THE WIN MAXIMIZATION MODEL RECONSIDERED

Flexible Talent Supply and Efficiency Wages

Prof. dr. Stefan Kesenne
Economics Department, University of Antwerp
and
Physical Education Department, University of Leuven
Belgium

1. Introduction

In the USA, the profit maximization assumption is generally accepted by sports economists as an adequate description of the behaviour of professional sports clubs (Rottenberg, 1956; Noll, 1974; Fort and Quirk, 1995). In Europe, Sloane (1971) argued that the objective of most professional football (soccer) clubs should rather be called utility maximization, with different variables, beside profits, entering the utility functions, such as playing success, attendances and league health. Kesenne (1996, 2000) simplified the Sloane model by selecting the maximization of the winning percentage as the most important club objective, and compared some of the results of the profit and the win maximization hypothesis. However, the win maximization model has been criticized on a number of points. The aim of this paper is to reconsider the original win maximization model and to pay attention to a few extensions of the model. In section 2 the main criticisms of the original model are shortly discussed. Section 3 introduces the "flexible talent" assumption into the model, which is more relevant for the open European player labour market. Section 4 concentrates on the determination of the salary level and introduces the efficiency wages theory. Conclusions are given in section 5.

2. Criticizing the win maximization model

The behaviour of many club owners and managers in European football suggests that the maximization of playing success is their main objective. In its most simplified version, the objective of win maximization includes that clubs are trying to maximize the season winning percentage of their team under the breakeven constraint. Club managers estimate the expected total season revenue, and attract the best players they can afford with this budget, i.e.:

$$\begin{aligned} &\max \ w \\ &\text{sub} \ R - C = 0 \end{aligned} \qquad (1)$$

where w is the winning percentage of a team, C is the total season cost and R is total season revenue. The revenue function is assumed to be concave in the winning percentage. Since its first appearance, this win maximization model has been criticized on several points.

A first criticism of model (1) is that win maximization is not an operational objective, because clubs cannot control their winning percentage. Clubs can only maximize the talents of the team. The best guarantee for a high winning percentage is fielding a performing team by attracting the best players. So, we follow Kesenne's (1996) original approach to win maximization, starting from the maximization of playing talents.

Also the relevance of the breakeven constraint in (1) has been under attack. In a recent paper, Fort and Quirk (2004) argue: "*If the justification for the win maximization case is identification of the sportsman owner with his team's success, why would such an owner follow a self-imposed zero profit restriction?*" However, it should be clear that the win maximization model does not completely disregard profits or losses. Only, profits are not maximized in this approach. A certain profit rate can be necessary to satisfy the owners or the shareholders of the club, or to invest in a new stadium or in talent development. To the extent that rich club owners consider their favourite sport as a consumption good, they can as well be prepared to put in some money with no expected return. Also, in European football, club losses have to stay within certain limits, as required by national or international leagues licence systems. All these scenarios can be handled by the win maximization approach, and do not affect its main conclusions regarding the competitive balance, the salary level or the impact of revenue sharing. So, the constraint in (1) can simply be adjusted to $R - C = \pi^{\circ}$, where π° is a certain amount of profits, but not maximum profits. Also in the USA, some sports economists have their doubts about the profit maximizing condition (see Zimbalist, 2003).

In the same paper, Fort and Quirk (2004) also question one of the results of Kesenne (1996) who asserted that in a win maximization league, the distribution of talent is more unequal than in a profit maximization league. They showed that nothing can be derived concerning the competitive balance in a win and a profit maximization league if no simplifying assumptions on the club revenue functions are made beyond concavity. The authors' assertion is correct. It is clear, even for a simple quadratic revenue function, that the competition can be more balanced in a win maximization league, if one considers the type of imbalance where the small market team dominates the large market team. However, for the more likely competitive imbalance, where the large market team dominates the small market team, the competition will be more unbalanced in the win maximization league (see Kesenne, 2004). Dobson and Goddard (2001) have shown that the competitive balance is the same in both scenarios if the revenue functions are Cobb-Douglas. However, a Cobb-Douglas specification is not a suitable revenue function for professional sports clubs, because the third-order partial derivative with respect to the winning percentage is positive, where it should be negative, or zero at the utmost. Indeed, the diminishing effect of the winning percentage on the marginal revenue must be stronger the more a team's winning percentage approaches 100 %. In other words, the marginal revenue curves should be concave to the origin, and this condition is not fulfilled in a Cobb-Douglas specification. This concavity condition of the marginal revenue curves also implies that the win maximization model is not necessarily identical to a restricted revenue maximization model.

Most empirical tests in the literature seem to confirm the profit maximizing hypothesis (see Ferguson, D., Stewart, K., Jones, J., Le Dressay, A., 1991; Alexander, 2001). However, all these tests, to the best of our knowledge, are based on the (ticket) pricing rule or the estimated price elasticities, while it can be shown that the pricing rule of a win maximizing club is the same as the pricing rule of a profit maximizing club. So, these tests support both the profit and the win maximization hypothesis.

Another serious critique of both the conventional win and profit maximization models comes from recent papers by Szymanski and Kesenne (2004) and Szymanski (2004). They assert that the (implicit or explicit) hypothesis of the fixed supply of playing talent is not a reasonable assumption for European football, given the open European player market after the Bosman verdict. In 1995, the European Court of Justice not only abolished the existing transfer system, but also the so-called 3+2 Rule, which limited the number of foreign players a club can field. Moreover, the authors argued that the conventional Walrasian model with a constant talent supply, considered to be a realistic assumption for the closed US player market, is not a meaningful model. The main reason is that this model does not fulfil the conditions of a Nash equilibrium, because the incorporation of the constant talent supply in the objective function leaves one team without a choice of strategy. Even if one questions the validity of the latter assertion, it is true that the European market of professional football players is an open market, showing a high degree of international player mobility. So, the win maximization model should be adjusted for the flexible supply case in Europe, which is done in section 3.

One last remark concerns the wage formation. If the Nash equilibrium model starts from a given marginal cost of talent, how is it determined? One hypothesis is that all clubs are wage takers and that the player cost is determined by demand and supply on a competitive player market. However, if the most important objective of a team is playing success, team managers are probably more likely to pay higher salaries in order to attract the better players (adverse selection), or to prevent the good players from leaving the club (labour turnover model). The winning percentage does not only depend on the talents of the players, but also on the effort players are willing to make. So, it is worthwhile to investigate what the implications for the behaviour of win maximizing teams are if the *efficiency wage theory* is introduced in the model, which is done in section 4.

3. Win maximization with a flexible talent supply

In this section, we specify the win maximization model by taking into consideration the flexible talent supply in an open European labour market for professional players. Assume that a club is maximizing the number of playing talents, given a fixed profit rate:

$$\max \; x_i$$

$$\text{sub} \; R_i[m_i, w_i] - cx_i = \pi_i^0 \tag{2}$$

where x_i is the number of playing talents of team *i*, π_i^0 is a fixed amount of (positive or negative) profits, added to a fixed capital cost, and *c* is the marginal cost of talent which is the same for every team. The two most important factors determining a club's season revenue R_i are the market size m_i, determining a club's drawing potential for players and spectators, and the teams' winning percentage w_i . Both variables have a positive impact on revenue, but the winning percentage shows decreasing marginal returns. The cross second-order derivative of the revenue function is assumed to be positive, because the marginal revenue of a win is higher for a club with the larger market. The winning percentage is supposed to be a simple function of the relative talents of the team and can be defined as:

$$w_i = \frac{n}{2} \frac{x_i}{\sum_j^n x_j} \tag{3}$$

where *n* is the number of teams in the league. As can be easily seen, the sum of the winning percentages of all teams is always equal to $\frac{n}{2}$.

From the first-order conditions for a maximum number of talents under constraint (2) a club's demand curve for talent is given by the net average revenue curve:

$$\frac{R_i[m_i, w_i] - \pi_i^0}{x_i} = c \qquad \text{for all } i \tag{4}$$

Expression (4) can be interpreted as the set of reaction functions of the teams, because each club's demand for talent depends on the demand for talent of all other clubs, so that the Nash-Cournot Equilibrium can be found, which determines the distribution of talents among the clubs and the competitive balance in the league.

This solution can be illustrated using a simple 2-club model with the following quadratic revenue functions:

$$R_i = m_i w_i - w_i^2 \qquad \text{with} \qquad w_i = \frac{x_i}{\sum_j x_j} \qquad \textit{for i: 1,2} \qquad (5)$$

Let the market of club *1* be larger than the market of club *2*: i.e.: $m_1 > m_2$. In order to keep revenues and winning percentages positive, also the following restrictions must hold: $m_1 > 1$ and $m_1 - m_2 < 1$. The reaction functions for the two teams can now be written as:

$$\frac{m_i}{\sum_j x_j} - \frac{x_i}{(\sum_j x_j)^2} - \frac{\pi_i^0}{x_i} = c \qquad \textit{for i: 1, 2} \qquad (6)$$

Furthermore, if the simplifying assumption is made that the wage/turnover ratio (*k*) is the same for every club:

$$\frac{cx_i}{R_i} = k \qquad \text{or} \qquad \pi_i^0 = (1-k)R_i \qquad \textit{for i: 1, 2} \qquad (7)$$

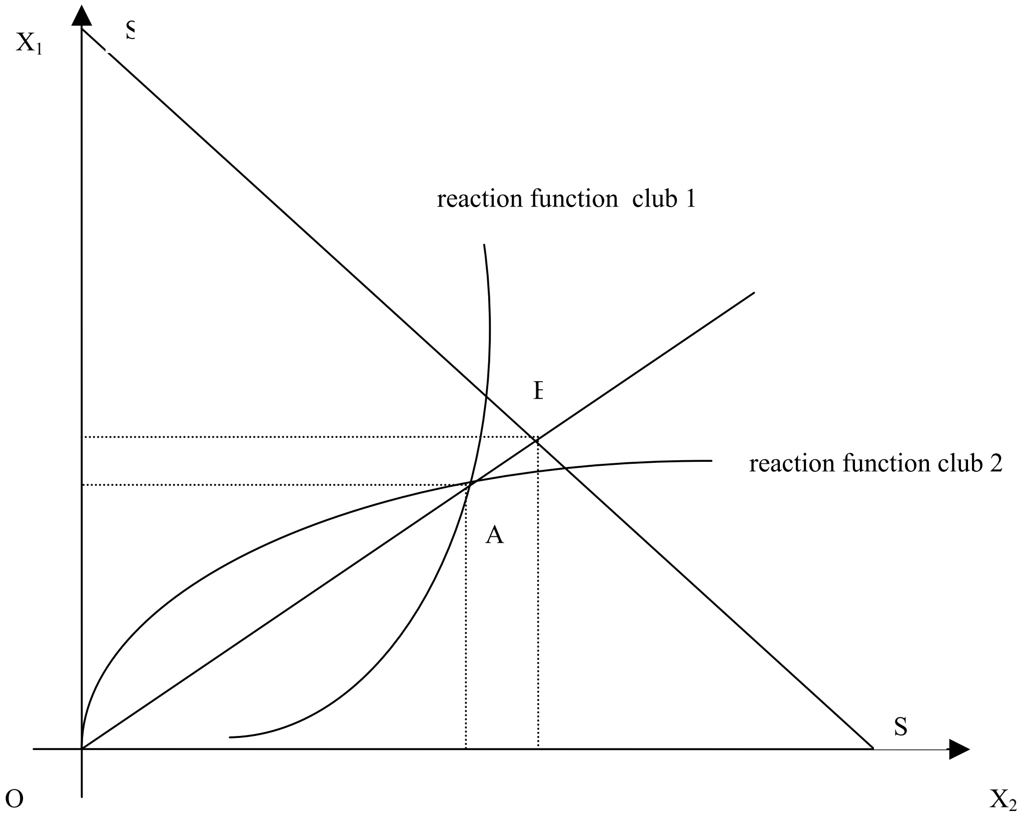
the Cournot equilibrium point, where the reaction curves (6) intersect, can be found with the following competitive balance:

$$\frac{w_1}{w_2} = \frac{x_1}{x_2} = \frac{1+(m_1 - m_2)}{1-(m_1 - m_2)} \qquad (8)$$

It follows that the team with the largest market will have the highest winning percentage, ceteris paribus.

This Cournot equilibrium can be represented graphically in figure 1 where the playing talents of the clubs are measured on the axes. The two reaction curves are drawn and intersect at point A, so that the competitive balance is given by the slope of the line connecting the origin O and point A.

Figure 1. The Cournot Equilibrium



Compared with the Cournot equilibrium under profit maximization:

$$\frac{w_1}{w_2} = \frac{x_1}{x_2} = \frac{m_1}{m_2} \qquad (9)$$

win maximization turns out to yield a more unequal distribution of talent.

It is also worth mentioning that the Cournot equilibrium for a win maximization league yields the same talent demand and competitive balance than the fixed-supply Walrasian equilibrium, whereas these solutions are different for a profit maximization league.

It has been shown by Szymanski and Kesenne (2004) that Revenue sharing worsens the competitive balance in a 2-club Nash-contest model if clubs are profit maximizers. However, in a league where clubs are win maximizers, it is obvious that any revenue sharing arrangement, that includes a net transfer of club revenue from the large market club to the small market club, will improve the competitive balance because the small market club will increase its demand for talent while the large market club reduces its demand, ceteris paribus.

4. How is the salary level determined?

In deriving the Cournot equilibrium, the marginal cost of talent, or the player salary level, was assumed to be exogenously given. The central question is how this salary level is determined. If the player labour market is a competitive market, with a given constant supply of talent, the salary level equals the market clearing cost of talent. If the supply of talent in the simplified 2-club model above is fixed, equal to *s*, and fully used in a competitive player market, the equilibrium unit cost of talent can be calculated as:

$$c = k\frac{m_1 + m_2 - 1}{2s} \qquad (10)$$

In a profit maximization league, the market clearing unit cost of talent can be found as:

$$c = \frac{m_1^2 m_2 + m_1 m_2^2 - 2m_1 m_2}{s(m_1 + m_2)^2} \qquad (11)$$

From (10) and (11) it can be derived it that, if k = 1, the equilibrium salary level in a win maximization league is higher than in a profit maximization league; it turns out

that, only for a extremely low value of the wage/turnover ratio (k), the salary level will be lower.
The fixed supply of talent can be represented in figure 1 by the linear curve SS, so that the number of playing talents can be found at point B at the intersection of SS and the line through the origin. The distribution of talent is the same as in the flexible supply model.

However, one can criticize the hypothesis of a perfect competitive player labour market with wage taking clubs. If the main objective of sports club is to maximize the winning percentage, and if a better pay leads to a better performance, the salary level can be used by club managers to enhance the effort that players are willing to make and to increase the winning percentage. Most sports clubs have some kind of bonus system besides the fixed salary level. Clubs will also try to attract the best players and to prevent their best players to move to another club by paying higher salaries. These phenomena are known as the 'shirking', the 'adverse selection' and the 'labour turnover' explanations of the efficiency wage theory. By introducing the efficiency wage theory, we try to find out how the clubs' demand for talent and the player salary level are affected.

We assume that the winning percentage is not only affected by the relative talent of a team but also by an index representing the effort the team is willing to make, where effort is a function of a club's salary level**:**

$$w_i = e(c_i)\frac{x_i}{\sum_j^n x_j} \qquad (12)$$

The effort function e is an increasing function of the club's salary level with decreasing marginal returns, i.e.: $e^{'} > 0$, and $e^{''} < 0$. Most applications of the efficiency wage theory introduce some kind of relative salary in the effort functions. The argument is that a player is only willing to make an extra effort if his salary level is higher than what he can expect to be paid in another club, or higher than the equilibrium salary level in a competitive player market. So, c_i can be interpreted as the level of the salary, relative to the market clearing salary level.

In this scenario, club managers have to decide on both the optimal talent level and the optimal salary level. Under the conventional hypothesis of profit maximization, the first-order partial derivatives of the profit function with respect to the salary level and number of the playing talents have to equal zero, i.e.:

$$MR_{ci} = \frac{\partial R_i}{\partial w_i} e^{'}(c_i) \frac{x_i}{\sum_j^n x_j} = x_i \tag{13}$$

$$MR_{xi} = \frac{\partial R_i}{\partial w_i} e(c_i) \frac{\sum_{j \neq i}^n x_j}{(\sum_j^n x_j)^2} = c_i \tag{14}$$

From these conditions a variant of the so-called Solow Condition (1979) can be derived, indicating that the wage elasticity of effort is:

$$\varepsilon_i^{\pi} = \frac{\sum_{j \neq i}^n x_j}{\sum_j^n x_j} < 1 \tag{15}$$

which means that profit maximizing clubs in the team sports industry are willing to pay higher efficiency wages than profit maximizing firms with the same effort function in other industries, where $\varepsilon_i^{\pi} = 1$.

Turning to the win maximization problem in (2), the first-order conditions can now be written as:

$$1 + \lambda_i MR_{xi} - \lambda_i c_i = 0 \tag{16}$$

$$MR_{ci} - x_i = 0 \tag{17}$$

$$R_i[m_i, w_i] - c_i x_i - \pi_i^0 = 0 \tag{18}$$

where λ_i is the positive Lagrange multiplier. From (16) and (17) the effort elasticity can now be derived as:

$$\varepsilon_i^{w} = \frac{c_i}{c_i - 1/\lambda} \frac{\sum_{j \neq i}^n x_j}{\sum_j^n x_j} > \varepsilon_i^{\pi} \tag{19}$$

which is clearly higher than the effort elasticity in a profit maximizing team.

The equations (15) and (19) point out that a club's efficiency wage per unit of talent is set at a higher level the stronger the team is compared with its opponents in the league. So, for a given effort function, that is the same for all clubs, the talented clubs are paying higher efficiency wages than the less talented clubs. This might suggest that efficiency wages, set by profit or win maximizing clubs, result in a more balanced competition than the equilibrium wages that are determined in a competitive market. If the more talented clubs set higher efficiency wages than the less talented clubs, one would expect that also the demand for talent of the better teams will be reduced more than the demand of the lesser teams, ceteris paribus. However, for at least two reasons, little can be derived from this model concerning the competitive balance: firstly: with efficiency wages there is no longer any guarantee that the demand curves for talent are downward sloping, and secondly: the relative talent of a club is no longer a reliable indicator of its winning percentage because of the impact of the effort function in (12).

Comparing (15) with (19) one can also see that for a win maximizing club, the effort elasticity is higher, so that the efficiency wage is lower. Does this imply that we can expect the efficiency wage to be lower in a win maximization league compared with a profit maximizing league? This is most unlikely because, in efficiency wage applications, it is not the absolute salary level that matters, but, for obvious reasons, the salary level relative to a reference level, which can be the market clearing salary in a competitive market. So, what this finding suggests is that win maximizing clubs are inclined to pay lower *relative* salary levels compared with profit maximizing clubs, but not necessarily lower *absolute* salary levels, because, as has been shown above, the competitive market clearing wage level in a win maximization league is higher than in a profit maximization league.

Once the efficiency wage level is fixed, the demand for playing talent of a win maximizing club is determined by the net average revenue curve, as indicated by the first-order condition (18). It follows that the demand for talent in a win maximizing club, for a given salary level, will still be higher than in a profit maximizing club because the net average revenue curve lies always to the right of the marginal revenue curve, even if both curves are upward sloping because of the effort effect. However, little can be concluded regarding the actual levels of demand and the competitive balance, because these variables depend also on the levels of the efficiency wages.

The solutions (15) and (19) can also explain the existing unemployment among professional players because one can expect the optimal and rigid efficiency salary levels in all clubs to be above the market clearing level. The excess supply of talent will not tempt team owners to lower the salary level, because it will lower the clubs' profits or the teams' winning percentage.

In section 3 it has been shown that revenue sharing improves the competitive balance in a win maximization league. It is also interesting to find out how the efficiency wages are affected by a revenue sharing arrangement. Let us start from a simple pool sharing arrangement:

$$R_i^* = \mu R_i + (1-\mu)\overline{R} \tag{20}$$

which can also be written as:

$$R_i^* = \frac{(n-1)\mu+1}{n} R_i + \frac{1-\mu}{n}\sum_{j\neq i}^{n} R_j \tag{21}$$

where R^* is the revenue after sharing and $\overline{R}$ is the average club revenue in the league. Assuming that the efficiency salary level of one team does not affect the effort of another team, the first-order conditions can now be derived as:

$$\frac{\partial R_i^*}{\partial c_i} = \frac{(n-1)\mu+1}{n}\frac{\partial R_i}{\partial c_i} = x_i \tag{22}$$

$$\frac{\partial R_i^*}{\partial x_i} = \frac{(n-1)\mu+1}{n}\frac{\partial R_i}{\partial x_i} + \frac{1-\mu}{n}\sum_{j\neq i}^{n}\frac{\partial R_j}{\partial x_i} = c_i \tag{23}$$

so that one can find the effort elasticity to be:

$$\varepsilon_i^{w*} = \left(\frac{c_i}{c_i - 1/\lambda - \frac{1-\mu}{n}\sum_{j\neq i}^{n}\frac{\partial R_j}{\partial x_i}}\right)\frac{\sum_{j\neq i}^{n} x_j}{\sum_{j}^{n} x_j} < \varepsilon_i^{w} \qquad (24)$$

Because $\frac{\partial R_j}{\partial x_i}$ is negative, this lower effort elasticity suggests is that a higher efficiency wage level is set by the owners, due to the revenue sharing arrangement. Also, the more revenue is shared, (i.e. the lower *μ)*, the higher will be the efficiency wage. Expression (24) also suggests that small market clubs, due to the sharing arrangement, will increase the salary level per unit of talent more than large market clubs. Indeed, if club 1 is a large market club and club 2 is a small market club, it is clear that $\frac{\partial R_1}{\partial x_2} > \frac{\partial R_2}{\partial x_1}$, given the properties of revenue function (2) and winning percentage (12).

Whether or not revenue sharing improves the competitive balance in a win maximization league will depend on the size of the efficiency wage changes of large and small market clubs, caused by the sharing arrangement. However, the difference between these wage effects has to be extremely large in order to offset the difference in club revenue, caused by the sharing arrangement, so that it can be expected that the proposition that revenue sharing improves the competitive balance in a win maximization league still holds when efficiency wages are set by the owners.

5. Conclusion

The objective of this paper was to review the win maximization model in professional team sports, taking into consideration some criticism and remarks that has been put forward recently. Including some important adjustments, the central conclusion is that, in a competitive player market, the main results of the win maximization model, compared with profit maximization, still hold, such as a higher demand for talent, a higher salary level, a more unequal distribution of talent and a positive impact of revenue sharing on competitive balance. However, if efficiency wages are introduced, things get more complicated. Even if efficiency wages per unit of talent can be expected to be higher for large clubs than for small clubs, little can be derived regarding its impact on the demand for talent and the competitive balance. Moreover, revenue sharing seems to have an increasing effect on efficiency wages, and more so for small market than for large market clubs, which complicates the impact of revenue sharing on competitive balance.

Nevertheless, what this analysis might help to explain, beside the unemployment of grass root professional players, is the poor financial situation of many win maximizing football clubs in Europe who, in the rat race for the best players, are bidding up salary levels. Furthermore, club managers are facing a high degree of uncertainty, not only in the relationship between relative talent and winning percentage, but also in the relationship between salary and effort. However, this uncertainty does not justify the serious financial losses of many European football clubs, because risk and uncertainty has to be handled properly in a well-managed business. The heavy losses of most European soccer clubs are generally believed to be caused by poor management and the reckless overpayment of professional players, whose performances are staying far behind the expected or predicted levels.

References

Alexander D.L., (2001), Major League Baseball: Monopoly Pricing and Profit Maximizing Behavior, *Journal of Sports Economics*, 2/4, 341-255

Dobson S., Goddard J., (2000), *The Economics of Football*, Cambridge University Press, Cambridge, 458 p.

Ferguson, D., Stewart, K., Jones, J., Le Dressay, A., (1991), 'The pricing of sport events: do teams maximize profits?' *The Journal of Industrial Economics,*.3

Fort, R., Quirk, J., (1995), Cross-subsidization, Incentives and Outcomes in Professional Team Sports Leagues, *Journal of Economic Literature*, 33/3

Fort R., Quirk, J., (2004), Owner Objectives and Competitive Balance, *Journal of Sports Economics*, 5/1, 20-32

Kesenne, S., (1996), League Management in Professional Team Sports with Win Maximizing Clubs, *European Journal for Sports Management*, 2/2

Kesenne, S., (2000), Revenue sharing and competitive balance in professional team sports, *Journal of Sports Economics*, 1/1

Kesenne, S., Pauwels, W., (2002), Club Objectives and Ticket Pricing in Professional Team Sports, Research paper 2002-018, Economics Department, University of Antwerp

Kesenne, S., (2004), Competitive Balance and Revenue Sharing When Rich Clubs Have Poor Teams, *Journal of Sports Economics*, 5/2, 206-212

Noll, R., ed., (1974), *Government and the Sport Business,* Brookings Institution, Washington D.C.

Quirk, J., Fort, R., (1992), *Pay Dirt, The Business of Professional Team Sports*, Princeton University Press, Princeton

Rottenberg, S., (1956), 'The Baseball Players' Labor Market', *Journal of Political Economy*, 64

Sloane, P., (1971), 'The economics of professional football: the football club as a utility maximizer', *Scottish Journal of Political Economy*, 7/2

Solow R., (1979) Another Possible Source of Wage Stickiness, *Journal of Macroeconomics*,1

Szymanski S., Kesenne S., (2004), Competitive balance and revenue sharing in team sports, *Journal of Industrial Economics*, LII/1, 165-177

Szymanski S., (2004), Professional Team Sports Are a Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium, and the Invariance Principle, *Journal of Sports Economics*, 5/2, 111-126

Zimbalist A., (2003), Sport as Business, *Oxford Review of Economic Policy*, 19/4, 503-511