I. INTRODUCTION

The inclusive cross section for jets produced with high transverse momenta in proton-proton collisions is described by quantum chromodynamics (QCD) in terms of parton-parton scattering. The partonic cross section $\hat{\sigma}_{\text{jet}}$ is convolved with the parton distribution functions (PDFs) of the proton and is computed in perturbative QCD (pQCD) as an expansion in powers of the strong coupling constant, $\alpha_S$. In practice, the complexity of the calculations requires a truncation of the series after a few terms. Next-to-leading-order (NLO) calculations of inclusive jet and dijet production were carried out in the early 1990s \cite{1,2,3}, and more recently, progress towards next-to-next-to-leading-order (NNLO) calculations has been reported \cite{4}. Jet cross sections at the parton level are not well defined unless one uses a jet algorithm that is safe from collinear and infrared divergences, i.e., an algorithm that produces a cluster result that does not change in the presence of soft gluon emissions or collinear splittings of partons. Analyses conducted with LHC data employ the anti-$k_T$ jet algorithm \cite{5}, which is collinear and infrared safe. At the Tevatron, however, only a subset of analyses done with the $k_T$ jet algorithm \cite{6,7,8,9} are collinear and infrared safe. Nonetheless, the inclusive jet measurements with jet size parameters $R$ on the order of unity performed by the CDF \cite{10,11,12} and D0 \cite{13,14,15} Collaborations at 1.8 and 1.96 TeV center-of-mass energies are well described by NLO QCD calculations. Even though calculations at NLO provide at most three partons in the final state for jet clustering, measurements with somewhat smaller anti-$k_T$ jet radii of $R = 0.4$ up to 0.7 by the ATLAS \cite{16,17}, CMS \cite{18,19,20}, and ALICE \cite{21} Collaborations are equally well characterized for 2.76 and 7 TeV center-of-mass energies at the LHC.

The relative normalization of measured cross sections and theoretical predictions for different jet radii $R$ exhibits a dependence on $R$. This effect has been investigated theoretically in Refs. \cite{22,23}, where it was found that, in a collinear approximation, the impact of perturbative radiation and of the nonperturbative effects of hadronization and the underlying event on jet transverse momenta scales for small $R$ roughly with $\ln R$, $-1/R$, and $R^2$ respectively. As a consequence, the choice of the jet radius parameter $R$ determines which aspects of jet formation are emphasized. In order to gain insight into the interplay of these effects, Ref. \cite{22} suggested a study of the relative difference between inclusive jet cross sections that emerge from two different jet definitions:

$$\left( \frac{d\sigma_{\text{alt}}}{dp_T} - \frac{d\sigma_{\text{ref}}}{dp_T} \right) / \left( \frac{d\sigma_{\text{ref}}}{dp_T} \right) = R(\text{alt}, \text{ref}) - 1. \quad (1)$$

Different jet algorithms applied to leading-order (LO) two-parton final states lead to identical results, provided partons in opposite hemispheres are not clustered together. Therefore, the numerator differs from zero only for three or more partons, and the quantity defined in Eq. (1) defines a three-jet observable that is calculable to NLO with terms up to $\alpha_s^4$ with NLOJET++ \cite{24,25} as demonstrated in Ref. \cite{26}.

The analysis presented here focuses on the study of the jet radius ratio, $R(0.5, 0.7)$, as a function of the jet $p_T$ and rapidity $y$, using the anti-$k_T$ jet algorithm with $R = 0.5$ as the alternative and $R = 0.7$ as the reference jet radius. It is expected that QCD radiation reduces this ratio below unity and that the effect vanishes with the increasing collimation of jets at high $p_T$. 

---

* Full author list given at the end of the article.

Published by the American Physical Society under the terms of the Creative Commons Attribution 3.0 License. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.
The LO Monte Carlo (MC) event generators PYTHIA6 [27] and HERWIG++ [28] are used as a basis for comparison, including parton showers (PS) and models for hadronization and the underlying event. As in the previous publication [20], they are also used to derive nonperturbative (NP) correction factors for the fixed-order predictions, which will be denoted LO ⊗ NP and NLO ⊗ NP as appropriate. In addition, jet production as predicted with POWHEG at NLO [29] and matched to the PS of PYTHIA6 is compared to measurements.

A similar study has been performed by the ALICE Collaboration [21], and the ZEUS Collaboration at the HERA collider investigated the jet ratio as defined with two different jet algorithms [30]. Comparisons to predictions involving POWHEG have been presented previously by ATLAS [16].

II. THE CMS DETECTOR

A detailed description of the CMS experiment can be found elsewhere [31]. The CMS coordinate system has the origin at the center of the detector. The $z$ axis points along the direction of the counterclockwise beam, with the transverse plane perpendicular to the beam. Azimuthal angle is denoted $\phi$, polar angle $\theta$ and pseudorapidity is defined as $\eta \equiv -\ln(\tan(\theta/2))$.

The central feature of the CMS apparatus is a superconducting solenoid, of 6 m internal diameter, providing a field of 3.8 T. Within the field volume are a silicon pixel and strip tracker, a crystal electromagnetic calorimeter (ECAL) and a sampling hadron calorimeter (HCAL). The ECAL is made up of lead tungstate crystals, while the HCAL is made up of layers of plates of brass and plastic scintillator. These calorimeters provide coverage up to $|\eta| < 3.0$. An iron and quartz-fiber Cherenkov hadron forward (HF) calorimeter covers $3.0 < |\eta| < 5.0$. The muons are measured in the range $|\eta| < 2.4$, with detection planes made using three technologies: drift tubes, cathode strip chambers, and resistive-plate chambers.

III. JET RECONSTRUCTION

The particle-flow (PF) event reconstruction algorithm is meant to reconstruct and identify each single particle with an optimal combination of all subdetector information [32]. The energy of photons is directly obtained from the ECAL measurement, corrected for zero-suppression effects. The energy of electrons is determined from a combination of the track momentum at the main interaction vertex, the corresponding ECAL cluster energy, and the energy sum of all bremsstrahlung photons attached to the track. Muons are identified with the muon system and their energy is obtained from the corresponding track momentum. The energy of charged hadrons is determined from a combination of the track momentum and the corresponding ECAL and HCAL energy, corrected for zero-suppression effects, and calibrated for the nonlinear response of the calorimeters. Finally the energy of neutral hadrons is obtained from the corresponding calibrated ECAL and HCAL energy.

Jets are reconstructed offline from the PF objects, clustered by the anti-$k_T$ algorithm with jet radius $R = 0.5$ and 0.7 using the FastJet package [33]. The jet momentum is determined as the vectorial sum of all particle momenta in the jet. An offset correction is applied to take into account the extra energy clustered into jets due to additional proton-proton interactions within the same bunch crossing. Jet energy corrections are derived from the simulation separately for $R = 0.5$ and 0.7 jets, and are confirmed by in situ measurements with the energy balance of dijet, $Z +$ jet, and photon + jet events using the missing $E_T$ projection fraction method, which is independent of the jet clustering algorithm [34]. Additional selection criteria are applied to each event to remove spurious jet-like features originating from isolated noise patterns in certain HCAL regions.

The offset correction is particularly important for the jet radius ratio analysis, because it scales with the jet area, which is on average twice as large for $R = 0.7$ jets than for 0.5 jets, while most other jet energy uncertainties cancel out. The offset subtraction is performed with the hybrid jet area method presented in Ref. [34]. In the original jet area method [35] the offset is calculated as a product of the global energy density $\rho$ and the jet area $A_{jet}$, both of which are determined using FastJet. In the hybrid method $\rho$ is corrected for (1) the experimentally determined $\eta$ dependence of the offset energy density using minimum bias data, (2) the underlying event energy density using dijet data, and (3) the difference in offset energy density inside and outside of the jet cone using simulation.

The average number of pileup interactions in 2011 was between 7.4 and 10.3, depending on the trigger conditions (as discussed in Sec. VA). This corresponds to between 5.6 and 7.5 good, reconstructed vertices, amounting to a pileup vertex reconstruction and identification efficiency of about 60%–65%. The global average energy density $\rho$ was between 4.8 and 6.2 GeV/\text{rad}^2, averaging to about 0.5 GeV/\text{rad}^2 per pileup interaction on top of 1.5 GeV/\text{rad}^2 for the underlying event, noise, and out-of-time contributions. The anti-$k_T$ jet areas are well approximated by $\pi R^2$ and are about 0.8 and 1.5 \text{rad}^2 for $R = 0.5$ and 0.7, respectively. This sets the typical offset in the range of 3.8–4.9 GeV (7.2–9.3 GeV) for $R = 0.5$ (0.7). Most of the pileup offset is due to collisions within the same bunch crossing, with lesser contributions from neighboring bunch crossings, i.e. out-of-time pileup.

IV. MONTE CARLO MODELS AND THEORETICAL CALCULATIONS

Three MC generators are used for simulating events and for theoretical predictions:
V. MEASUREMENT OF DIFFERENTIAL INCLUDING JET CROSS SECTIONS

The measurement of the jet radius ratio $\mathcal{R}(0.5,0.7)$ is calculated by forming the ratio of two separate measurements of the differential jet cross sections with the anti-$k_T$ clustering parameters $R = 0.5$ and $0.7$. These measurements are reported in six 0.5-wide bins of absolute rapidity for $|y| < 3.0$ starting from $p_T > 56$ GeV for the lowest single jet trigger threshold. The methods used in this paper closely follow those presented in Ref. [20] for $R = 0.7$, and the results fully agree with the earlier publication within the overlapping phase space. The results for $R = 0.5$ also agree with the earlier CMS publication [18] within statistical and systematic uncertainties. Particular care is taken to ensure that any residual biases in the $R = 0.5$ and 0.7 measurements cancel for the jet radius ratio, whether coming from the jet energy scale, jet resolutions, unfolding, trigger, or the integrated luminosity measurement. The statistical correlations between the two measurements are properly taken into account, and are propagated to the final uncertainty estimates for the jet radius ratio $\mathcal{R}$.

A. Data samples and event selection

Events were collected online with a two-tiered trigger system, consisting of a hardware level-1 and a software high-level trigger (HLT). The jet algorithm run by the trigger uses the energies measured in the ECAL, HCAL, and HF calorimeters. The anti-$k_T$ clustering with radius parameter $R = 0.5$ is used as implemented in the FASTJet package. The data samples used for this measurement were collected with single-jet HLT triggers, where in each event at least one $R = 0.5$ jet, measured from calorimetric energies alone, is required to exceed a minimal $p_T$ as listed in Table I. The triggers with low $p_T$ thresholds have been prescaled to limit the trigger rates, which means that they correspond to a lower integrated luminosity $\mathcal{L}_{\text{int}}$, as shown in Table I.

The $p_T$ thresholds in the later analysis are substantially higher than in the HLT to account for differences between jets measured with only the calorimetric detectors and PF jets. For each trigger threshold the efficiency turn-on as a function of $p_T$ for the larger radius parameter $R = 0.7$ is less sharp than for $R = 0.5$. This is caused by potential splits of one $R = 0.7$ jet into two $R = 0.5$ jets and by additional smearing from pileup for the larger cone size. The selection criteria ensure trigger efficiencies above 97% (98.5%) for $R = 0.7$ at $p_T = 56$ GeV ($p_T > 114$ GeV as in Ref. [20]) and above 99.5% for $R = 0.5$ at $p_T = 56$ GeV. The analysis $p_T$ thresholds, which closely follow those reported in Ref. [20], are reproduced in Table I.

B. Measurement of the cross sections and jet radius ratio

The jet $p_T$ spectrum is obtained by populating each bin with the number of jets from the events collected with the

<table>
<thead>
<tr>
<th>$p_T$ threshold (GeV)</th>
<th>30</th>
<th>60</th>
<th>110</th>
<th>190</th>
<th>240</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum $p_T$ for analysis (GeV)</td>
<td>56</td>
<td>97</td>
<td>174</td>
<td>300</td>
<td>362</td>
<td>507</td>
</tr>
<tr>
<td>$\mathcal{L}_{\text{int}}$ (pb$^{-1}$)</td>
<td>0.0149</td>
<td>0.399</td>
<td>7.12</td>
<td>150</td>
<td>513</td>
<td>4960</td>
</tr>
</tbody>
</table>
associated trigger as described in the previous section. The yields collected with each trigger are then scaled according to the respective integrated luminosity as shown in Table I.

The observed inclusive jet yields are transformed into a double-differential cross section as follows:

$$\frac{d^2\hat{\sigma}}{dp_T dy} = \frac{1}{\epsilon \cdot L_{\text{int}} \Delta p_T \Delta y} N_{\text{jets}},$$

where $N_{\text{jets}}$ is the number of jets in the bin, $L_{\text{int}}$ is the integrated luminosity of the data sample from which the events are taken, $\epsilon$ is the product of the trigger and event selection efficiencies, and $\Delta p_T$ and $\Delta y$ are the transverse momentum and rapidity bin widths, respectively. The widths of the $p_T$ bins are proportional to the $p_T$ resolution and thus increase with $p_T$.

Because of the detector resolution and the steeply falling spectra, the measured cross sections ($\bar{\sigma}$) are smeared with respect to the particle-level cross sections ($\sigma$). Gaussian smearing functions are obtained from the detector simulation and are used to correct for the measured differences in the resolution between data and simulation [34]. These $p_T$-dependent resolutions are folded with the NLO $\otimes$ NP theory predictions, and are then used to calculate the response matrices for jet $p_T$. The unfolding is done with the RooUNFOLD package [54] using the D’Agostini method [55]. The unfolding reduces the measured cross sections at $|y| < 2.5$ ($2.5 \leq |y| < 3.0$) by 5%–20% (15%–30%) for $R = 0.5$ and $R = 0.7$. The large unfolding factor at $2.5 \leq |y| < 3.0$ is a consequence of the steep $p_T$ spectrum combined with the poor $p_T$ resolution in the region outside the tracking coverage. The larger unfolding factor for $R = 0.7$ than for $R = 0.5$ at $p_T < 100$ GeV is caused by the fact that jets with a larger cone size are more affected by smearing from pileup.

The unfolding procedure is cross-checked against two alternative methods. First, the NLO $\otimes$ NP theory is smeared using the smearing function and compared to the measured data. Second, the RooUNFOLD implementation of the singular-value decomposition (SVD) method [56] is used to unshear the data. All three results (D’Agostini method, forward smearing, and SVD method) agree within uncertainties.

The unfolded inclusive jet cross section measurements with $R = 0.5$ and 0.7 are shown in Fig. 1. Figure 2 shows the ratio of data to the NLO $\otimes$ NP theory prediction using the CT10 NLO PDF set [57]. The data agree with theory within uncertainties for both jet radii. For $R = 0.5$ the new measurements benefit from significantly improved jet energy scale (JES) uncertainties compared to the previous one [18] and the much larger data sample used in this analysis increases the number of jets available at high $p_T$. Contrarily, at low $p_T$ the larger single jet trigger prescales reduce the available number of jets. For $R = 0.7$ the data set is identical to Ref. [20], but the measurement is extended to lower $p_T$ and to higher rapidity. The total uncertainties in this analysis are reduced with respect to the previous one as discussed in Section V C1.

The jet radius ratio, $R(0.5, 0.7) = \sigma_5/\sigma_7$, is obtained from the bin-by-bin quotient of the unfolded cross sections, $\sigma_5$ and $\sigma_7$, for $R = 0.5$ and 0.7, respectively. The statistical uncertainty is calculated separately to account for the correlation between the two measurements. The details of the error propagation are discussed in Appendix A.

C. Systematic uncertainties

The main uncertainty sources and their impact is summarised in Table II. The dominant experimental uncertainties come from the subtraction of the pileup offset in the JES correction and the jet $p_T$ resolution. The total systematic uncertainty on $R(0.5, 0.7)$ varies from about

![FIG. 1 (color online). Unfolded inclusive jet cross section with anti-$k_T$ $R = 0.5$ (left) and 0.7 (right) compared to an NLO $\otimes$ NP theory prediction using the CT10 NLO PDF set. The renormalization ($\mu_R$) and factorization ($\mu_F$) scales are defined to be the transverse momentum $p_T$ of the jets.](attachment:image.png)
0.4% at $p_T = 1$ TeV to 2% at $p_T = 60$ GeV for $|y| < 0.5$, and from about 1.5% at $p_T = 600$ GeV to 3.5% at $p_T = 60$ GeV for $2.0 \leq |y| < 2.5$. Outside the tracker coverage at $2.5 \leq |y| < 3.0$, the uncertainty increases to between 3% at $p_T = 300$ GeV and 8% at $p_T = 60$ GeV. The statistical uncertainties vary from a few per mil to a couple of percent except at the highest $p_T$ (around the TeV scale), where they grow to 10%. The theory uncertainties amount typically to 1% to 2%, depending on the region. They are composed of

- the scale dependence of the fixed-order perturbative calculations, of the uncertainties in the PDFs, of the non-perturbative effects, and of the statistical uncertainty in the cross section ratio prediction.

- The luminosity uncertainty, which is relevant for the individual cross section measurements, cancels out in the jet radius ratio, as do most jet energy scale systematic uncertainties except for the pileup corrections. The trigger efficiency, while almost negligible for separate cross
section measurements, becomes relevant for the jet radius ratio when other larger systematic effects cancel out and the correlations reduce the statistical uncertainty in the ratio. Other sources of systematic uncertainty, such as the jet angular resolution, are negligible.

The trigger efficiency uncertainty and the quadratic sum of all almost negligible sources are assumed to be fully uncorrelated versus \( p_T \) and \( y \). The remaining sources are assumed to be fully correlated versus \( p_T \) and \( y \) within three separate rapidity regions, but uncorrelated between these regions: barrel (\(|y| < 1.5\)), endcap (1.5 \( \leq |y| < 2.5\)), and outside the tracking coverage (2.5 \( \leq |y| < 3.0\)).

### 1. Pileup uncertainty

The JES is the dominant source of systematic uncertainty for the inclusive jet cross sections, but because the \( R = 0.5 \) and 0.7 jets are usually reconstructed with very similar \( p_T \), the JES uncertainty nearly cancels out in the ratio. A notable exception is the pileup offset uncertainty, because the correction, and therefore the uncertainty, is twice as large for the \( R = 0.7 \) jets as for the \( R = 0.5 \) jets. The pileup uncertainty is the dominant systematic uncertainty in this analysis over most of the phase space.

The JES pileup uncertainties cover differences in offset observed between data and simulation, differences in the instantaneous luminosity profile between the single jet triggers, and the \( \hat{\sigma} \) stability versus the instantaneous luminosity, which may indicate residual pileup-dependent biases. The earlier CMS analysis [18] also included JES uncertainties based on simulation for the \( p_T \) dependence of the offset and the difference between the reconstructed offset and the true offset at \( p_T \sim 30 \text{ GeV} \). These uncertainties could be removed for the jet radius ratio analysis because of improvements in the simulation.

The leading systematic uncertainty for \(|y| < 2.5\) is the stability of \( \hat{\sigma} \) versus the instantaneous luminosity, while for \(|y| \geq 2.5\) the differences between data and simulation are dominant. The \( \hat{\sigma} \) stability uncertainty contributes 0.4%–2% at \(|y| < 0.5\) and 1%–2% at \(2.0 \leq |y| < 3.0\), with the uncertainty increasing towards lower \( p_T \) and higher rapidity. The data/MC differences contribute 0.5%–1.5% at \(2.0 \leq |y| < 2.5\) and 2%–5% at \(2.5 \leq |y| < 3.0\), and increase towards low \( p_T \). They are small or negligible for lower rapidities. Differences in the instantaneous luminosity profile contribute less than about 0.5% in the barrel at \(|y| < 1.5\), and are about the same size as the data/MC differences in the end caps within tracker coverage at 1.5 \( \leq |y| < 2.5\). Outside the tracker coverage at \(2.5 \leq |y| < 3.0\) they contribute 1.0%–2.5%.

The uncertainty sources are assumed fully correlated between \( R = 0.5 \) and 0.7, and are simultaneously propagated to the \( R = 0.5 \) and 0.7 spectra before taking the jet radius ratio, one source at a time.

### 2. Unfolding uncertainty

The unfolding correction depends on the jet energy resolution (JER) and the \( p_T \) spectrum slope. For the inclusive jet \( p_T \) spectrum, the relative JER uncertainty varies between 5% and 15% (30%) for \(|y| < 2.5 \( 2.5 \leq |y| < 3.0\).

The JER uncertainty is propagated by smearing the NLO \( \otimes \) NP cross section with smaller and larger values of the JER, and comparing the resulting cross sections with the cross sections smeared with the nominal JER. The relative JER uncertainty is treated as fully correlated between \( R = 0.5 \) and 0.7, and thus the uncertainty mostly cancels for the jet radius ratio. Some residual uncertainty remains mainly at \( p_T < 100 \text{ GeV} \), where the magnitude of the JER differs between \( R = 0.5 \) and 0.7, because of additional smearing for the larger cone size from the pileup offset. The unfolding uncertainty at \( p_T = 60 \text{ GeV} \) varies between about 1% for \(|y| < 0.5\), 2% for \(2.0 \leq |y| < 2.5\), and 5%–7% for \(2.5 \leq |y| < 3.0\). It quickly decreases to a sub-dominant uncertainty for \( p_T = 100 \text{ GeV} \) and upwards, and is practically negligible for \( p_T > 200 \text{ GeV} \) in all rapidity bins.

### 3. Trigger efficiency uncertainty

The trigger turn-on curves for \( R = 0.7 \) are less steep than for \( R = 0.5 \), which leads to relative inefficiencies near the trigger \( p_T \) thresholds. The trigger efficiencies are estimated in simulation by applying the trigger \( p_T \) selections to \( R = 0.5 \) jets measured in the calorimeters, and comparing the results of a tag-and-probe method [58] for data and MC. The tag jet is required to have 100% trigger efficiency, while the unbiased PF probe jet is matched to a \( R = 0.5 \) jet measured by the calorimetric detectors to evaluate the trigger efficiency. Differences between data and MC trigger efficiencies are at most 0.5%–1.5% and are taken as a systematic uncertainty, assumed to be fully correlated between bins in \( p_T \) and \( y \).

The maximum values of the trigger uncertainty are found near the steep part of the trigger turn-on curves, which are also the bins with the smallest statistical uncertainty. For the other bins the trigger uncertainty is small or negligible compared to the statistical uncertainty. Adding the trigger and the statistical contributions in quadrature results in a total uncorrelated uncertainty of 0.5%–2.0% for most \( p_T \) bins, except at the highest \( p_T \).

### 4. Theory uncertainties in the NLO pQCD predictions

The scale uncertainty due to the missing orders beyond NLO is estimated with the conventional recipe of varying...
the renormalization and factorization scales in the pQCD calculation for the cross section ratio $R(0.5, 0.7)$. Six variations around the default choice of $\mu_R = \mu_F = \sqrt{p_T}$ for each jet are considered: $(\mu_R/p_T, \mu_F/p_T) = (0.5, 0.5), (2, 2), (1, 0.5), (1, 2), (0.5, 1), (2, 1)$. The maximal deviation of the six points is considered as the total uncertainty.

The PDF uncertainty is evaluated by using the eigenvectors of the CT10 NLO PDF set [57] for both cross sections, with $R = 0.5$ and 0.7. The total PDF uncertainty is propagated to $R(0.5, 0.7)$ by considering it fully correlated between $R = 0.5$ and 0.7. The uncertainty induced by the strong coupling constant is of the order of 1%–2% for individual cross sections and vanishes nearly completely in the ratio.

The uncertainty caused by the modeling of nonperturbative effects is estimated by taking half the difference of the PYTHIA6 and HERWIG++ predictions.

The scale uncertainty of the cross sections exceeds 5% and can grow up to 40% in the forward region, but it cancels in the ratio and can get as small as 1%–2%. It is, nevertheless, the overall dominant theoretical uncertainty for the ratio analysis. Similarly, the PDF uncertainty for the ratio is very small, while the NP uncertainty remains important at low $p_T$, since it is sensitive to the difference in jet area between $R = 0.5$ and 0.7 jets. Finally, the

![Figure 3](color online). Jet radius ratio $R(0.5, 0.7)$ in six rapidity bins up to $|y| = 3.0$, compared to LO and NLO with and without NP corrections (upper panel) and versus NLO $\otimes$ NP and MC predictions (lower panel). The error bars on the data points represent the statistical and uncorrelated systematic uncertainty added in quadrature, and the shaded bands represent correlated systematic uncertainty. The NLO calculation was provided by G. Soyez [26].
VI. RESULTS

The results for the jet radius ratio $R(0.5,0.7)$ are presented for all six bins of rapidity in Fig. 3. Each source of systematic uncertainty is assumed to be fully correlated between the $R = 0.5$ and 0.7 cross section measurements, which is supported by closure tests. Systematic uncertainties from the trigger efficiency and a number of other small sources are considered as uncorrelated and are added in quadrature into a single uncorrelated systematic source. The statistical uncertainty is propagated from the $R = 0.5$ and 0.7 measurements taking into account the correlations induced by jet reconstruction, dijet events, and unfolding. The uncorrelated systematic uncertainty and the diagonal component of the statistical uncertainty are added in quadrature for display purposes to give the total uncorrelated uncertainty, as opposed to the correlated systematic uncertainty.

In the central region, $|y| < 2.5$, which benefits from the tracker coverage, the systematic uncertainties are small and strongly correlated between different $y$ bins. In contrast, the forward region, $2.5 \leq |y| < 3.0$, relies mainly on the calorimeter information and suffers from larger uncertainties. The central and forward regions are uncorrelated in terms of systematic uncertainties.

The jet radius ratio does not exhibit a significant rapidity dependence. The ratio rises toward unity with increasing $p_T$. From the comparison to pQCD in the upper panel of Fig. 3 one concludes that in the inner rapidity region of $|y| < 2.5$, the theory is systematically above the data with little rapidity dependence, while the NLO $\otimes$ NP prediction is closer to the data than the LO $\otimes$ NP one. The pQCD predictions without nonperturbative corrections are in clear disagreement with the data. Nonperturbative effects are significant for $p_T < 1$ TeV, but they are expected to be reliably estimated using the latest tunes of PYTHIA6 and HERWIG++, for which the nonperturbative corrections agree. Because of the much larger uncertainties in the outer rapidity region with $2.5 \leq |y| < 3.0$, no distinction between predictions can be made except for pure LO and NLO, which also here lie systematically above the data.

In the lower panel of Fig. 3 the data are compared to different Monte Carlo predictions. The best overall agreement is provided by POWHEG+PYTHIA6. Comparing the parton showering predictions of PYTHIA6 and HERWIG++ to data exhibits agreement across some regions of phase space, and disagreement in other regions. The PYTHIA6 tune Z2 prediction agrees with data at the low $p_T$ end of the measurement, where nonperturbative effects dominate. This is where PYTHIA6 benefits most from having been tuned to the LHC underlying event data. The HERWIG++ predictions, on the other hand, are in disagreement with the low $p_T$ data, which is expected to be primarily due to the limitations of the underlying event tune 2.3 in HERWIG++. This disagreement between the underlying event in data and HERWIG++ has been directly verified by observing that for the same pileup conditions the energy density $\rho$ [35] is larger by 0.3 GeV/GeV rad$^2$ in HERWIG++ than in data, while PYTHIA6 describes well the energy density in data. At higher $p_T$ the situation is reversed, with HERWIG++ describing the data and PYTHIA6 disagreeing. This fact might be related to the better ability of HERWIG++ to describe the high-$p_T$ jet substructure with respect to PYTHIA6 [59].

VII. SUMMARY

The inclusive jet cross section has been measured for two different jet radii, $R = 0.5$ and 0.7, as a function of the jet rapidity $y$ and transverse momentum $p_T$. Special care has been taken to fully account for correlations when the jet radius ratio $R(0.5,0.7)$ is derived from these measurements. Although the cross sections themselves can be described within the theoretical and experimental uncertainties by predictions of pQCD at NLO (including terms up to $\alpha^3_s$), this is not the case for the ratio $R(0.5,0.7)$. The cancellation of systematic uncertainties in the ratio poses a more stringent test of the theoretical predictions than the individual cross section measurements do. For this three-jet observable $R(0.5,0.7)$, which looks in detail into the pattern of QCD radiation, NLO (including terms up to $\alpha^3_s$), even when complemented with nonperturbative corrections, is in clear disagreement with the data. This is not unexpected, since at most four partons are available at this order to characterize any $R$ dependence.

The MC event generators PYTHIA6 and HERWIG++, which rely on parton showers to describe three-jet observables, are in better accord with the measured jet radius ratio $R(0.5,0.7)$ than the fixed-order predictions. The best description of this ratio is obtained by matching the cross section prediction at NLO with parton showers, as studied here using POWHEG with PYTHIA6 for the showering, underlying event, and hadronization parts. The observations above hold for all regions with $|y| < 2.5$, while for $|y| \geq 2.5$ the experimental uncertainty limits the ability to discriminate between different predictions.

In summary, it has been demonstrated that jet radius $R$ dependent effects, measurable in data, require pQCD predictions with at least one order higher than NLO or a combination of NLO cross sections matched to parton shower models to be sufficiently characterized by theory.

ACKNOWLEDGMENTS

We would like to thank G. Soyez for providing us with the NLO predictions for the jet radius ratio. We congratulate our colleagues in the CERN accelerator departments for the excellent performance of the LHC and thank the
Appendix: Error Propagation

The procedure of extracting from data the jet radius ratio \( R(0.5,0.7) \) and its covariance matrix consists of the following steps: the data are in the form of exclusive jet radius–pair production cross sections \( m_{\rho\rho}^{ij} \), \( m_{\rho\rho}^{ij} \), \( m_{\rho\rho}^{ij} \), for jet radius pairs \( R = 0.5, 0.7 \), \( R = 0.5, 0.5 \), and \( R = 0.7, 0.7 \), respectively, with given number \( q \) and \( p \) of jets in \( p_T \) bins with indices \( i \) and \( j \), respectively. From these the inclusive jet cross sections \( \tilde{\sigma}_5 \) and \( \tilde{\sigma}_7 \) are extracted as functions of \( p_T \), using

\[
\tilde{\sigma}_{5,i} = \sum_{p,q} p \cdot m_{\rho\rho}^{ij} \quad \sum_{p,q} p \cdot m_{\rho\rho}^{ij},
\]

for any \( j \).

\[
\tilde{\sigma}_{7,j} = \sum_{p,q} q \cdot m_{\rho\rho}^{ij} \quad \sum_{p,q} q \cdot m_{\rho\rho}^{ij},
\]

for any \( i \).

As a result of unfolding, \( \tilde{\sigma}_5 \) and \( \tilde{\sigma}_7 \) are converted into particle-level cross sections \( \sigma_5 \) and \( \sigma_7 \), from which the jet radius ratio \( R(0.5,0.7) \) is computed for each \( p_T \) bin.
The error propagation can be summarized in matrix notation:

\[
W_{55,ij} = \sum_{p,q} pq \cdot \text{Var}[m_{5,pq}^{ij}],
\]

\[
W_{77,ij} = \sum_{p,q} pq \cdot \text{Var}[m_{7,pq}^{ij}],
\]

\[
W_{57,ij} = \sum_{p,q} pq \cdot \text{Var}[m_{5,7,pq}^{ij}],
\]

(A2)

\[
B_{5,ij} = \frac{\partial \sigma_{5,i}}{\partial \sigma_{5,j}},
\]

\[
B_{7,ij} = \frac{\partial \sigma_{7,i}}{\partial \sigma_{7,j}}
\]

(evaluated numerically)

(A3)

\[
V_{55} = B_{5}W_{55}B_{5}^T,
\]

\[
V_{77} = B_{7}W_{77}B_{7}^T,
\]

\[
V_{57} = B_{5}W_{57}B_{7}^T,
\]

(A4)

giving

\[
V = \begin{bmatrix} V_{55} & V_{57} \\ (V_{57})^T & V_{77} \end{bmatrix},
\]

(A5)

\[A_{ik} = \begin{cases} \mathcal{R}_{i} \frac{1}{\sigma_{ik}} & \text{if } k = i, \text{ and } i \leq n, \\ -\mathcal{R}_{i} \frac{1}{\sigma_{ik}} & \text{if } k = i + n, \text{ and } i \leq n, \\ 0 & \text{otherwise}, \end{cases}\]

(A6)

\[U = A V A^T.\]

(A7)

The \(W\) matrices in Eq. (A2) give the correlations of the jet cross sections in the various \(p_T\) bins, for \((R = 0.5, 0.5)\), \((R = 0.7, 0.7)\), and \((R = 0.5, 0.7)\) jets; the correlations in the first two arise from dijet events, and the correlations in the last one primarily from the fact that a single jet can appear in both \(R = 0.5\) and 0.7 categories. Most of the jets are reconstructed with both \(R = 0.5\) and 0.7 clustering parameters, and often fall in the same \((p_T,y)\) bin. The measured correlation between \(\sigma_{5,i}\) and \(\sigma_{7,j}\) for bin \(i = j\) in data is about 0.4 at \(p_T = 50\) GeV, rising to 0.65 at \(p_T = 100\) GeV, and finally to 0.85 at \(p_T \geq 1\) TeV. The correlation is almost independent of rapidity for a fixed \(p_T\). At low \(p_T\) there is fairly strong correlation of up to 0.4 between bins \(i = j - 1\) and \(j\), and of up to 0.1 between bins \(i = j - 2\) and \(j\). A small correlation of up to 0.1 between bins \(i = j + 1\) and \(j\) is also observed at high \(p_T\) at \(|y| < 1.0\) because of dijet events contributing to adjacent \(p_T\) bins. This correlation is also present for jets reconstructed with the same radius parameter, and is considered in the error propagation. The correlation between other bins is negligible and only bin pairs coming from the same single-jet trigger are considered correlated.

The \(B\) matrices in Eq. (A3) transform the covariance matrices \(W\) of the measured spectra \(\sigma_{5}\) and \(\sigma_{7}\) to the covariance matrices \(V\) for the unfolded spectra \(\sigma_{5}\) and \(\sigma_{7}\). Equations (A4) and (A7) follow from standard error propagation, as in Eq. (1.55) of Ref. [60]. The partial derivatives \(\partial \sigma_{i}/\partial \sigma_{j}\) in Eq. (A3) are evaluated by numerically differentiating the D’Agostini unfolding, where the \(\sigma_{5,i}\) and \(\sigma_{7,i}\) are the unfolded cross sections, \(\sigma_{5,j}\) and \(\sigma_{7,j}\) are the corresponding smeared cross sections, and \(\mathcal{R}_{i} = \sigma_{5,i}/\sigma_{7,i}\) is the jet radius ratio. The matrices \(V_{55}\) and \(V_{77}\) agree to within 10% of those returned by RooUNFOLD for \(R = 0.5\) and 0.7 \(p_T\) spectra, respectively, but also account for the bin-to-bin correlations induced by dijet events.

![FIG. 4 (color online). (Left) Covariance matrix \(U\) for the jet radius ratio \(\mathcal{R}(0.5, 0.7)\), normalized by the diagonal elements to show the level of correlation. Dashed horizontal and vertical lines indicate the analysis \(p_T\) thresholds corresponding to different triggers. The size of the boxes relative to bin size is proportional to the correlation coefficient in the range from \(-1\) to \(1\). The diagonal elements are \(1\) and thus indicative of the variable bin size. The crossed boxes corresponds to anticorrelation, while the open boxes correspond to positive correlations between two bins. (Right) Comparison of the square root of the covariance matrix diagonals with a random sampling estimate using the delete-\(d\) (\(d = 10\%\)) jackknife method. The differences between the full data set and the ten delete-\(d\) samples are shown by the full circles.](https://www.journals.aps.org/prd/abstract/10.1103/PhysRevD.90.072006)
For the purposes of error propagation, the $\tilde{c}_3$ and $\tilde{c}_7$ data are represented as a single $2n$ vector with $\tilde{c}_3$ at indices 1 to $n$ and $\tilde{c}_7$ at indices $n + 1$ to $2n$. The matrix $V$ in Eq. (A5) therefore has dimensions of $2n \times 2n$ and the matrix $A$ in Eq. (A6) has dimensions $n \times 2n$.

Finally, the covariance matrix $U$ in Eq. (A7) for the jet radius ratio $R(0.5, 0.7)$ is calculated using the error propagation matrix $A$ and the combined covariance matrix $V$ for the unfolded jet cross sections with $R = 0.5$ and 0.7.

The resulting covariance matrix $U$ is shown in Fig. 4 (left) for $|y| < 0.5$. The strong anticorrelation observed between neighboring bins is similar to that observed for individual spectra, and is mainly an artifact of the D’Agostini unfolding. The statistical uncertainty for each bin of $R(0.5, 0.7)$ is illustrated as the square root of the corresponding diagonal element of the covariance matrix in Fig. 4 (right). Given the relative complexity of the error propagation, the statistical uncertainties are validated using a variant of bootstrap methods called the delete-$d$ jackknife [61]. In this method the data are divided into ten samples, each having a nonoverlapping uniformly distributed fraction $d = 10\%$ of the events removed. The ten sets of jet cross sections are used to obtain a covariance matrix, which is scaled by $(1 - d)/d = 9$ to estimate the (co)variance of the original sample. The variances obtained by error propagation agree with the jackknife estimate in all rapidity bins within the expected jackknife uncertainty.

[34] CMS Collaboration, JINST 6, P11002 (2011).
MEASUREMENT OF THE RATIO OF INCLUSIVE JET ...  PHYSICAL REVIEW D 90, 072006 (2014)
MEASUREMENT OF THE RATIO OF INCLUSIVE JET


072006-15

(CMS Collaboration)

1Yerevan Physics Institute, Yerevan, Armenia
2Institut für Hochenergiephysik der OeAW, Wien, Austria
3National Centre for Particle and High Energy Physics, Minsk, Belarus
4Universiteit Antwerpen, Antwerpen, Belgium
5Vrije Universiteit Brussel, Brussel, Belgium
6Université Libre de Bruxelles, Bruxelles, Belgium
7Ghent University, Ghent, Belgium
8Université Catholique de Louvain, Louvain-la-Neuve, Belgium
9Université de Mons, Mons, Belgium
10Centro Brasileiro de Pesquisas Fisicas, Rio de Janeiro, Brazil
11Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil
12Universidade Estadual Paulista, São Paulo, Brazil
13Universidade Federal do ABC, São Paulo, Brazil
14Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria
15University of Sofia, Sofia, Bulgaria
16State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China
17Universidad de Los Andes, Bogota, Colombia
18Technical University of Split, Split, Croatia
19University of Split, Split, Croatia
20Institute Rudjer Boskovic, Zagreb, Croatia
21University of Cyprus, Nicosia, Cyprus
22Charles University, Prague, Czech Republic
23Academy of Scientific Research and Technology of the Arab Republic of Egypt, Egyptian Network of High Energy Physics, Cairo, Egypt
24National Institute of Chemical Physics and Biophysics, Tallinn, Estonia
25Department of Physics, University of Helsinki, Helsinki, Finland
26Helsinki Institute of Physics, Helsinki, Finland
27Lappeenranta University of Technology, Lappeenranta, Finland
28DSM/IRFU, CEA/Saclay, Gif-sur-Yvette, France
29Laboratoire Leprince-Ringuet, Ecole Polytechnique, In2P3-CNRS, Palaiseau, France
30Institut Pluridisciplinaire Hubert Curien, Université de Strasbourg, Université de Haute Alsace Mulhouse, CNRS/IN2P3, Strasbourg, France
31Centre de Calcul de l’Institut National de Physique Nucleaire et de Physique des Particules, CNRS/IN2P3, Villeurbanne, France
32Université de Lyon, Université Claude Bernard Lyon 1, CNRS-IN2P3, Institut de Physique Nucléaire de Lyon, Villeurbanne, France
33Institute of High Energy Physics and Informatization, Thilisi State University, Thilisi, Georgia
34RWTH Aachen University, I. Physikalisches Institut, Aachen, Germany
35RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany
36RWTH Aachen University, III. Physikalisches Institut B, Aachen, Germany
37Deutsches Elektronen-Synchrotron, Hamburg, Germany
38University of Hamburg, Hamburg, Germany
39Institut für Experimentelle Kernphysik, Karlsruhe, Germany
40Institute of Nuclear and Particle Physics (INPP), NCSR Demokritos, Aghia Paraskevi, Greece
41University of Athens, Athens, Greece
42University of Ioannina, Ioannina, Greece

S. CHATRCHYAN et al. PHYSICAL REVIEW D 90, 072006 (2014)

072006-18
University of Canterbury, Christchurch, New Zealand
National Centre for Physics, Quaid-I-Azam University, Islamabad, Pakistan
National Centre for Nuclear Research, Swierk, Poland
Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Warsaw, Poland
Laboratório de Instrumentação e Física Experimental de Partículas, Lisboa, Portugal
Joint Institute for Nuclear Research, Dubna, Russia
Petersburg Nuclear Physics Institute, Gatchina (Saint Petersburg), Russia
Institute for Nuclear Research, Moscow, Russia
Institute for Theoretical and Experimental Physics, Moscow, Russia
P.N. Lebedev Physical Institute, Moscow, Russia
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
State Research Center of Russian Federation, Institute for High Energy Physics, Protvino, Russia
University of Belgrade, Faculty of Physics and Vinca Institute of Nuclear Sciences, Belgrade, Serbia
Centro de Investigaciones Energéticas Medioambientales y Tecnológicas (CIEMAT), Madrid, Spain
Universidad Autónoma de Madrid, Madrid, Spain
Universidad de Oviedo, Oviedo, Spain
Instituto de Física de Cantabria (IFCA), CSIC-Universidad de Cantabria, Santander, Spain
CERN, European Organization for Nuclear Research, Geneva, Switzerland
Paul Scherrer Institut, Villigen, Switzerland
Institute for Particle Physics, ETH Zurich, Zurich, Switzerland
Universität Zürich, Zurich, Switzerland
National Central University, Chung-Li, Taiwan
National Taiwan University (NTU), Taipei, Taiwan
Chulalongkorn University, Bangkok, Thailand
Cukurova University, Adana, Turkey
Middle East Technical University, Physics Department, Ankara, Turkey
Bogazici University, Istanbul, Turkey
Istanbul Technical University, Istanbul, Turkey
National Scientific Center, Kharkov Institute of Physics and Technology, Kharkov, Ukraine
University of Bristol, Bristol, United Kingdom
Rutherford Appleton Laboratory, Didcot, United Kingdom
Imperial College, London, United Kingdom
Brunel University, Uxbridge, United Kingdom
Baylor University, Waco, USA
The University of Alabama, Tuscaloosa, USA
Boston University, Boston, USA
Brown University, Providence, USA
University of California, Davis, Davis, USA
University of California, Los Angeles, USA
University of California, Riverside, Riverside, USA
University of California, San Diego, La Jolla, USA
University of California, Santa Barbara, Santa Barbara, USA
California Institute of Technology, Pasadena, USA
Carnegie Mellon University, Pittsburgh, USA
University of Colorado at Boulder, Boulder, USA
Cornell University, Ithaca, USA
Fairfield University, Fairfield, USA
Fermi National Accelerator Laboratory, Batavia, USA
University of Florida, Gainesville, USA
Florida International University, Miami, USA
Florida State University, Tallahassee, USA
Florida Institute of Technology, Melbourne, USA
University of Illinois at Chicago (UIC), Chicago, USA
The University of Iowa, Iowa City, USA
Johns Hopkins University, Baltimore, USA
The University of Kansas, Lawrence, USA
Kansas State University, Manhattan, USA
Lawrence Livermore National Laboratory, Livermore, USA
University of Maryland, College Park, USA
Massachusetts Institute of Technology, Cambridge, USA
aDeceased.
bAlso at Vienna University of Technology, Vienna, Austria.
cAlso at CERN, European Organization for Nuclear Research, Geneva, Switzerland.
dAlso at Institut Pluridisciplinaire Hubert Curien, Université de Strasbourg, Université de Haute Alsace Mulhouse, CNRS/IN2P3, Strasbourg, France.
eAlso at National Institute of Chemical Physics and Biophysics, Tallinn, Estonia.
fAlso at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia.
gAlso at Universidade Estadual de Campinas, Campinas, Brazil.
hAlso at California Institute of Technology, Pasadena, California, USA.
iAlso at Laboratoire Leprince-Ringuet, Ecole Polytechnique, IN2P3-CNRS, Palaiseau, France.
jAlso at Zewail City of Science and Technology, Zewail, Egypt.
kAlso at Suez University, Suez, Egypt.
lAlso at British University in Egypt, Cairo, Egypt.
mAlso at Cairo University, Cairo, Egypt.
nAlso at Fayoum University, El-Fayoum, Egypt.
oAlso at Ain Shams University, Cairo, Egypt.
pAlso at Université de Haute Alsace, Mulhouse, France.
qAlso at Joint Institute for Nuclear Research, Dubna, Russia.
rAlso at Brandenburg University of Technology, Cottbus, Germany.
sAlso at The University of Kansas, Lawrence, Kansas, USA.
tAlso at Institute of Nuclear Research ATOMKI, Debrecen, Hungary.
uAlso at Eötvös Loránd University, Budapest, Hungary.
vAlso at King Abdulaziz University, Jeddah, Saudi Arabia.
wAlso at University of Visva-Bharati, Santiniketan, India.
xAlso at University of Ruhuna, Matara, Sri Lanka.
yAlso at Isfahan University of Technology, Isfahan, Iran.
zAlso at Sharif University of Technology, Tehran, Iran.
aaAlso at Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, Iran.
abAlso at Laboratori Nazionali di Legnaro dell’INFN, Legnaro, Italy.
acAlso at Università degli Studi di Siena, Siena, Italy.
dAlso at Centre National de la Recherche Scientifique (CNRS)—IN2P3, Paris, France.
eAlso at Purdue University, West Lafayette, Indiana, USA.
fAlso at Universidad Michoacana de San Nicolas de Hidalgo, Morelia, Mexico.
gAlso at Institute for Nuclear Research, Moscow, Russia.
hAlso at St. Petersburg State Polytechnical University, St. Petersburg, Russia.
iAlso at Faculty of Physics, University of Belgrade, Belgrade, Serbia.