Job raiding raises human capital investments

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abstract

This paper studies job raiding and its effect on incentives to invest in human capital. A firm can offer more attractive wages to new hires than to its current employees, thereby raiding a rival’s workers. Our model shows that firms prefer to raid in equilibrium when given the opportunity to do so. As rational workers foresee that job raids increase expected job earnings, they are willing to increase their ex ante investment in human capital. This insight has important implications for any industry where human capital is a scarce input and important aspects of personnel output are observable. Examples include the publications of academic researchers, the performance of professional athletes, and lawsuits won by lawyers. Our conclusions indicate that limiting organizations’ freedom to offer higher wages to new hires vis-à-vis equally productive incumbent employees inhibits investments in human capital.

Key words: job raiding, human capital, labor markets.
JEL-classification: L13, J63.

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1. Introduction

When the number of potential employees falls short of market demand, labor markets can best be described as “sellers’ markets.” When firms cannot easily attract currently unemployed workers to fill vacancies, they often try to lure employees from other companies. In this event, firms directly compete for scarce workers although they may not be competitors in the output market. Qualified IT-personnel can be considered as a prime example, especially during the software boom in the nineties. Turnover and labor poaching appeared to be quite prevalent in this segment of the labor market. A 1998 study by the Hay Group, a Chicago-based management consulting firm, shows that, again during the nineties, IT employers often offered a signing-on bonus, as well as a hefty compensation premium in order to lure employees away from their current position. Nowadays, healthcare personnel — particularly nurses — are in short supply. This has led to higher competition and turnover rates for nurses (United States General Accounting Office, July 2001). Consequently, “…nurses with some experience are often being wooed with one-time sign-on bonuses that sometimes exceed $5,000”. The market for CEO’s can also be characterized as a seller’s market and firms often compete head-on for them. One survey by the American Compensation Organization revealed that more than two-thirds of the companies surveyed offer sign-on bonuses of $10,000 and up for upper management positions. According to the Wall Street Journal (15 September, 1998), upfront awards, including sign-on bonuses for top executive personnel have grown in importance over the last decade. Finally, our model also applies to universities. Ransom (1993) describes a survey by the American Council on Education which reported that “more than half of the doctoral-granting institutions surveyed responded that in some departments they had hired new, junior faculty members at a salary above that of some senior faculty members in the same department”.

We model competition for scarce labor in an imperfectly competitive labor market and investigate how raiding impacts on productivity-increasing investments in human capital. More precisely, future workers make once and for all costly investments in non-
contractible human capital before entering the labor market. Two firms compete for scarce human capital during two consecutive periods by setting wages. These firms hire workers, conducting a one-period business project whose outcome is publicly observable. Investment in human capital increases the probability of the successful completion of a business project. Only employees with successfully completed projects are offered a new contract by the two firms in the next period. Although both firms are identical ex ante, employees incur a switching or mobility cost in period two when changing employer. At this stage, each firm can adopt two wage policies. The first wage policy consists of uniform wages; that is, identical wages are offered to current employees and new hires. With the other wage policy, the firm offers discriminatory wages to its current employees and its rival’s employees. By setting more attractive wages for new hires, firms effectively poach its rival’s employees. This job raiding intensifies wage competition. Our model shows that the discriminatory wage setting emerges as the unique equilibrium outcome. As a result, employees expect higher equilibrium wages if raiding is allowed for. And so, future employees are willing to increase their ex ante investment in human capital. Our conclusions indicate that limiting organizations’ possibility of offering higher wages to new hires vis-à-vis equally productive incumbent employees, inhibits ex ante investment in human capital. The results hold for different wage contracts such as fixed salaries and outcome dependent wages.

Another reason raids may occur is to affect competition in the product market. The New York Times (8 May, 1997) described Microsoft’s raid on key employees of Borland. In the course of two and a half years Microsoft recruited at least 34 of its top software architects, engineers, and marketing managers. Another illustration of poaching activities was observed when Amazon.com targeted a large number of technical associates from Wal-Mart (Internet World, April 19, 1999). Both cases resulted in law suits (see Gardner, 2003) where the plaintiffs claimed the defendants raided their employees so as to gain a competitive advantage in the output market. Similarly, professional sports teams often attract rival teams’ players, often mainly to impede those teams. Although important, we abstract from this. In our model, firms do not compete in the output market and the sole
reason job poaching takes place is to attract valuable personnel who are difficult to find elsewhere.

This paper makes two distinct contributions. Our first contribution is that job mobility—
in particular job raiding—may result from strategic firm behavior in an imperfectly
competitive labor market. In contrast with most models following Jovanovic (1979), we
do not rely on job matching to explain labor turnover. In the same spirit, Lazear (1986)
and Bernhardt and Scoones (1993) study job raiding as a specific form of labor turnover
using a matching model. In these models employers raid their rival’s workers only
because they are more productive than with their current employer. In our model,
however, workers are equally productive with both firms. Yet, raids on talent occur in
equilibrium. As workers incur costs when switching employers, as in Acemoglu and
Pischke (1999b), Black and Loewenstein (1991) or Ransom (1993), firms extract surplus
from their current employees. This allows new employers to profitably raid those rival’s
employees with relatively low switching costs. Although the empirical findings of Topel
and Ward (1992) are consistent with job matching models, our results point in the same
direction. As such, our model contributes to the literature on labor turnover.

As a second contribution, our model links job raiding with human capital accumulation.
Human capital, as embodied in education, substantially impacts firms’ productivity as
shown by Black and Lynch (1996), particularly in non-manufacturing sectors. Similarly,
Ho and Jorgenson (2001) find that labor quality improvements accounted for 40% of
labor input growth between 1948 and 1968 and in the period after 1980. In addition,
quality improvements in labor can be largely attributed to the rise in average levels of
educational attainment. Our specification that projects pay off with a higher probability at
higher levels of human capital is one way of modeling this. In our model, workers build
up human capital before entering the labor market, and so, we primarily think of human
capital as educational achievement. We show that different modes of wage competition
explain average education levels. If both firms set discriminatory wages, workers will
receive higher wages, because the ability to poach each other’s workers intensifies the
competition for talent. Higher expected wages increase incentives to invest in human
capital, and so the average level of worker qualifications will be higher with wage discrimination. Moreover, once allowed for, raiding becomes a dominant strategy. Although, given workers’ qualifications, overall profits will be lower for both firms when both raid, each firm always has an incentive to do so.

Our set-up shares a number of common features with Waldman (1990). He shows that employees invest more in general human capital with “up-or-out” contracts compared to standard spot-market contracts. As in our model, workers live for two periods and have the possibility to make a one-time investment in human capital. His main result is that the option of signing up-or-out contracts mitigates information asymmetries between inside and outside employers about a worker’s productivity. This increases competition for high-ability workers. As a result, employees have a greater incentive to raise productivity by human capital investment. Instead of examining how labor contracts impact human capital, we analyze how job raids through discriminatory wage policies affect human capital build-up. Moreover, we abstract from asymmetric information.

Our paper also relates to Acemoglu and Pischke (1998, 1999a) and Moen and Rosen (2003). They conclude that, with imperfectly competitive labor markets, general training will be suboptimally provided for. Training is general (as opposed to specific) as long as it equally impacts productivity across all firms in the industry. In our model we interpret human capital investments as a form of general training paid for by the worker, albeit in an imperfectly competitive labor market. We do not focus on who bears the cost of human capital investments, as these investments are decided on prior to entry in the labor market. Therefore, we make the natural assumption that future employees incur the costs, e.g. in the form of tuition fees.

As current employees find switching costly, their wage sensitivity will differ from that of new hires. This allows firms to practice third-degree wage discrimination. Oligopolistic third-degree discrimination has been well documented in output markets (see Stole (2003) for an overview). Similar insights, however, also apply for input markets, such as the labor market. In the words of Holmes (1989), current employees belong to the “strong
market” and the rival’s employees to the “weak market.” Consequently, firms offer relatively low wages to current personnel compared to the rival’s employees.

This model relates to Chen’s (1997), and adds to it in several ways. Most importantly, we apply it to the labor market and include an extra stage where human capital investments are decided on. We also specify the most likely outcome of first-stage competition when both firms compete with uniform wages, i.e. the payoff-dominant equilibrium. Finally, we endogenize discrimination by allowing firms the choice between discriminatory wages or uniform wages and come to the conclusion that as long as firms cannot credibly commit to a wage policy, both firms will discriminate in equilibrium. Ransom (1993) as well as Black and Loewenstein (1991) also consider labor market competition with mobility costs. In both models, new hires are offered a premium. We derive outside offers as ensuing from imperfect competition, whereas Ransom takes outside offers as given and Black and Loewenstein assume competition for new hires is perfectly competitive. Moreover, we compare competition when firms do not wage discriminate and link wage competition with human capital build-up.

After the presentation of the model, Section 3 deals with competition that arises when either both firms set uniform wages, or when both firms raid in period two. Section 4 examines competition in the first period, again with respect to the two distinct wage regimes. Section 5 proves that human capital investment will be superior with discriminatory wages. In Section 6 we show that discriminating constitutes a dominant strategy and so always occurs if not restricted or forbidden. Section 7 presents a discussion of the results. Finally, Section 8 concludes.

2. The model

Consider a unit mass of potential employees. They can be hired on the labor market by two firms $A$ and $B$ to work on one-period projects during two consecutive periods. The
two firms are competitors in the input market for labor only. That is, they do not compete in the output market. Firms as well as employees are risk-neutral. In order to capture the idea that labor is scarce, we assume that the number of potential, profitable projects exceeds the number of workers. This means that expected profits increase with the number of employees. Projects can either succeed, yielding a payoff of \( q \), or they can fail, yielding zero payoff.

In our model, workers can either be of a “good” or “bad” type, without the employees knowing to which class they belong. That a proportion \( 0 \leq k \leq 1 \) of all employees is of the good type is common knowledge. Bad workers always fail while good workers have a positive success probability \( p_G \) at both periods. If all workers are hired, the overall chance of success therefore equals \( p \equiv kp_G \). Employers cannot directly observe the type of their personnel and project outcomes are publicly observable, as in Holmström (1999). Firms and workers update their beliefs after the first period according to Bayes’ rule. Workers who have failed will likely be of the bad type, while all the successful workers necessarily have to be of good quality.

Firms can only offer one-term contracts to their workers in both periods, stipulating a positive wage \( W \). We suppose that firms only pay out the contracted wage in the event of success. Hiring a worker entails a cost equal to \( c \), irrespective of the quality of this employee. This could be the cost of renting an office, providing a computer, etc… Without this cost, everyone will always be hired (as an employer has then nothing to lose by engaging someone with a minor chance of succeeding). Profits increase linearly with the number of “good” workers a firm can attract. The expected profit per worker at period one therefore equals \( (q - W)p - c \), while the expected utility for workers equals \( pW \).

We impose the following restriction

\[
pq \geq c \geq (1 - p_G) pq.
\] (1)

If \( pq < c \), the first inequality is not satisfied and the labor market collapses. No firm would be willing to engage a worker at wages she would accept. If with a wage of zero
firms still would expect losses, then firms would rather not hire anyone at all.⁸ As we are not primarily concerned with such equilibria, we study the market only when \( pq \geq c \). For these parameter values, firms are willing to engage workers in the first period. It is not yet clear whether they are willing to offer contracts to all employees in the second period though. Previously successful workers are of the good type, with probability one. They have a chance of succeeding once more with probability \( p_G \) and will therefore certainly be attractive for both firms. Unsuccessful employees, however, could be of either type. The expected success probability at period two of someone who failed in period one equals \( k(1-p_G)p_G \), which is obviously smaller than the expected chance in period one. As long as the expected payoff of hiring someone who failed before, that is \( k(1-p_G)p_Gq \), is smaller than \( c \), both firms will be unwilling to do so. From (1), only good employees remain in the market at stage two; this increases the average chance of success from \( kp_G \) to \( p_G \). Good — but unlucky — workers are driven out of the market because they are pooled with all the low-ability workers.⁹

Workers can affect their chance at success by investing in human capital.¹⁰ Workers of the bad type fail whatever their human capital level, but good workers’ success probability is positively influenced by the investment undertaken. We assume the level of human capital to equal \( p_G \) as well, so that the variable of choice equals the success probability of good workers. Other functions will lead to similar results, as long as they exhibit a monotonic positive relationship between \( p_G \) (or \( p \)) and human capital. This level of human capital is chosen once and for all, prior to the first-period contract, is non-contractible and non-observable.¹¹

Investing in human capital is costly. Disutility of human capital build-up is defined as \( V(p_G) \), with \( \infty > V'(\cdot) \geq 0 \) and \( V''(\cdot) > 0 \). In addition, we assume that \( V'(0) = 0 \) and impose that \( V'(1) \geq k[q(1+2\delta p_G) - \delta c] \) where \( \delta \) equals the common discount factor.

This technical assumption ensures that the chance of conducting successful projects never exceeds one. This is proven in Appendix 1. Choosing the optimal level of investment
involves a trade-off. On the one hand it is costly, but on the other hand it increases the chance of a project succeeding and thus receiving a wage.

The two firms compete in wages simultaneously at periods one and two. At the start of period two, workers experience a cost \( s \) only when switching employers. Workers are in other words indifferent between employment by either firm ex ante (before the start of competition), but prefer employment with the firm they currently work for, other things being equal. Switching costs may be present for reasons such as the cost of establishing a relationship with a new employer, moving, adopting new work practices, integrating in a new work environment… \(^{12}\) There is little reason why this cost should be the same for every employee. Some may find it very easy to change jobs, while this may be more costly for others. Assuming dispersed switching costs is probably closer to the reality than assuming one identical cost for all employees, even if the actual distribution of costs in the model is chosen mainly for simplicity. For this reason, we assume that this switching cost \( s \) is uniformly and independently distributed among employees on the interval \([0, \bar{s}]\) (see Chen, 1997). Employees as well as employers only know the distribution of switching costs at the start of the game. The employee “learns” her switching cost after undertaking a project and before starting a second project, while the firm does not.\(^{13}\) After firms set wages, job turnover may occur and new projects are undertaken. Again, employees receive wages only if successful. The game ends after this second stage. Figure 1 summarizes the timing of the model.

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**Figure 1**: The timeline of the game.
3. Period-two competition

3.1. Discriminatory wages

We start the analysis at the second period of the game. At this point, we take success probability, \( p_G \), as given. If both firms set equal period-one wages, workers are indifferent between both firms, and we assume that a fraction \( 0 \leq \sigma \leq 1 \) visits firm A and the remaining part firm B. If wages at period one differed between the two firms, one firm will have an incumbency position at the start of period two, while the other will be an entrant in the labor market.\(^{14}\) We first study competition with an incumbent and an entrant firm, where the superscript \( I \) refers to the former and \( E \) to the latter. Subscripts denote the periods at which competition takes place. We assume that employees do not switch when they are indifferent between both firms at the second stage.

Market demand \( D_2^I \) for the incumbent equals

\[
D_2^I \left( W_2^I, W_2^E \right) = \begin{cases} 
0 & \text{if } \frac{p_G W_2^E - \bar{s}}{p_G} > W_2^I \\
\frac{\bar{s} - s^*}{\bar{s}} & \text{if } W_2^E > W_2^I \geq \frac{p_G W_2^E - \bar{s}}{p_G} \\
p & \text{if } W_2^I \geq W_2^E ,
\end{cases}
\]  

(2.1)

(2.2)

(2.3)

where \( s^* \equiv p_G (W_2^E - W_2^I) \) is defined as the switching cost for which an employee of the incumbent is indifferent between staying with the current employer and contracting with the entrant. Note that total market size at period two equals \( p \) instead of unity. By assumption, firms are unwilling to hire previously unsuccessful workers and only the workers who have performed well in period one are employed in this period.

Job demand for firm \( I \) is zero, as shown in part (2.1), when even the hires for whom switching is most costly find it in their interest to change jobs. If, however the wage of the incumbent is sufficiently high for the employees with the lowest switching costs to
stay with this firm, firm I retains its monopoly. Part (2.3) depicts this. The middle part (2.2) represents the intermediate case where only those workers with relatively low switching costs leave for firm E and the others stay with firm I. Evidently, market demand for the entrant is the complementary share.

The best-response for the incumbent firm equals

\[
W'_2 = \begin{cases} 
q - \frac{c}{p_G} & \text{if } W_2^E > q - \frac{c - \bar{s}}{p_G} \\
\frac{p_G W_2^E + p_G q - c - \bar{s}}{2p_G} & \text{if } q - \frac{c - \bar{s}}{p_G} \geq W_2^E \geq q - \frac{c + \bar{s}}{p_G} \\
W_2^E & \text{if } q - \frac{c + \bar{s}}{p_G} > W_2^E.
\end{cases}
\] (3.1) (3.2) (3.3)

The highest wage firm I can possibly adopt, without certainly making non-positive profits, is \(q - c/p_G\). If the entrant were to offer a higher wage than \(q - (c - \bar{s})/p_G\), firm I could not improve by setting a higher or a lower wage. By offering a lower wage, market share of firm I remains zero, while a higher wage leads to negative profits. This covers (3.1) of the best response of the incumbent. The second part (3.2) is derived by multiplying the market share, \(p(\bar{s} - s)/\bar{s}\) by the profits per employee to obtain the overall profits. From the sufficient first-order conditions and solving for \(W'_2\), we acquire the optimal response to the entrant’s wages. The wage thus obtained is optimal for somewhat lower values of \(W_2^E\) than in part (3.1). For low values of \(W_2^E\) the incumbent optimally matches the entrant’s wage. In this way, the incumbent extends the monopoly at the lowest wage possible. The wage of firm E for which this strategy proves superior to the former (i.e. focusing only on the employees with relatively high switching costs) can be easily gauged by comparing profits resulting from parts (3.2) and (3.3). This summarizes the reaction function of the incumbent.
Proposition 1. There exists a unique Nash equilibrium in pure strategies when both firms wage discriminate. The following discriminatory wages

\[
W_{2D}^I \equiv q - \frac{c}{p_G} - \frac{2\bar{s}}{3p_G} \quad \text{and} \quad W_{2D}^E \equiv q - \frac{c}{p_G} - \frac{\bar{s}}{3p_G}
\]

belong to this equilibrium. The incumbent retains 2/3th of its employees, while the remaining one-third switches to the entrant. Every employee with a switching cost lower than \(\bar{s}/3\) switches in equilibrium. Profits then become \(\Pi_{2D}^I \equiv 4p\bar{s}/9\) and \(\Pi_{2D}^E \equiv p\bar{s}/9\).

One can directly verify from the sufficient first-order conditions that only the wages of Proposition 1 satisfy the above best response functions. The entrant sets a higher wage than its competitor to compensate for the disadvantage stemming from the switching costs. The incumbent’s profits are larger because of the higher mark-up per employee as well as the larger pool of employees. Note also that the profits of both duopolists increase as \(\bar{s}\) becomes larger.

If both firms set the same wage at the first stage of the game, workers are indifferent and a fraction \(\sigma\) gets to work with firm \(A\) and \(1-\sigma\) with firm \(B\). Each firm behaves as an incumbent for its current employees, while setting a different wage for poaching the rival’s employees. Total labor demand for firm \(A\) therefore equals \(\left[2\sigma/3+(1-\sigma)/3\right]p\) and \(\left[2(1-\sigma)/3+\sigma/3\right]p\) for firm \(B\). This leads to second-period profits of \(\Pi_{2D}^I \equiv \sigma\Pi_{2D}^I + (1-\sigma)\Pi_{2D}^E\) and \(\Pi_{2D}^E \equiv (1-\sigma)\Pi_{2D}^I + \sigma\Pi_{2D}^E\).
3.2. Uniform wages

If one firm engages all workers in period one, it will set the incumbent’s wage in period two, while the other offers the entrant’s wage. As a result, the analysis will be no different from the discriminatory case. In our model, firms only want to discriminate between different groups of employees if wage elasticities differ among these groups. Current employees have a lower wage elasticity than others because of the switching costs. However, if one firm employs everyone, workers are not differentiated anymore and discrimination loses its appeal. So the distinction between uniform and discriminatory wages becomes interesting only if both firms have a strictly positive market share in period one.

Without loss of generality, we now assume that market share of firm $A$ equals $0 < \sigma \leq 0.5$. Second-period labor demand for firm $A$ equals

$$D^A_2(W^A_2, W^B_2) \equiv \begin{cases} 
    p & \text{if } W^A_2 > \frac{p_G W^B_2 + \bar{s}}{p_G} \\
    \sigma + (1 - \sigma) \frac{s^I}{\bar{s}} & \text{if } \frac{p_G W^B_2 + \bar{s}}{p_G} \geq W^A_2 > W^B_2 \\
    \sigma - \frac{s''}{\bar{s}} - p & \text{if } W^B_2 > W^A_2 \geq \frac{p_G W^B_2 - \bar{s}}{p_G} \\
    0 & \text{if } \frac{p_G W^B_2 - \bar{s}}{p_G} > W^A_2.
\end{cases}$$

We denote by $s^I \equiv p_G (W^A_2 - W^B_2)$ and $s'' \equiv p_G (W^B_2 - W^A_2)$ the switching cost of the indifferent employee of firm $B$ and $A$, respectively.

For a wage high enough such that even the employees of $B$ with the highest switching cost find it worthwhile to switch, (5.1) shows that firm $A$ captures the full market (of size
In contrast, when the competitor’s wage is high enough, firm $A$ is left without personnel at period 2. This is captured by (5.5). Only when wages for both firms are identical, as with (5.3), will switching not occur. For wages that do not differ much, only employees with relatively low switching costs will change jobs; in (5.2) there is switching from $B$ to $A$ whereas the opposite takes place in (5.4).

Obviously, the demand curve of firm $B$ is the mirror image of $A$’s. Note that the demand curves of both competitors exhibit a kink where wages are equal to one another. This implies that the labor demand and hence the profit function of the duopolist with fewer workers (firm $A$) is not concave in the decision value. The opposite holds for the firm with the most workers. For this reason, firm $B$’s best response function is easier to characterize and will be handled first.

\[
W_2^B = \begin{cases} 
\frac{q-c}{p_G} & \text{if } W_2^A > q - \frac{c-\overline{s}}{p_G} \\
\frac{p_G W_2^A + p_G q - c - \overline{s}}{2p_G} & \text{if } q - \frac{c-\overline{s}}{p_G} \geq W_2^A \geq q - \frac{c + \overline{s}}{p_G} \\
\frac{p_G W_2^A + p_G q - c - (1-\sigma) \overline{s}}{2p_G} & \text{if } q - \frac{c + \overline{s}((1-\sigma)/\sigma)}{p_G} \geq W_2^A \geq q - \frac{c + \overline{s}((1-\sigma)/\sigma)}{p_G} \\
\frac{p_G W_2^A + \overline{s}}{p_G} & \text{if } q - \frac{c + \overline{s}(2+(1-\sigma)/\sigma)}{p_G} > W_2^A.
\end{cases}
\]  

Again, when $W_2^A$ is extremely high, that is over $q - (c - \overline{s})/p_G$, firm $B$ cannot improve upon demanding the break-even wage $q - c/p_G$. On the other hand, when $W_2^A$ becomes sufficiently low, the best one can do is to set a wage in order to employ all workers. For all wages of $B$ equal or over $\left(p_G W_2^A + \overline{s}\right)/p_G$, this happens. In that case, setting the wage equal to $\left(p_G W_2^A + \overline{s}\right)/p_G$ is always optimal, as profits per employee will then be at the highest, while still poaching all the workers.
So far, we know that for very high and very low values of the competitor’s wage, firm \( B \) either employs no worker at all or everyone. For values in between, \( B \) can pursue two possible strategies: either offer a higher wage and target part of firm \( A \)’s work force (at the expense of having to pay current employees more), or propose a lower wage, thereby losing some of the current employees (but receiving high profits on the remaining hires). In contrast with discriminatory competition, both firms cannot possibly invade each other’s work force at the same time.

In the “high wage” case \( B \) poaches \( A \)’s employees, as in (6.4). This best response can be calculated from the first-order condition. Notice that this expression is decreasing in \( B \)'s current market share \( \sigma^{-} \). This means that the more employees firm \( B \) has in the first period, the less aggressive it will behave in the second period. With a large existing personnel force, it pays less to set a rather high wage: the profit on a relatively small number of new hires does not counterbalance the loss one incurs on all the existing employees.

For sufficiently low wages of \( A \), firm \( B \) would be better off setting \( \left( p_g W_2^A + \bar{s} \right) / p_g \) as a wage and attracting all employees. By comparing profits resulting from wages as set in (6.4) and (6.5), one can find the threshold value of \( W_2^A \) for which firm \( B \) prefers to lure away all the rival’s employees instead of some of them.

From the first-order conditions, setting a lower wage than firm \( A \) implies a best response as in (6.2). In contrast with (6.4), this wage does not directly depend on the market share \( 1 - \sigma \). Actually, the firm behaves as an incumbent in its market share and thus responds in the same fashion as the incumbent did in Section 3.

There is a range of wages within which it is optimal for firm \( B \) to set its wage equal to that of its competitor. To see this, suppose that firm \( A \) offers a low wage and firm \( B \)’s first-period market share approximates one. In this event, it does not pay off for firm \( B \) to offer a strictly higher wage; entering the other’s market does not compensate for the loss
of mark-up on current employees. Likewise, setting an even lower wage entails the switching of part of the current work force. As profits per employee are high with low wages, this loss does not weigh up against the extra mark-up on the remaining workers. Matching wages, though, guarantees a maximal mark-up per employee without loss of workers. When \((1 - \sigma)\) equals 1, setting an equal wage is the best response for all values of the competitor’s wage below \(q - \frac{(c + \bar{s})}{p_G}\). As the market share of firm B decreases, the range of \(W_2^A\) for which firm B prefers to offer matching wages, shrinks. Because firm A’s market increases, invading it by setting a strictly higher wage becomes more attractive. Remark that when \(\sigma = 0.5\), part (6.3) of firm B’s best response disappears. This summarizes the best response of the larger firm B.

Firm A’s best response function, given that its current market share is smaller than one half, is similar in a lot of ways. The main difference is that the labor demand curve (and hence the profit function) is no longer concave over the domain. The result is that the best response function will be discontinuous. For one particular wage \(\tilde{W}_2^B\), firm A is indifferent between offering a higher wage and hence invading B’s market, and offering a lower wage and being invaded. Solving for this wage \(\tilde{W}_2^B\) is straightforward, but difficult and cumbersome computationally. For this reason, we do not specify this wage but instead focus on the optimal wage of the small employer, given the wage of the large firm. If the wage of B changes by a small amount around this point, the response will shift discontinuously, as can be gauged from the following expression:

\[
W_2^A = \begin{cases} 
q - \frac{c}{p_G} & \text{if } W_2^B > q \frac{c + \bar{s}}{p_G} \\
\frac{p_G W_2^B + p_G q - c - \bar{s}}{2p_G} & \text{if } q \frac{c + \bar{s}}{p_G} \geq W_2^B \geq \tilde{W}_2^B \\
\frac{p_G W_2^B + \bar{s}}{p_G} & \text{if } \tilde{W}_2^B \geq W_2^B \geq q - \frac{c + \bar{s} \left(2 + \frac{1 - \sigma}{\sigma}\right)}{p_G} \\
\frac{p_G W_2^B + \bar{s}}{p_G} & \text{if } q - \frac{c + \bar{s} \left(2 + \frac{1 - \sigma}{\sigma}\right)}{p_G} > W_2^B . 
\end{cases}
\]
The next Proposition summarizes the main findings of this Section. The proof of this Proposition is relegated to Appendix 2.

**Proposition 2.** For any $0 \leq \sigma \leq 0.5$, there is a unique Nash equilibrium in pure strategies if both employers set a uniform wage

\[
W_{2U}^A = q - \frac{c}{P_A} - \frac{\bar{S}}{3P_A} \left[1 + 2 \frac{\sigma}{(1 - \sigma)}\right] \quad \text{and} \quad W_{2U}^B = q - \frac{c}{P_B} - \frac{\bar{S}}{3P_B} \left[2 + \frac{\sigma}{(1 - \sigma)}\right].
\]

The number of workers switching from the large employer to the smaller equals

\[
p \left[1 - \frac{2}{3} \sigma\right].
\]

This means the average expected switching cost for those employees that change jobs equals

\[
\delta \left(1 - \frac{2}{3} \sigma\right)^2 \bar{S}p.
\]

Profits then become

\[
\Pi_{2U}^A = \frac{\bar{S}p}{9} \left(1 + 2 \frac{\sigma}{(1 - \sigma)}\right)(\sigma + 1)
\]

and

\[
\Pi_{2U}^B = p(1 - \sigma) \left[2 + \frac{\sigma}{(1 - \sigma)}\right]^2.
\]

A firm’s first-period market share will influence its second-period wage setting when it practises a uniform wage policy. In general, setting a low wage has two opposite effects on profits. On the one hand, it allows rents to be extracted from current employees who do not switch. On the other hand, this comes at the cost of losing some workers and/or not attracting new ones. The first effect becomes less important when the market share to begin with is rather low. There is little point in taking advantage of the loyalty of current employees when they are but few. A better strategy for small firms would be to offer a high wage in order to attract new personnel, instead of skimming the employees already engaged. Increasing market share is more important to small firms. Large firms, however, have a far greater incentive to offer low wages, as they are largely uninterested in targeting the small pool of workers of the small firm. These observations are in line with the results obtained by Chen (1997) and Fudenberg and Tirole (2000), albeit in a different context.
Wages decrease with market share and the firm with the highest market share always offers the lowest wage. In the terminology of Fudenberg and Tirole (1984), the large firm is a so-called “fat cat.” In a market where wage discrimination is impossible and one duopolist has obtained a large “captive” market, this player becomes more reluctant to match wages. This facilitates the entry of others (as indeed the smaller firm profitably attracts some of the larger firm’s workers). Note that the equilibrium wages converge to the wages of Proposition 1 if the market share of firm A converges to zero. Finally, note that profits are not symmetric for both duopolists. This asymmetry is introduced by period-one market shares.

From Proposition 2, the industry profits can be written as

\[ \Pi_{2u}^T = \begin{cases} \frac{s}{9} p \frac{2\sigma^2 - 5 - 2\sigma}{\sigma - 1} & \text{if } \sigma < 0.5 \\ \frac{s}{9} p \frac{2\sigma^2 + 5 - 2\sigma}{\sigma} & \text{if } \sigma \geq 0.5, \end{cases} \]

where the superscript T refers to total industry profits. Examining the first derivative of this expression yields the following corollary.

**Corollary 1.** Industry profits \( \Pi_{2u}^T \) are monotonically increasing in \( \sigma \) until \( \sigma = 0.5 \), and monotonically decreasing thereafter. The profit function exhibits a maximum at \( \sigma = 0.5 \) and a minimum at \( \sigma = 0 \) and \( \sigma = 1 \), at which profits add up to \( p5\bar{s}/9 \).

The more evenly the market is split in two, the less an incentive there is for firm A to forego high margins on current employees in order to target the rival’s employees. The less aggressive behaviour by A allows firm B to keep wages down also.

We now compare the above results with the conclusions that emerge under discriminatory wage policy.
**Corollary 2.** Second-period industry profits are, given the level of human capital, always superior with uniform wages than with discrimination. This difference increases as \( \sigma \to 0.5 \).

Total industry profits are \( p5\bar{s}/9 \) for all possible values of \( \sigma \) if both duopolists discriminate. Corollary 2 then instantly follows from Corollary 1.

**Corollary 3.** The profit of both firms with uniform wages exceeds the profits with discrimination, given the level of human capital.

One can easily verify this by comparing the profits from both wage regimes for one firm, say firm \( A \), for all possible market shares. It can indeed be shown that

\[
\begin{cases}
  p\left(1+2\sigma/(1-\sigma)\right)(\sigma+1)\bar{s}/9 \geq p(1+3\sigma)\bar{s}/9 & \text{if } \sigma < 0.5 \\
  p\sigma\left[2+(1-\sigma)/\sigma\right]^2 \bar{s}/9 \geq p(1+3\sigma)\bar{s}/9, & \text{if } \sigma > 0.5
\end{cases}
\]

where the left-hand sides refer to profits with uniform wages and the right-hand sides represent the profits from discrimination. Competition is intensified by discriminating, because firms can now poach employees without harming profits from the current work force. Because both firms target each other’s employees, all wages are driven up. This is in contrast with a monopolistic setting, where profits are never lower when discrimination is allowed for (Varian, 1989). In one of the first papers about price competition in oligopoly, Holmes (1989) describes the effect of discrimination on profits as ambiguous in general. Corts (1998) extends this model by studying asymmetric models of competition also, meaning that firms differ in their ranking by consumer groups according to their demand elasticities at a given rival’s price. Then firms may have incentives to lower prices for all consumers, leading to “all-out competition.” This asymmetry is present in our model. Firms have a competitive advantage with respect to their own employees and a disadvantage with respect to the rival’s; this has as a consequence that both are worse off by discriminating.
4. Period-one competition

4.1. Discriminatory wages

In the first period, there is no friction in the market and so the firm offering the highest wage captures the full market. Total profits for firm $A$, $\Pi_D$, can be summarized as

$$
\Pi_D^A = \begin{cases} 
(p(q-W_{1D}^A)-c) + \delta \Pi_2^I & \text{if } W_1^A > W_1^B \\
\sigma(p(q-W_{1D}^A)-c) + \delta (\sigma \Pi_2^I + (1-\sigma) \Pi_{2D}^E) & \text{if } W_1^A = W_1^B \\
\delta \Pi_{2D}^E & \text{if } W_1^A < W_1^B
\end{cases}
$$

(8.1) (8.2) (8.3)

If firm $A$ sets a strictly higher wage than $B$, it will employ all the workers and take on the role of incumbent in the following period. If it sets a lower wage, it will be priced out of the market in the first period but still has the possibility of profitably entering later. If the wages of the two competitors are equal, the market is shared between them. Firm $A$ then behaves as an incumbent, and receives the profits of an incumbent on a $\sigma$-part of the market and earns an entrant’s profits on the remaining part. The overall profits of firm $B$ are similar.

**Proposition 3.** The wages $W_1^A = W_1^B = q - \frac{c}{p} + \delta \bar{s}/3$, denoted by $(W_{1D}^A, W_{1D}^B)$ form the unique Nash-equilibrium of this game with discriminatory wage setting. Overall profits for both duopolists equal $\delta \Pi_2^E = \Pi^*$.

Each firm is prepared to incur losses in the first period, equal to the difference in the (discounted) profits one could achieve as an incumbent and the profits as an entrant. The first-period wage of both should thus be $\left[pq-c-\delta(\Pi_2^I-\Pi_2^E)\right]/p$, or $q-c/p+\delta \bar{s}/3$, with $\Pi_2^I-\Pi_2^E$. 


which leads to overall profits of $\Pi'$. If one firm deviates and introduces a lower wage, it is sure to lose the first-period competition and acquires $\Pi'$ as profits anyway. By proposing a higher wage, one becomes incumbent, but at too high a cost. Its overall profit will then be lower than $\Pi'$ and thus the above wages constitute a Nash equilibrium.

To show the uniqueness of this equilibrium, we consider all other possible wage configurations and show that none of these actually constitutes an equilibrium. Equilibria with one or both firms offering a higher wage than $W_{1D}$ can be ruled out, as at least one firm would then make strictly lower profits than $\Pi'$. Irrespective of the wage of the competitor, either firm could guarantee itself a higher profit by setting a wage lower than or equal to $W_{1D}$.

For wages lower than $W_{1D}^A$, the firm offering the lowest wage (or any firm in case of a tie) could improve by setting an infinitesimally higher wage than its competitor. In this way, this firm would ensure an incumbency position for itself at a minor cost and would achieve a higher profit overall. Suppose one firm puts forward wage $W_{1D}^i$ and the other some lower wage $W^-$. Then, the former can do better by setting its wage equal to $W^- + \varepsilon$, with $\varepsilon$ an arbitrarily small but positive number. In this way, it still “wins” the competition, but at a lower cost.

The prospect of high incumbency profits generally leads to fierce competition in the first period. If profitable second-period entry is not possible, all the profits for the incumbent in the second period would be fully competed away in the first period. Competition for the incumbency position would then ensure that profits over the whole game would be equal to zero. In our model with dispersed switching costs, second-period entry entails positive profits and so one does not compete as harshly in the first period. Losing the competition in the first period still guarantees a profit of $\Pi'$, and so firms will not be willing to incur too much loss in the first period in order to set the higher wage. This contrasts with a model with identical switching costs across workers.
4.2. Uniform wages

In contrast with discrimination, the first-period labor demand with uniform wage setting is not perfectly elastic. From Proposition 2 we know that the firm with the highest first-period market share always offers the lowest wage in period two. Moreover, the larger a firm’s market share becomes in period one, the lower the wage of this firm will be in the following period and rational workers foresee this. Suppose, therefore, that one firm sets a slightly higher wage than the other. Not all employees are going to work with this firm, because they know that it will set lower wages in the following period. This differs from discriminatory wage setting, as workers now care about the market share of each firm in period one.

The expected wage in period two is a function of the first-period market share. More specifically, suppose first-period wages equal each other. If more workers were to contract with firm $A$ than with firm $B$, the former would offer lower wages in the next period and consequently workers would not be indifferent between both employers. Only when both firms attract half of the market will employees be indifferent between the two firms, for then both firms’ wages will equal each other both in period one as well as in period two. In other words, when $W_1^A = W_1^B$ we find that $\sigma = 1 - \sigma = 0.5$ are the only market shares consistent with employees’ rationality.

Similarly, if first-period wages differ to some extent, workers choose employers such that they become indifferent between firm $A$ and $B$ in equilibrium. The firm offering the highest wage will attract more workers, but not all of them, because workers trade off the current high wage with the future lower wage related to a higher market share. First-period market shares become fully endogenous and depend on the difference between first-period wages. Market share $\sigma$ is described by

$$pW_1^A + pp_g \delta W_{2U}^A = pW_1^B + p\delta \left( \frac{1}{3} - \frac{2}{3} \sigma \right) \left( p_g W_{2U}^A - \frac{1}{2} \left( \frac{1}{3} - \frac{2}{3} \sigma \right) \right) + \left[ 1 - \left( \frac{1}{3} - \frac{2}{3} \sigma \right) \right] p_g W_{2U}^B.$$
The left-hand side reflects the utility of being employed with firm \( A \) and the right hand side with \( B \). The first term of each side reflects the expected wages from working for firm \( A \) or firm \( B \) in period one. The second term on both sides expresses the expected wage of period two, depending on period-one wages. As \( A \) is the small firm by assumption, it will set the highest wage at the second stage and therefore no employee will switch from \( A \) to \( B \). As a result, employees of \( A \) earn \( W_{2U}^A \). Some of \( B \)’s employees will switch firms, however. As a consequence, their expected second-period wage equals a weighted average of \( W_{2U}^A \) and \( W_{2U}^B \), where the weights are the probabilities with which employees switch from \( B \) to \( A \) or stay with \( B \), respectively. Note that workers have to pay a switching cost when switching to firm \( A \) so that the average switching cost has to be subtracted from the earned wage.

Rewriting the above equality yields

\[
W_{1B}^U - W_{1A}^U = \delta \left( 2^{1+\frac{\sigma}{3}} - \frac{\sigma}{1-\sigma} + \frac{1}{2} \left( \frac{1-\frac{2}{3}\sigma}{3} \right)^2 \right). \tag{8}
\]

The first derivative of the right-hand expression with respect to \( \sigma \) is negative. This means that the lower the difference in the wages of both firms becomes, the higher \( \sigma \) becomes in order for employees to have the same expected utility. If wages equal each other, \( \sigma \) becomes one half.

Several first-period wages constitute a subgame perfect Nash equilibrium. Following Harsanyi and Selten (1988), we focus on the “payoff-dominant” equilibrium, i.e. the subgame perfect equilibrium that yields the highest profits of all possible equilibria. This commonly preferred equilibrium acts as a focal point. As a result, firms should find it easy to coordinate their expectations on this payoff-dominant equilibrium. In order to find this equilibrium, we can restrict our attention to symmetric equilibria as Proposition 2 shows that second-period profits for both firms are uniquely maximized when \( \sigma = 0.5 \).

Given worker qualifications, profits for both firms increase with lower period-one wages. The equilibrium with the lowest wages yields the highest profits for both firms. Selecting this equilibrium leads to Proposition 4. We relegate the proof to Appendix 4.
Proposition 4. The wages \( W_1^A = W_1^B = q - \frac{c}{p} - \delta \bar{\sigma} \), denoted by \((W_{1u}^A, W_{1u}^B)\), form the unique payoff-dominant subgame perfect Nash-equilibrium of this game with uniform wage setting. Overall profits for both duopolists equal \( \delta p \bar{\sigma} \equiv \Pi^{**} \).

5. Equilibrium investment in human capital

In this section we compare the equilibrium level of human capital under the two wage policies.

Proposition 5. When both firms wage discriminate, equilibrium human capital is larger than when both firms apply a uniform wage policy.

We first compute the equilibrium level of human capital under discrimination and compare this to the equilibrium level with uniform wage setting. This comparison results in Proposition 5.

The worker’s expected utility with equilibrium discriminatory wages equals

\[
U(p_G) = kp_c W_{1d} + \delta kp_c \left( \frac{2}{3} W_{2d}^I + \frac{1}{3} W_{2d}^E \right) - \delta \frac{1}{3} kp_c \bar{\sigma} - V(p_G)
\]

with \( i = A, B \). The first term of this expression is the worker’s expected wage in period 1. The second term shows the expected wage in the second stage which is a weighted average of the wage the incumbent firm charges and the entrant’s wage. As one-third will switch from their current employer (offering a wage \( W_{2d}^I \)) to the other firm (offering a wage of \( W_{2d}^E \)) and workers do not know their cost of switching when investing in human capital, we can interpret this as the probability of earning \( W_{2d}^E \) instead of \( W_{2d}^I \). Ex ante, a worker does not know if she will ever reach the second stage, so the payoff of working in period two only comes to pass if one succeeds in period one (with probability \( p \)) and in
the period thereafter (with probability \( p_G \)). The third term expresses the average cost of switching. The average ex ante switching cost equals the product of the chance of switching and the cost if one switches. As one-third of all employees change employers, they have an average cost of \( \bar{s}/6 \), multiplied by \( p \) (as one can only switch after succeeding). Finally, human capital build-up is costly, according to the function \( V \).

By optimizing utility, we know the equilibrium human capital level is implicitly defined by

\[
k \left[ q + \delta \left( 2kp_Gq - c - \frac{5}{18} \bar{s} \right) \right] = \frac{\partial V(p_G)}{\partial p_G}. \tag{12}
\]

For the parameter values allowed, more investment in human capital will be put forth when successful projects yield high revenues (\( q \) is high). This happens because expected wages increase in \( q \). When the discount rate nears unity, the human capital investment also expands. The latter comes to pass because the advantages of a one-time investment in human capital accrues over several periods. In addition, high marginal costs for employing a worker drives wages and, therefore, human capital down.

Finally, and most important, the optimal human capital investment decreases as the market becomes more segmented (\( \bar{s} \) is high), for this entails more market power for the employers and hence lower expected wages. In other words, higher labor mobility increases incentives to invest in human capital as long as the employee bears the cost of this investment.\(^\text{16}\)

The worker’s expected utility with equilibrium uniform wages equals

\[
U(p_G) = pW_{iU} + \delta p p_G W_{2U} - V(p_G).
\]

Optimization of this function yields

\[
k \left[ q + \delta \left( 2kp_Gq - c - 2\bar{s} \right) \right] = \frac{\partial V(p_G)}{\partial p_G}. \tag{13}
\]
From (12) and (13) we see that equilibrium human capital level is higher with job raids. As workers earn more in both periods with a discriminatory wage policy, they have a much higher incentive to succeed, that is, to invest in human capital. This illustrates Becker’s (1975) observation that “[t]he most important single determinant of the amount invested in human capital may well be its profitability or rate of return…” (p. 45). Our paper adds to this fundamental insight by explicitly modeling oligopolistic competition in the market for scarce labor.

6. Discrimination as a dominant strategy

This section presents the central result of our paper. We assume that firms cannot commit to a wage policy.

**Proposition 6.** As a strategy, wage discrimination between current and new employees dominates wage uniformity. Consequently, workers accumulate more human capital when wage discrimination is allowed for.

By a dominant strategy, we mean that firms always prefer to discriminate between current and new employees, irrespective of the wage setting policy of the competitor. To prove this, we need to check whether the outcomes of Propositions 1 and 2 constitute an equilibrium when firms can choose their wage policy. In other words, we have to check whether the conjectures adhered to by both firms are consistent in equilibrium. Suppose that both firms expect each other to set uniform wages as in Proposition 2. Is it not possible for one of the firms to deviate profitably by applying discriminatory wages taking the other’s uniform wage as given? If so, the outcome of Proposition 2 is not an equilibrium when firms can choose their wage policy. We must apply a similar reasoning for the outcome of Proposition 1. In addition, we also need to find out whether there is an asymmetric equilibrium when only one firm wage discriminates.
Proof of Proposition 6:

(i) Suppose firm $A$ sets the uniform wage $W_{2U}^{A}$ as in Proposition 2. Firm $B$’s optimal response then corresponds with (3.2) and (4.2). As a result, firm $B$ maximizes its profits by offering the wages

$$W_{2B}^{U} = q - \frac{c}{p_g} - \frac{\bar{\sigma}}{3p_g} \left[ 2 + \frac{\sigma}{1-\sigma} \right] \quad \text{and} \quad W_{2B}^{E} = q - \frac{c}{p_g} - \frac{\bar{\sigma}}{6p_g} \left[ 1 + \frac{2\sigma}{1-\sigma} \right]$$

to its current employees and new hires, respectively. The corresponding profits become

$$\Pi_{2B}^{U} \equiv \frac{p \bar{\sigma}}{9} \left\{ (1-\sigma) \left[ 2 + \frac{\sigma}{(1-\sigma)} \right]^2 + \sigma \left[ \frac{1}{2} + \frac{\sigma}{1-\sigma} \right]^2 \right\}.$$ 

Comparison shows $\Pi_{2D}^{U} \geq \Pi_{2U}^{B}$ for all $0 \leq \sigma \leq 0.5$. Similarly, it can be shown that it is optimal to deviate when $0.5 \leq \sigma \leq 1$. Consequently, there is no equilibrium where both firms have a uniform wage policy.

(ii) Suppose firm $A$ applies a discriminatory wage policy whereas firm $B$ pays its personnel a uniform wage. Is it optimal for firm $B$ to deviate and discriminate between current and newly hired personnel as well? From Appendix 5 it follows that firm $B$’s profit-maximizing discriminatory wages equal

$$W_{2B}^{I} = q - \frac{c}{p_g} - \frac{8-\sigma}{12p_g} \bar{\sigma} \quad \text{and} \quad W_{2B}^{E} = q - \frac{c}{p_g} - \frac{5-\sigma}{12p_g} \bar{\sigma}.$$ 

By substituting these wages into $B$’s profit function, we find that the profit from deviating equals

$$\Pi_{2D}^{B'} \equiv \frac{p \bar{\sigma}}{9} \left\{ \sigma \left( \frac{5-\sigma}{12} \right)^2 + (1-\sigma) \left[ \frac{8-\sigma}{12} \right]^2 \right\}$$

which is larger than the corresponding profit from setting a uniform wage for all $0 \leq \sigma \leq 1$. As a result, an asymmetric equilibrium does not exist when one firm offers discriminatory wages and the other a uniform wage.

(iii) Suppose firm $A$ sets the discriminatory wages $W_{2D}^{I}$ and $W_{2D}^{E}$ as in Proposition 1. From Appendix 5 it follows that firm $B$’s profit-maximizing uniform wage equals
\[ W_{2U}^B = q - c \frac{(2 - \sigma)S}{s} \cdot \frac{3p_G}{p_c}. \]

The corresponding profit becomes

\[ \Pi_{2U}^B = \frac{P_S}{9} \left[ (1 - \sigma)(2 + \sigma) + \sigma^3 \right] (2 - \sigma). \]

A comparison shows that \( \Pi_{2U}^B \leq \Pi_{2D}^B \) and so firm B has no incentive to deviate. Likewise, A will have no incentive either. This proves that the unique equilibrium consists of both firms discriminating between current and newly hired personnel. Moreover, it is a dominant strategy to do so. \( \blacksquare \)

If employers have the choice between discriminating or not, they will discriminate in period two, as, given the investment in human capital and wages in period one (which are bygones) and irrespective of the wage policy of the competitor, one will always improve profits by doing so. Because profits are higher with uniform wages, the ability to wage discriminate yields a prisoner’s dilemma situation.\(^{18}\)

7. Discussion

One may wonder why firms do not offer a higher wage to the workers being poached by the competing employer. After all, in models such as Lazear (1986) and Bernhardt and Scoones (1993), employees cannot be profitably poached, unless they are more productive with rival firms than with their current employer. As workers are equally productive with both firms in our model, then why are raids on talent an equilibrium phenomenon? In our setup, firms would not be willing to counter-offer even if it were possible, as long as employees cannot credibly signal their switching costs. The optimal counter-offer if all current employees are offered a wage \( W_{2D}^E \), is presented by (3.2), and so even if firms have the possibility of renegotiating wages, they will still not be prepared to do so. As in Lazear (1993), firms may optimally adopt a policy of “no offer matching” if workers have a strong preference for their current firm. This is precisely captured by
our switching costs approach. Most workers are unwilling to change employers, even if offered a higher wage $W_{2D}$. For this reason, firms will not raise wages if their personnel threaten to leave. If workers could somehow credibly display their seriousness to leave (i.e. what switching costs they have), firms would bid up individual wages to $q - c/p_g - s$, which means every employee receives a different wage and no turnover takes place. As a special case, suppose every employee has the same switching cost. Then, a no-switching equilibrium would emerge, even when outside firms can poach. Firms will retain all their workers by setting wages equal to the marginal labor productivity minus the switching cost. Banerjee and Gaston (2001) have constructed a model in which two types of workers send signals about their type to the labor market. As long as they can perfectly signal their type (or productivity) and all have the same switching costs, the above equilibrium would pertain.

Our main results still hold true when the number of competing firms $N$ is greater than two. Because of perfect competition between raiding firms, second-period profits from raiding become zero for all $N \geq 3$. As a result, firms break even overall and human capital investment is maximal. In contrast, when job raiding is impossible every firm makes strictly positive profits. All firms that have a positive market share in period one, make strictly positive profits in period two as they enjoy market power over their current employees. Furthermore, from (8), a firm’s demand elasticity with respect to wages in period one is imperfectly elastic. In other words, second-period profits will not be completely bidded away in the first period. As a result, overall profits are positive and therefore, investments in human capital are always below the maximum.

The equilibrium wages as described in Propositions 1 and 3 reveal that wages can decrease over time. Although Baker et al. (1994a,b) do observe some evidence for real wage declines, they most often rise over time. Note, however, that our set-up abstracts from an on-the-job accumulation of general productivity-increasing skills. We could capture this effect by increasing second-period payoffs from $q$ to $q'$. For sufficiently high $q'$ wages always rise over time.
Proposition 1 illustrates that equally productive employees may receive different wages within the same firm depending on their moment of entry into the firm. Firms set higher wages for new hires to compensate for switching costs. Baker et al. (1994a,b) have observed that starting wages may differ significantly from year to year after correcting for composition effects. Moreover, employees earning more on entry maintain their advantage over time. They refer to this as the “cohort effect”. A two-period model cannot explain wage patterns over long periods of time. However, we can extend our analysis to a multi-period model as follows. Suppose switching costs are drawn independently from the same distribution in each period. In this event, our second-period analysis repeats, and cohort effects are absent. In contrast, when switching costs are positively correlated over time, employers infer information about employees’ switching costs from past behavior. That is, employees who changed jobs before are more prone to switch again in the periods following and are offered higher wages for this reason. As such, our model predicts a cohort effect: raided employees receive persistently higher wages than other employees.20

We considered a labor market consisting of employees of two types only. Suppose now that there are several types, possibly a continuum. Bad-type workers always fail, whereas other types succeed with probability $p_G$ yielding a payoff $\tilde{q}$ distributed on the interval $[q, \tilde{q}]$, all satisfying condition (1). Switching costs are distributed identically and independently over all types. If payoffs are common knowledge after period one, firms will offer different contracts to all types. More specifically, discriminating firms offer

$$W^{I}_{2D}(\tilde{q}) \equiv \tilde{q} - \frac{2\bar{x}}{3p_G} \quad \text{and} \quad W^{E}_{2D}(\tilde{q}) \equiv \tilde{q} - \frac{c}{3p_G}$$

to their own workers and their rival’s workers, respectively. As wages increase with productivity in a one-to-one relationship, second-period profits remain as in Proposition 1. Hence, first-period competition will also be no different, as before. Under uniform competition, wages and profits are as in Proposition 2. Thus, the main results still hold when considering a market with a continuum of types. Throughout the analysis, we assumed that output is perfectly observable by both the incumbent and rival employer.
However, it is natural to assume that inside employers have better information about their employees’ output than outside employers. Suppose, therefore, that in contrast to the inside employer, the outside employer cannot distinguish high from low output employees. Of course, the incumbent employer optimally sets lower wages for low output employees. As a result, the raiding firm always hires a higher proportion of these low output employees and consequently offers a low wage. For this reason, the incumbent employer also sets low wages. Therefore, both firms’ wages are lower when output is imperfectly observable. Anticipating this, workers invest less in human capital.

How would results change if workers were better suited to one firm than to the other, as in Jovanovic (1979) or Lazear (1986)? Suppose that all previously successful workers of firm I yield payoff \( q \), whereas the payoff with the competing firm equals \( q + \varepsilon \), with \( \varepsilon \) symmetrically distributed around zero on the interval \([\varepsilon, \bar{\varepsilon}]\), such that condition (1) is always satisfied. We think of \( \varepsilon \) as a finite negative number and \( \bar{\varepsilon} \) as a finite positive number. This implies that some workers are more valuable with the firm they are currently employed with while others with the other employer. Analysis shows that for sufficiently small values of \( \varepsilon \), wages under discrimination equal

\[
W_{2D}^E = q - c \left( \frac{\bar{\varepsilon}}{3p_G} + \frac{2\varepsilon}{3} \right) \quad \text{and} \quad W_{2D}^I = q - c \left( \frac{2\bar{\varepsilon}}{3p_G} + \frac{\varepsilon}{3} \right)
\]

with \( W_{2D}^E \) and \( W_{2D}^I \) the wages offered by the raiding employer and the current employer, respectively. The raiding employer is willing to offer a wage premium for employees who are more valuable to him, while the raider discounts when \( \varepsilon < 0 \). As wages are strategic complements, the current employer acts accordingly, although to a lesser extent. Consequently, more employees switch if they have a better match with the firm they are switching to. Note that employees with \( \varepsilon > 0 \) remain with their current employer if their switching cost is high enough, while some employees with \( \varepsilon < 0 \) do change jobs if their switching cost is rather low. For \( \varepsilon \) sufficiently large and positive, all workers will switch employers. In contrast, when \( \varepsilon \) is sufficiently large and negative, no raiding occurs. As \( \varepsilon \) is symmetrically distributed around zero, average wages equal the wages of Proposition 1. When firms set the same wages for all current employees as for all the new
hires (i.e. uniform wages in a matching context), firms will optimally set the same wages as in Proposition 2 and hence we find that human capital investments are superior when job raids are allowed for.

In the analysis so far, firms can only offer one-period contracts. How do our results change if firms can credibly commit to two-period contracts? This means that, in period one, firms commit to both period-one and period-two wages. If these contracts are not enforceable, both wages coincide with one-period wage offers. Suppose now firms first commit before stage one on the terms of the contracts—short-term or long-term—after which they announce the wages. Long-term contracts can reflect “deferred compensation schemes” in which employees receive low wages in period one and high wages in the following, conditional on success. If one firm commits to long-term contracts, it can offer a deferred compensation scheme such that none of its employees switches to the rival firm. In other words, this prevents the rival firm from raiding. This results in standard Bertrand competition in period one. Hence, zero profits accrue for both firms. Anticipating this, both firms prefer short-term contracts so as to dampen period-one competition. Although long-term contracts restrict labor mobility, wages are actuarially fair if at least one firm offers long-term contracts, leading to maximal human capital investment.

8. Conclusion

This paper explains sign-on bonuses when firms compete for labor. We demonstrate that if labor is sufficiently scarce and labor output easily gauged by outsiders, firms offer premiums to new employees. If employees incur a cost when changing jobs, employers optimally set higher wages for new hires in order to attract them. This results in labor turnover, even in the absence of matching. If firms have the possibility of actively poaching rival’s employees, the labor market becomes more competitive and wages are driven up. For this reason, workers have a higher incentive to perform well. In our model,
workers can invest in human capital prior to labor market entry, thereby increasing their performance. Thus, on the one hand, turnover is socially wasteful in our model, because of the switching cost involved, but on the other hand, human capital levels increase because of this turnover.

This insight has important implications for any industry where human capital is an essential and scarce input and where important aspects of personnel output are observable. We think of workers and their projects as lawyers who close cases successfully for their law firms, athletes performing well for their team, faculty members publishing in top journals, engineers working on projects… We expect intense competition for these scarce workers and, as long as switching or relocation costs are significant, premiums in order to attract new hires. As an example, according to the Employment Outlook 2001 from Chemical & Engineering News, almost all companies are using stock options, sign-on bonuses, and relocation packages to sweeten deals in hiring chemists in the pharmaceutical and the biotech industry.

Our model allows for some predictions. In most western countries, academic institutions will experience a shortage of university professors in the foreseeable future. We expect turnover and competition in this market to increase, driving wages and qualifications up in the long run. Our model also shows that labor mobility in general and wage discrimination in particular stimulates people to invest in human capital. Limiting organizations’ possibility to offer higher wages to new hires vis-à-vis equally productive incumbent employees inhibits ex ante investment in human capital. For example, the Age Discrimination Act of 1975 in the US prohibits discrimination on the basis of age in programs or activities receiving Federal financial assistance. As age is strongly correlated with seniority, this makes it difficult to raid new (often young) employees from another firm and this may have consequences with respect to human capital. Also, in some sectors, especially in high tech areas where, arguably, skilled workers are in short supply, labor mobility is restricted in order to protect intellectual property. “Covenants not to compete” are often in place, effectively hindering raids on firms in the same sector and region (see Motta and Rønde, 2002). Although we do not investigate these covenants in
any detail, we expect these kind of covenants indeed protect firms’ intellectual property, but decrease average worker productivity.

We can extend our model in several directions as described in the discussion. One particularly interesting avenue for further research concerns a setting where employees are charged with several tasks. As an example, academics usually have both research and teaching assignments. The research output is probably more observable to outsiders than the teaching component. In such a setting employees may underinvest in the less observable tasks, thereby conflicting with employers’ preferences. It would be interesting to know how firms can align these incongruent objectives in an imperfectly competitive labor market. As another avenue, firms that compete for the same workers, often also compete in the output market (as in e.g. the earlier mentioned Borland-Microsoft case) and it appears sometimes that job raids are primarily motivated by output competition considerations. Considering this particular two-sided competition would be another particularly appealing extension of our model.
Footnotes

1 We refer to http://healthcare.monster.com/articles/bonus.

2 These models, as ours, do not study job changes with unemployment spells as in Greenwald (1986). We are primarily interested in competition for highly-skilled employees. As Lazear (1986, p. 141) remarks, “highly educated workers rarely suffer unemployment, even though job changes are common.” This supports the idea of a seller’s market for labor.

3 Although non-pecuniary factors play an important role in determining job satisfaction and therefore voluntary job turnover, firms compete in wages only. Topel and Ward (1992) find evidence that turnover decisions are indeed strongly affected by wage differences. They find that, on average, the net wage gain of a change in employer equals 10% of the current wage.

4 This is also the case in Hart (1983) and Greenwald (1986).

5 Long-term contracts are discussed in Section 7.

6 We take the assumption that employees have no initial wealth and cannot borrow money. In addition, employees consume their income at the end of the first working period.

7 The basic results of this paper are left unchanged if workers receive a positive fixed wage and a bonus depending on results. Furthermore, this is also the case when workers receive fixed wages, but are fired after period one if they haven’t performed well (and receive zero wages afterwards). As long as workers are rewarded on completing a project successfully and/or punished otherwise, our main conclusion will hold.

8 If the reservation wage of workers would not equal zero, but some positive $W_0$, the condition looks like $p(q-W_0) > c > (1-p_G)p(q-W_0)$.

9 If $k(1-p_G)p_Gq > c$, previously unsuccessful employees would still be hired, but at a different wage than successful ones. This would not change the main results of our paper.

10 Following Waldman (1990), we prefer the term “human capital” to “effort” for two reasons. The first reason is that the “training” occurs before employment. The second reason is that the effect on expected labor productivity remains for several periods. “Effort” would suggest that workers can work hard or slacken on a particular task and perform differently later on.

11 The line-up of the game is related to Padilla & Pagano (2000). They consider competition in the credit market between two banks for a mass of borrowers. As with our model, these borrowers may take action before competition begins, increasing the chance of successfully completing a business project in the next period. The cost of this ex ante investment is chosen once and for all in anticipation of interest rates (and is thus a sunk cost) and is neither contractible nor observable. Instead of studying the impact of different wage regimes on incentives to invest in human capital, they investigate how different information sharing regimes among banks influence borrowers’ incentives to exert effort.
12 See Klemperer (1995) for an overview of markets with switching costs.

13 Were switching costs known beforehand, analysis would be thoroughly complicated, as every worker would invest differently in human capital according to her switching cost.

14 All workers are equal at period one, because neither their type nor their switching cost are known. For this reason, standard Bertrand competition emerges.

15 In Appendix 3 we derive one other symmetric equilibrium.

16 The reverse occurs if the employer would bear the costs of investing in human capital. In that case, the better the bargaining position of an employer is, the less problematic the underinvestment in human capital will be (see Stevens, 1996).

17 This results from selecting the payoff-dominant equilibrium. For most payoff-dominated equilibria, such as the equilibrium derived in Appendix 4, this result holds true. There are, however, payoff-dominated equilibria for which our result is reversed. Because these equilibria are payoff-dominated, we attach less economic significance to them.

18 Thisse and Vives (1988) obtain a similar result in a product market with spatial competition.

19 If competition between raiding firms is imperfect, e.g. because of firm-specific switching costs, firms may still make positive profits.

20 Of course, employees with intermediate switching costs may behave strategically. They increase earnings from switching because they are then pooled with more mobile employees and henceforth receive higher future wages. Nevertheless, cohort effects remain.
References


**Appendix 1**

The probability of success has to be strictly lower than one in equilibrium. If with actuarially fair wages $q - c/p$ and $q - c/p_G$, optimal human capital build-up remains low enough such that the success chance never exceeds one, this chance will certainly be lower than one in equilibrium with imperfect competition. With actuarially fair wages, the expected utility of an employee would be:

\[ p \left( q - \frac{c}{p} \right) + \delta p p_G \left( q - \frac{c}{p_G} \right) - V(p_G). \]

Utility in period one equals the average chance at success, $p$, times the mark-up when doing well. The same holds for period 2. As failure occurs less in period 2, wages are higher. Further, only with probability $p$ will the second stage ever be reached, since all failures are banned from the market. Second-period utility is also discounted, with $\delta$ the discount factor. Finally, human capital build-up entails a disutility cost $V(p_G)$. Differentiating the above expression with respect to $p_G$ and equating this to zero yields
\[ V'(p_G) = k\left[q(1 + 2\delta p_G) - \delta c\right] \]

As \( p_G \) has to lie below unity, \( V'(1) \geq k\left[q(1 + 2\delta p_G) - \delta c\right] \) is the appropriate condition.

\[ \Box \]

**Appendix 2**

By solving (6.2) and (7.3) we obtain the wages \( W^A_{2U} \) and \( W^B_{2U} \) as in Proposition 2. It can be easily seen that \( q - \frac{c - \delta}{p_G} \geq W^A_{2U} \geq q - \frac{c + \delta}{p_G} \) and \( W^B_{2U} \geq q - \frac{c + \delta(2 + (1 - \sigma)/\sigma)}{p_G} \). We still need to check whether \( \tilde{W}^B_2 \geq W^B_{2U} \) holds. In other words, firm \( A \) should not make more profits with a wage resulting from the best-response as in (7.2), given \( W^B_{2U} \). A comparison of profits with “the high-wage best-response” and the “low-wage best-response” shows this holds for all splitting rules between zero and one half. All other combinations of best-responses described in (6) and (7) yield an inconsistency and cannot therefore constitute an equilibrium.

\[ \Box \]

**Appendix 3**

Without loss of generality, assume \( \sigma \leq 0.5 \). From Proposition 2, second-period profits for both firms are uniquely maximized when \( \sigma = 0.5 \). Therefore, consider the one-period game, i.e. where second-period profits are not taken into account, but \( \sigma \) is still defined by (8). If we can find a Nash-equilibrium for this one-period game, resulting in equal market shares, this would then surely be a (subgame perfect) Nash equilibrium for the full two-period game as well.\(^1\) The profit functions \( \Pi^A_1 \) and \( \Pi^B_1 \) for the one-period game equal

\[ \Pi^A_1(W^A_1, W^B_1) = \sigma \left[p(q - W^A_1) - c\right] \quad \text{and} \quad \Pi^B_1(W^A_1, W^B_1) = (1 - \sigma)\left[p(q - W^B_1) - c\right], \]

respectively. The Nash equilibrium of this one-period game is characterized by

---

\(^1\) This approach follows Chen (1997).
\[ \frac{\partial \Pi_i^A}{\partial W_i^A} = \frac{\partial \sigma}{\partial q} \left[ p(q - W_i^A) - c \right] - \sigma p = 0, \]  
(9)

\[ \frac{\partial \Pi_i^B}{\partial W_i^B} = \frac{\partial (1 - \sigma)}{\partial q} \left[ p(q - W_i^B) - c \right] - (1 - \sigma) p = 0, \]  
(10)

together with (8).

From implicitly differentiating (8), we know that

\[ \frac{\partial \sigma}{\partial W_i^A} = \frac{\partial (1 - \sigma)}{\partial W_i^B} = -\frac{9}{2} \frac{\sigma^2 - 2\sigma + 1}{\sigma^3 - 3\sigma^2 - 1}, \]  
so that equating (9) with (10) yields

\[ W_i^B - W_i^A = \frac{2}{9} \frac{\delta s (2\sigma^3 - 3\sigma^2 - 1)}{\sigma^2 - 2\sigma + 1} (1 - 2\sigma). \]

Equating this last expression with (8) yields us the unique solution \( \sigma = 0.5 \).

By substituting \( \sigma = 0.5 \) in (9) and (10), we find that

\[ W_i^A = W_i^B = q - \frac{c}{p} - \frac{2}{3} \delta s \]

is the unique Nash-equilibrium to the one-period game. As period-two profits are uniquely maximized at \( \sigma = 0.5 \), these wages are also part of the equilibrium for the corresponding two-period game.

### Appendix 4

We denote the payoff-dominant symmetric Nash equilibrium by \( W_i^A = W_i^B \equiv W_i^- \). At these wages, firms must not have an incentive to deviate, while at wages \( W_i^{A^*} = W_i^{B^*} = W_i^- - \epsilon, \ \epsilon > 0 \), they must have an incentive to do so. Profits from setting a higher wage than the competitor equal

\[ \Pi^A(W_i^A, \sigma) = \sigma \left[ p(q - W_i^A) - c \right] + \delta p \sigma \frac{\overline{s}}{9} \left[ 2 + \frac{1 - \sigma}{\sigma} \right]^2. \]
This expression is a function of $\sigma$ and $W_1^A$ only.² Profits in period two are uniquely maximized as in Proposition 2 and only depend on period-one market shares. Note that as firm $A$ sets a (marginally) higher wage in period one, $\sigma \geq 0.5$ holds.

The marginal effect of increasing wages on $A$’s profits is expressed by

$$
\frac{\partial \sigma}{\partial W_1^A} \left[ p(q-W_1^A) - c \right] - p\sigma - \delta p \frac{\partial \sigma}{9 \partial W_1^A} \left[ \left( 2 + \frac{1-\sigma}{\sigma} \right)^2 - 2\sigma \left( 2 + \frac{1-\sigma}{\sigma} \right) \left( \frac{1}{\sigma} + \frac{1-\sigma}{\sigma^2} \right) \right].
$$

(11)

The first term evaluates profits per employee, while the second term describes the effect on market share, both with respect to the first period. The third term depicts the marginal effect of a wage increase on second-period profits.

We only consider symmetric outcomes and check for the wage for which firm $A$ is indifferent between marginally “undercutting” its rival or setting the same wage. This implies that we evaluate the above function around the points $W_1^A = W_1^B$, or equivalently, around $\sigma = 0.5$. Substituting $\sigma = 0.5$ in the above expression yields

$$
\frac{3}{4\delta s} \left[ p(q-W_1^A) - c \right] - \frac{P}{2} - \frac{P}{8}.
$$

Evaluated at symmetric outcomes, the second and third effect are constants and do not depend on the value of $W_1^A = W_1^B$, from which the marginal effect is gauged. Marginal changes in $A$’s wages have the same effect on second-period profits, given that we start from symmetry, because second-period profits only depend on the absolute difference between first-period wages and not on their level. The same applies for first-period market shares. The first term, on the other hand, monotonically decreases with $W_1^A = W_1^B$. Deviating - i.e. offering a slightly higher wage than one’s rival - from a “low” symmetric starting point brings more profits than deviating from a “high” symmetric outcome.

² From (8), $W_2^B$ is implied from $W_2^A$ and $\sigma$. Profits can also be written as a function of $W_2^A$ and $W_2^B$, in which case market shares are implied from (8).
When wages are high enough, each firm makes losses per employee, and marginally increasing one’s wage entails incurring even more losses. If wages are low, though, profits per employee are high, and so a firm gains a lot by increasing its market share by increasing its wages.

From setting (11) to zero, it follows that firm $A$ (and hence also firm $B$) is indifferent between setting wages $W_i^A = W_i^B = q - c/p - \delta\bar{s}$ and offering a marginally higher wage. If this wage constellation constitutes an equilibrium, it necessarily follows that $W_i^- = q - c/p - \delta\bar{s}$. At lower wages both firms will have an incentive to offer higher wages unilaterally. Higher wages can possibly make up equilibria as well, but are payoff-dominated.

It remains to be determined whether $W_i^A = W_i^B = q - c/p - \delta\bar{s}$ is indeed an equilibrium. Can firm $A$ increase profits by unilaterally deviating and offering a lower wage? Overall profits for firm $A$, given that $W_i^B = W_i^-$ and $A$’s wage is lower than or equal to $B$’s, equal

$$\sigma \left[ p \left( q - W_i^A \right) - c \right] + \delta p \frac{\bar{s}}{9} \left( 1 + \frac{2}{1 - \sigma} \right) \left( \sigma + 1 \right)$$

where the second term comes from Proposition 2, for $\sigma \leq 0.5$.

Substituting (8) in the profit function yields

$$\sigma p \delta \left[ 2 + \frac{1}{3} \left( 1 - \frac{\sigma}{1 - \sigma} \right) \right] + \delta p \bar{s} \left( 1 - \frac{2}{3} \right) \left( \frac{1}{1 - \sigma} \right) + \delta \frac{\bar{s}}{9} \left( 1 + \frac{2}{1 - \sigma} \right) \left( \sigma + 1 \right).$$

Profits monotonically increase in $\sigma$ for $\sigma \in [0,0.5]$. This implies that, given $B$’s wage, offering a wage that leads to an equal split of the market generates optimal profits for firm $A$, or equivalently, that $A$ cannot improve upon setting the same wage as $B$. 
Applying the same reasoning, if $W_1^B = W_1^-$ it does not pay for firm $A$ to deviate by setting a higher wage (and thus not just marginally higher wages) as profits monotonically decrease in $\sigma$ for $\sigma \in [0.5, 1]$. ■

**Appendix 5**

This appendix offers the equilibrium wages when firm $A$ discriminates whereas firm $B$ applies a uniform wage policy.

**Lemma.** Given a market share of $\sigma \in [0, 1]$ for firm $A$ and the complementary share for firm $B$, the following wages satisfy a unique Nash equilibrium in pure strategies if firm $A$ discriminates and firm $B$ sets a uniform wage:

\[
W_{2l}^A = q - \frac{c}{p_G} - \frac{(5-\sigma)s}{6p_G}, \quad W_{2k}^A = q - \frac{c}{p_G} - \frac{(2-\sigma)s}{6p_G} \quad \text{and} \quad W_2^B = q - \frac{c}{p_G} - \frac{(2-\sigma)s}{3p_G}
\]

The profits for firm $A$ equal $\frac{3p}{36}(4+17\sigma-5\sigma^2)$ and profits for firm $B$ are $\frac{p}{9}(2-\sigma)^2$. The number of employees switching is given by: $\sigma \frac{1+\sigma}{6} + (1-\sigma) \frac{2-\sigma}{6}$.

As in Section 3.1 the best-response of the wage-discriminating firm $A$ equals

\[
W_{2l}^A = \begin{cases} 
q - \frac{c}{p_G} & \text{if } W_2^B > q - \frac{c-s}{p_G} \\
\left(\frac{p_G W_2 + p_G q - c-s}{2p_G}\right) & \text{if } q - \frac{c-s}{p_G} \leq W_2^B \leq q - \frac{c-s}{p_G} \\
W_2^B & \text{if } W_2^B < q - \frac{c-s}{p_G}
\end{cases}
\]

and
The analysis for the nondiscriminating firm is more complicated, principally because its wage depends on both $W_{2E}^A$ and $W_{2I}^A$. Note that $W_{2I}^A \leq W_{2E}^A \leq \left( \frac{p_G W_{2I}^A + \bar{s}}{p_G} \right)$ always holds (since it follows from the best response analysis of firm $A$) and so we restrict our attention to the analysis when this condition applies. Not doing so would seriously confound the analysis both for the market demand curve and the profit function for firm $B$. The first part of this condition implies that the discriminating firm offers a premium to new employees and/or extracts extra surplus from its captured workers. The second part states that this premium cannot be too large.

Market share of firm $B$ is the following

$$D_2^B \equiv \begin{cases} p & \text{if } W_2^B > \frac{p_G W_{2I}^A - \bar{s}}{p_G} \\ \left[ 1 - \sigma + \sigma \frac{\bar{s}}{\bar{s}} \right] p & \text{if } \frac{p_G W_{2I}^A - \bar{s}}{p_G} \geq W_2^B > W_{2I}^A \\ \left[ \sigma \frac{\bar{s}}{\bar{s}} + (1 - \sigma) \frac{\bar{s} - \bar{s}}{\bar{s}} \right] p & \text{if } W_{2E}^A \geq W_2^B > W_{2I}^A \\ (1 - \sigma) \frac{\bar{s} - \bar{s}}{\bar{s}} p & \text{if } W_{2I}^A > W_2^B \geq \frac{p_G W_{2E}^A + \bar{s}}{p_G} \\ 0 & \text{if } \frac{p_G W_{2E}^A + \bar{s}}{p_G} > W_2^B \end{cases}$$

(9.1) (9.2) (9.3) (9.4) (9.5)

with $\bar{s} \equiv p_G (W_2^B - W_{2I}^A)$ and $\bar{s} \equiv p_G (W_{2E}^A - W_2^B)$.

The first and last part of the demand represent respectively the regions where firm $B$ employs the whole market and nobody. In (9.2), firm $B$ does not lose any employees to the competitor, while attracting some workers of $A$ to itself. In the third part, firm $B$ sets a
wage in between the two wages of firm $A$, so that both firms poach some of each workers. In the fourth part, finally, the wage has become so low that $B$ ceases to induce any employees of $A$ to switch, while still retaining some of its current workers.

Instead of describing a full best-response, we restrict analysis to finding Nash equilibria. Note foremost that the wage of firm $B$ has to be strictly lower than the $W_{2\text{E}}^A$. Suppose $B$ matches $W_{2\text{E}}^A$. The appropriate best response of firm $A$ will then be to set: $W_{2\text{E}}^A = \left( p_G q - c + p_G W_{2\text{E}}^A \right)/2p_G$ and this means $W_{2\text{E}}^B = W_{2\text{E}}^A = q - c/p_G$. At this wage, firm $B$ does not make any profits (although $A$ will). As will be shown later, there are strategies that yield strictly positive profits to $B$, so matching $W_{2\text{E}}^A$ cannot be optimal. From this it follows also that (9.5) cannot be part of an equilibrium. For wages $W_{2\text{E}}^B = W_{2\text{E}}^A + \gamma$ with $\gamma > 0$, profits for $B$ will become negative, which obviously cannot be part of any equilibrium either. In other words, firm $B$ has to allow some employees to switch to the other firm.

Firm $B$ has the choice between setting a wage higher than that offered by firm $A$ in its incumbent market, thereby capturing part of the employees of the opponent (which we call the “high” wage strategy and represented by (9.3)) or forfeiting this and focussing solely on its current work force (the “low” wage strategy and shown by (9.4)). Given $W_{2\text{E}}^A$, the wage in the former case is higher than the wage in the latter. From the sufficient first-order conditions, we know that the optimal best response equals either $\left[ \sigma p_G W_{2\text{E}}^A + (1 - \sigma)p_G W_{2\text{E}}^A + p_G q - c - (1 - \sigma)\bar{x} \right]/2p_G$ (the high wage) or $\left( p_G W_{2\text{E}}^A + p_G q - c - \bar{x} \right)/2p_G$ (the low wage). The former will cease to be optimal when $q - c/p_G - (\sigma \bar{x} + \bar{s})/\sigma p_G < W_{2\text{I}}^A > q - c/p_G - (\bar{s} + \sigma \bar{x})/\sigma p_G$ and $W_{2\text{E}}^A > W_{2\text{E}}^B$ isn’t the case, while the latter requires the following conditions to hold: $(p_G q - c + \bar{x})/p_G \geq W_{2\text{I}}^A \geq (p_G q - c - \bar{x})/p_G$ and $W_{2\text{I}}^A > \left( p_G W_{2\text{E}}^A + p_G q - c - \bar{x} \right)/2p_G$. From the demand function – as depicted in the graph below- we gauge that setting a wage exactly equal to $W_{2\text{I}}^A$ can never be optimal.
It is straightforward to check that the proposed equilibrium wages satisfy the high-wage conditions and the conditions for firm $A$ at the same time. It is equally straightforward (but somewhat tedious) to check that, given the response of firm $A$, $B$ prefers to set wages according to

$$\left[\sigma p_G W_{2i}^A + (1-\sigma) p_G W_{2E}^A + p_G q - c - (1-\sigma)\bar{s}\right]/2p_G$$

and not to

$$\left(p_G R_{2E}^A + p_G q - c - \bar{s}\right)/2p_G$$. There is not any equilibrium where firm $B$ sets a lower wage than firm $A$ for its incumbent workers, as both firm $A$ and $B$ would have incentives to deviate from any such wage constellation. This concludes our proof of the lemma. $\blacksquare$