{\partial T_n^e}{\partial \tau_n} \frac{\partial L}{\partial a'}\right]}{1 - \rho}
\]

\[
\frac{dT_n^e}{d\tau_n} = \frac{T_n^e \frac{\partial L}{\partial \tau_n} + \left[L \frac{\partial T_n^e}{\partial \tau_n} + (1 - L) \frac{\partial T_n^u}{\partial \tau_n}\right] \left[1 - T_n^e \frac{\partial L}{\partial a'}\right]}{1 - \rho}
\]

Using (A3.4), (A3.6) and (A3.7), we work out the determinant of (A3.3). We find, after substantial but simple algebra:

\[
\begin{vmatrix}
\frac{dL}{d\tau_c} & \frac{dT_c^e}{d\tau_c} & \frac{dT_n^e}{d\tau_c} \\
\frac{dL}{d\tau_c} & \frac{dT_c^e}{d\tau_c} & \frac{dT_n^e}{d\tau_c} \\
\frac{dL}{d\tau_n} & \frac{dT_c^e}{d\tau_n} & \frac{dT_n^e}{d\tau_n}
\end{vmatrix} = \frac{1}{1 - \rho} \frac{L \frac{\partial L}{\partial \tau_c} \frac{\partial T_c^e}{\partial \tau_c} - L \frac{\partial T_c^e}{\partial \tau_c} \frac{\partial T_c^e}{\partial \tau_c} + (1 - L) \frac{\partial T_c^u}{\partial \tau_c}}{L \frac{\partial T_n^e}{\partial \tau_n} + (1 - L) \frac{\partial T_n^u}{\partial \tau_n}}
\]

Finally, we solve (A3.3) by Cramer’s rule. Working out the result and rearranging, we derive the optimal tax rules as follows:

\[
\gamma \tau_c + V^e - V^u = \left[L(1 - \gamma)\right] \left[1 - (1 - \gamma) \frac{\partial W}{\partial \tau_c}\right]
\]

\[
+ \frac{(1 - \gamma)}{L \frac{\partial L}{\partial \tau_c} \frac{\partial T_c^e}{\partial \tau_c}} \left[T_c^e \left[L \frac{\partial L}{\partial \tau_c} - T_c \frac{\partial L}{\partial \tau_c}\right] + L \frac{\partial T_c^e}{\partial \tau_c} \left(T_n^e - T_n^u\right) \left[L \frac{\partial L}{\partial \tau_n} - T_n \frac{\partial L}{\partial \tau_n}\right]\right]
\]

(A3.8)

\[
\tau_c = \frac{MEC}{\gamma} - \left(1 - \gamma\right) \frac{1}{L \frac{\partial L}{\partial \tau_c} \frac{\partial T_c^e}{\partial \tau_c}} \left[L \frac{\partial L}{\partial \tau_c} - T_c \frac{\partial L}{\partial \tau_c}\right] + L \frac{\partial T_c^e}{\partial \tau_c} \frac{\partial L}{\partial a'}
\]

(A3.9)

\[
\tau_n = \frac{MEC}{\gamma} - \left(1 - \gamma\right) \frac{1}{L \frac{\partial L}{\partial \tau_n} \frac{\partial T_n^e}{\partial \tau_n}} \left[L \frac{\partial L}{\partial \tau_n} - T_n \frac{\partial L}{\partial \tau_n}\right] + L \left[S \frac{\partial L}{\partial a'}\right]
\]

(A3.10)
where \( S = L \frac{\partial T^e_u}{\partial \tau_n} + (1 - L) \frac{\partial T^u_n}{\partial \tau_n} \) is the own price effect of total non-commuting demand.

These expressions hold independently of the specification of the utility function. In the main text we discuss the results for Cobb-Douglas specification (15) of union preferences. Note that in that case one easily shows, using the procedure described in Appendix 1:

\[
L \frac{\partial L}{\partial \tau_c} - T_c \frac{\partial L}{\partial \tau_L} = 0 \tag{A3.11}
\]

\[
L \frac{\partial L}{\partial \tau_n} - T_n \frac{\partial L}{\partial \tau_L} = -(1 - L)T^u_n \frac{\partial L}{\partial \tau_L} \tag{A3.12}
\]

Substituting into the general expression for the optimal tax rules (A3.8)-(A3.10), we find:

\[
\gamma T_L + V^e - V^u = \frac{[L(1 - \gamma)]}{\partial \tau_L} \left[ 1 - \frac{\partial w}{\partial \tau_L} \right] - \frac{(1 - \gamma)}{\hat{S}} \left( (T^e_n - T^u_n)T^u_n (1 - L) \right) \tag{A3.13}
\]

\[
\tau_c = \frac{MEC}{\gamma} - \left( \frac{1 - \gamma}{\gamma} \right) L \frac{\partial w}{\partial \tau_L} a' \tag{A3.14}
\]

\[
\tau_n = \frac{MEC}{\gamma} - \left( \frac{1 - \gamma}{\gamma} \right) \left[ \frac{L \frac{\partial w}{\partial \tau_L} a'}{\hat{S}} - \frac{T^u_n (1 - L)}{\hat{S}} \right] \tag{A3.15}
\]

Here \( \hat{S} \) is the own price effect of the demand for non-commuting transport. These expressions are further discussed in Section 4 of the paper. Expressions for the other specifications of union utility can be derived analogously.

Finally, note that union utility specification (15) implies, using similar derivations as in Appendix 1, that

\[
\frac{\partial V^e}{\partial \tau_L} = -\frac{\partial V^e}{\partial a} = \nu^e(T^e_c + T^e_n) .
\]

Using this together with the definition of marginal external cost

\[
MEC = -\left[ L \frac{\partial V^e}{\partial a} a' + (1 - L) \frac{\partial V^u}{\partial a} a' \right] = L\nu^e(T^e_c + T^e_n) + (1 - L)\nu^u T^u_n
\]

in expression (A3.14), it follows that the tax rule for commuting can be written as:

\[
\tau_c = MEC + \left( \frac{1 - \gamma}{\gamma} \right) (1 - L)\nu^u T^u_n a'
\]
APPENDIX 4: Multiple modes of transport

Given the notation described in the main body of the paper (Section 5) employed consumers maximize utility defined over consumption, leisure, commuting demand by car and rail, and working at home, respectively (we deleted the subscript e to simplify notation):

$$\text{Max } u(C, \ell, T_{\text{car}}, T_{\text{rail}}, H)$$

subject to the constraints

$$C = \left[w - \tau_L\right]\left(T_{\text{car}} + T_{\text{rail}} + \alpha H\right) - (1 + \tau_{\text{car}})T_{\text{car}} - (1 + \tau_{\text{rail}})T_{\text{rail}} + G$$

$$l + aT_{\text{car}} + (T_{\text{car}} + T_{\text{rail}} + H) = R$$

$$T_{\text{car}} + T_{\text{rail}} + \alpha H = 1$$

The first order conditions can be solved for the optimal demands; this results in indirect utility with the standard properties:

$$V^e(w - \tau_L, \tau_{\text{car}}, \tau_{\text{rail}}, a, \alpha, G)$$

The government’s problem is to

$$\text{Max } \pi V^e(w - \tau_L, \tau_{\text{car}}, \tau_{\text{rail}}, \alpha) + (1 - L)\pi^u + (1 - \tau_L)\left[f(L) - wL\right]$$

s.t.

$$\tau_{\text{car}}\left[LT_{\text{car}}\right] + \tau_{\text{rail}}\left[LT_{\text{rail}}\right] + \tau_L L + \tau_{\pi} \left[f(L) - wL\right] = G$$

$$\tau_{\pi} \leq \bar{\tau}_\pi$$

where we have normalized marginal utilities at one. Note that the utility of the unemployed is now simply treated as given ($\bar{\pi}^u$). Since they don’t demand transport they are unaffected by transport taxes and congestion. The first-order conditions can be written as

$$\begin{bmatrix}
\frac{dL}{d\tau_L} T_{\text{car}} \frac{dL}{d\tau_L} + L \frac{dT_{\text{car}}}{d\tau_L} \frac{dL}{d\tau_L} + L \frac{dT_{\text{rail}}}{d\tau_L} \\
\frac{dL}{d\tau_{\text{car}}} T_{\text{car}} \frac{dL}{d\tau_{\text{car}}} + L \frac{dT_{\text{car}}}{d\tau_{\text{car}}} \frac{dL}{d\tau_{\text{car}}} + L \frac{dT_{\text{rail}}}{d\tau_{\text{car}}} \\
\frac{dL}{d\tau_{\text{rail}}} T_{\text{car}} \frac{dL}{d\tau_{\text{rail}}} + L \frac{dT_{\text{car}}}{d\tau_{\text{rail}}} \frac{dL}{d\tau_{\text{rail}}} + L \frac{dT_{\text{rail}}}{d\tau_{\text{rail}}} \\
\end{bmatrix} \begin{bmatrix}
\gamma \tau_{\text{car}} + V^e - V^u \\
\gamma \tau_{\text{car}} - MEC \\
\gamma \tau_{\text{rail}} \\
\end{bmatrix} = \begin{bmatrix}
(1 - \gamma) L \left(1 - \tau_{\pi}\right) \frac{dw}{d\tau_L} \\
(1 - \gamma) L \left(T_{\text{car}} - \tau_{\pi}\right) \frac{dw}{d\tau_{\text{car}}} \\
(1 - \gamma) L \left(T_{\text{rail}} - \tau_{\pi}\right) \frac{dw}{d\tau_{\text{rail}}} \\
\end{bmatrix} \quad (A4.1)
We limit the analysis to the case where feedback effects of congestion on transport demand and income effects of transport demand are both zero. Working out the individual terms in the above matrix under these assumptions, we obtain the following results:

\[
\begin{align*}
\frac{dL}{d\tau_L} & = \frac{1}{1-\rho} \frac{\partial L}{\partial \tau_L} \\
\frac{dL}{d\tau_{car}} & = \frac{1}{1-\rho} \frac{\partial L}{\partial \tau_{car}} + Z \frac{\partial T_{car}}{\partial \tau_{car}} \\
\frac{dL}{d\tau_{rail}} & = \frac{1}{1-\rho} \frac{\partial L}{\partial \tau_{rail}} + Z \frac{\partial T_{car}}{\partial \tau_{car}}
\end{align*}
\]

\[
\begin{align*}
T_{car} \frac{dL}{d\tau_L} + L \frac{dT_{car}}{d\tau_{car}} & = \frac{1}{1-\rho} T_{car} \frac{\partial L}{\partial \tau_{car}} + (L + T_{car}Z) \frac{\partial T_{car}}{\partial \tau_{car}} \\
T_{car} \frac{dL}{d\tau_{rail}} + L \frac{dT_{car}}{d\tau_{car}} & = \frac{1}{1-\rho} T_{car} \frac{\partial L}{\partial \tau_{rail}} + (L + T_{car}Z) \frac{\partial T_{car}}{\partial \tau_{rail}} \\
T_{rail} \frac{dL}{d\tau_L} + L \frac{dT_{rail}}{d\tau_{car}} & = \frac{1}{1-\rho} T_{rail} \frac{\partial L}{\partial \tau_{car}} + (L + T_{rail}Z) \frac{\partial T_{rail}}{\partial \tau_{car}} \\
T_{rail} \frac{dL}{d\tau_{rail}} + L \frac{dT_{rail}}{d\tau_{car}} & = \frac{1}{1-\rho} T_{rail} \frac{\partial L}{\partial \tau_{rail}} + (L + T_{rail}Z) \frac{\partial T_{rail}}{\partial \tau_{rail}}
\end{align*}
\]

where \( Z = L \frac{\partial L}{\partial a} a' < 0 \), \( \rho = T_{car} \frac{\partial L}{\partial a} a' < 0 \).

Going through the same derivations as in Appendix 1, we easily show that bargaining implies that labour taxes and the two transport taxes raise wages and reduce employment. Using the Cobb Douglas specification for union preferences for the remainder of this appendix (similar results are obtained for the other specifications), the following relations between transport and labour tax effects are easily shown:
Using (A4.2)-(A4.5) into the system of first-order conditions (A4.1), the latter can be rewritten as follows:

\[
\begin{bmatrix}
\frac{\partial L}{\partial \tau_L} & \frac{1}{1-\rho} & \\
\frac{T_{car}}{\partial \tau_L} & \frac{1}{1-\rho} & \\
-Z \frac{\partial T_{car}}{\partial \tau_L} & -(L+T_{car}Z) \frac{\partial T_{car}}{\partial \tau_L} & \\
-Z \frac{\partial T_{rail}}{\partial \tau_L} & -(L+T_{car}Z) \frac{\partial T_{rail}}{\partial \tau_L} & \\
\end{bmatrix}
\begin{bmatrix}
\tau_L \\
T_{car} \\
T_{rail} \\
\end{bmatrix}
= \begin{bmatrix}
\gamma T_L + V^e - V^w \\
\gamma T_{car} - MEC \\
\gamma T_{rail} \\
\end{bmatrix}
\]

Solving by Cramer’s rule yields, after some algebra:

\[
\tau_L = -\frac{V^e - V^w}{\gamma} + \left(1 - \frac{1}{\gamma}\right) \frac{L}{\partial L} \left[1 - \frac{\partial w}{\partial \tau_L}\right] - \left[\frac{B}{K}\right] \left[\begin{bmatrix}
T_{car}(L+T_{car}Z) \frac{\partial T_{car}}{\partial \tau_{car}} + T_{rail}(L+T_{car}Z) \frac{\partial T_{car}}{\partial \tau_{car}}
\end{bmatrix}\right]
\]

\[
\tau_{car} = MEC + \left(1 - \frac{1}{\gamma}\right) \left[\begin{bmatrix}
MEC - \frac{L}{\partial w} \frac{\partial w}{\partial \tau_L}
\end{bmatrix} + \left[\frac{B}{K}\right] \left[\begin{bmatrix}
-(L+T_{rail}Z) \frac{\partial T_{rail}}{\partial \tau_{car}} + T_{rail}Z \frac{\partial T_{car}}{\partial \tau_{car}}
\end{bmatrix}\right]\right]
\]

\[
\tau_R = \left(1 - \frac{1}{\gamma}\right) \left[\frac{B}{K}\right] \left[\begin{bmatrix}
\frac{\partial T_{car}}{\partial \tau_{car}}
\end{bmatrix}\right]
\]

where

\[
B = \tau_s \frac{\partial w}{\partial a} Z (T_{car} - T_{rail}) \frac{\partial T_{car}}{\partial \tau_{rail}}
\]

\[
K = (L + T_{rail}Z) \left[\begin{bmatrix}
\frac{\partial T_{car}}{\partial \tau_{car}} \frac{\partial T_{rail}}{\partial \tau_{rail}} - \frac{\partial T_{car}}{\partial \tau_{car}} \frac{\partial T_{rail}}{\partial \tau_{car}}
\end{bmatrix}\right]
\]
Finally, the tax rule for commuting by car can be simplified, using the definition of marginal external cost and the wage effects of congestion that follow from Cobb-Douglas union preferences. When only commuting demand is considered we have:

\[
MEC = -a' \left( L \frac{\partial V^e}{\partial a} \right) = -a' L v^e T_{car}
\]

Moreover, it is easily shown that Cobb-Douglas union preferences imply:

\[
L \frac{\partial w}{\partial a'} = a' \left( L \frac{\partial V^e}{\partial a} \right) = a' L v^e T_{car}
\]

Substitution of these results finally leads to the rule for the tax on car commuting:

\[
\tau_{car} = MEC + \left( \frac{1 - \gamma}{\gamma} \right) \left\{ \left[ \frac{B}{K} \right] \left[ -(L + T_{rail} Z) \frac{\partial T_{rail}}{\partial \tau_{car}} + T_{rail} Z \frac{\partial T_{car}}{\partial \tau_{car}} \right] \right\}
\]

This is the expression reported and further discussed in Section 5.