Probing the collective excitations of a spinor polariton fluid

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We propose a pump-probe setup to analyze the properties of the collective excitation spectrum of a spinor polariton fluid. By using a linear response approximation scheme, we carry on a complete classification of all excitation spectra, as well as their intrinsic degree of polarization, in terms of two experimentally tunable parameters only: the mean-field polarization angle and a rescaled pump detuning. We evaluate the system response to the external probe, and show that the transmitted light can undergo a spin rotation along the dispersion for spectra that we classify as diffusivelike. We show that in this case, the spin flip predicted along the dispersion is enhanced when the system is close to a parametrically amplified instability.

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I. INTRODUCTION

Strongly coupled matter-light systems, such as exciton-polariton microcavities, have recently witnessed an escalating interest thanks to the simultaneous versatility in manipulating and probing their intrinsic properties. Resulting from the strong coupling of cavity photons and quantum well excitons, exciton polaritons display unique properties deriving from the interest thanks to the simultaneous versatility in manipulating polariton microcavities, have recently witnessed an escalating polariton fluid. By using a linear response approximation scheme, we carry on a complete classification of all excitation spectra, as well as their intrinsic degree of polarization, in terms of two experimentally tunable parameters only: the mean-field polarization angle and a rescaled pump detuning. We evaluate the system response to the external probe, and show that the transmitted light can undergo a spin rotation along the dispersion for spectra that we classify as diffusivelike. We show that in this case, the spin flip predicted along the dispersion is enhanced when the system is close to a parametrically amplified instability.

In a previous recent work [18], the spectrum of elementary excitation for a resonantly excited pump-only polariton fluid including the spin degrees of freedom was analyzed. In that work, by considering for simplicity only the limiting cases of a purely linearly polarized fluid and a purely circularly polarized one, the focus was on the superfluid properties of the system and the possibility of reproducing a linear spectrum when the pump detuning compensates exactly the interaction-induced blueshift.

Here, we propose a pump-probe setup tailored for analyzing the properties of the collective excitation spectrum. We show that in the linear response approximation scheme, valid for a weak probe beam, the spectrum of excitations can be evaluated analytically even in the generic case of an elliptically polarized spinor polariton fluid. Further, for fixed interaction strengths, the spectrum can be completely classified in terms of only two experimentally tunable parameters: the mean-field polarization angle and a rescaled pump detuning. Now the number of different class sets of spectra is much larger compared with the single fluid case. Yet, depending how the two opposite circular polarization degrees of freedom mix together in the spectra, we can single out three larger sets where the behavior of the spectrum intrinsic degree of polarization is qualitatively different. We name them as gapped, 0-diffusive, and ω-diffusive; these regions in the two-parameter space are separated by conditions for which the spectrum can be linear. While for gapped spectra, there is no mixing of opposite circular polarization degrees of freedom, for both diffusivelike spectra, the mixing is responsible for flips of the intrinsic spin degree of polarization along the branches. Further, we evaluate the system response to the external probe and analyze the properties of the transmitted light and its relations to the collective excitation spectrum. In particular, we determine the properties of the spin flip along the branches for diffusivelike spectra and how the spin rotation is larger, the closer the system is to a parametrically amplified instability.

The paper is organized as follows: In Sec. II, we present the generalized Gross-Pitaevskii equation that describes the resonantly pumped spinor fluid and briefly discuss its mean-field solutions from existing literature. We introduce the proposed pump-probe scheme in Sec. III and discuss the linear response approximation scheme. The spectrum of collective
excitations is evaluated in Sec. III A, where we derive a “phase diagram” classifying all possible spectral categories. In Sec. III B we derive the emission properties of the intrinsic degree of polarization for each spectrum branch. Finally, in Sec. III C, we evaluate the spinor polariton fluid response to the additional probe beam and relate its properties to the spectrum intrinsic properties previously discussed.

II. MODEL

The dynamics of resonantly pumped polaritons is described by a Gross-Pitaevskii equation (GPE) for the polariton field generalized to include the effects of the polariton finite lifetime \(2\pi\hbar/\gamma\), as well as those of an external laser that resonantly injects polaritons into the microcavity [4]. Here, we consider a simplified model which involves the lower polariton (LP) branch only. The resonant pumping scheme we will consider imposes populating a specific LP state with low momentum, a simplified model which involves the lower polariton (LP) branch only. The resonant pumping scheme we will consider implies populating a specific LP state with low momentum, allowing us to neglect the occupation of the upper polariton branch. Further, we include the two degrees of freedom of the polariton polarization in the left \(|+\rangle\) and right \(|−\rangle\) circular polarization basis [1,19] (see Fig. 1 for a schematic representation of the Poincaré sphere for light polarization). The generalized GPE equation for the spinor LP field \(\Psi(\mathbf{r},t)\) reads as (\(\hbar = 1\) throughout)

\[
\frac{i}{\hbar} \frac{\partial \Psi}{\partial t} = \left[ \omega_{\text{LP}}(-i\nabla) - \frac{\gamma}{2} + \alpha_1 |\Psi|^2 + \alpha_2 |\Psi| \right] \Psi
+ \mathcal{F}(\mathbf{r},t).
\]

\(\alpha_2\) is weakly attractive and, in particular, we fix \(\alpha_2/\alpha_1 < -0.1\) [21]. Note that, for an equilibrium homogeneous spinor Bose-Einstein condensate at zero temperature (i.e., described by the same GPE with no pumping or decaying term and chemical potentials fixing the particle number in each condensate), attraction between opposite components implies a collapse of the system, i.e., mechanical instability, unless \(|\alpha_2/\alpha_1| < 1\) [22]. In this regime, the results obtained here for the polariton spinor fluid resonantly pumped by an external laser do not qualitatively depend on the particular value chosen for the ratio \(\alpha_2/\alpha_1\). Interestingly, the anisotropy of polariton-polariton interactions, characterized by the ratio \(\alpha_2/\alpha_1\), was shown to be responsible for the existence of effective magnetic monopoles in the form of half-integer topological defects [23] and screening of a magnetic field, the spin Meissner effect [24].

For a homogeneous pump-only scheme as in Eq. (2), the GPE dynamics (1) is solved by the following mean-field plane-wave steady-state solution:

\[
\Psi_{\pm}(\mathbf{r},t) \equiv \psi_{\pm} e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_{\text{LP}} t)},
\]

i.e., by assuming that the pump only populates the LP state with the very same momentum \(\mathbf{k}_p\) and energy \(\omega_{\text{LP}}\). Note that in this resonant pumping scheme, the polariton fluid phase is locked to that of the external laser. One can then find how the intensity of the emission in the two polarization states, \(|\psi_\pm|^2\), vary when changing the system parameters, e.g., by increasing the pump strength and its degree of polarization, via the parameters \(f_{\pm}\). Much work has been recently carried out to investigate the mean-field properties of spinor polariton fluids, including the possibility for multistable behavior (see, e.g., Refs. [14,25]). In particular, for a single-component resonantly pumped polariton fluid, multistability appears when the polariton population goes through a hysteresis loop as a function of the pump intensity: When resonantly pumping above the LP dispersion, an increase in the pump power
implies an interaction-induced blueshift of the LP energy towards resonance and thus a sudden increase of the LP population. When instead the pump power is lowered, a sudden drop occurs at lower pump powers, resulting in a region of multistability. For spinor fluids, it has been found that the most-populated spin component is subject to a hysteresis loop, while the less-populated component undergoes a smooth intensity increase [13,14,25]. Interestingly, this mechanism has been proposed for realizing an optical spin switch [15].

We refer the reader to the literature for the mean-field analysis, and instead assume here that the system is pumped in such a way as to induce a given degree of polarization for the mean-field solution. This is completely characterized, as any general elliptical polarization state, by both a polar angle \( \theta_0 \) and an azimuthal angle \( \phi_0 \) (see Fig. 1), respectively defined as

\[
\cos(2\theta_0) = \frac{|\psi_+|^2 - |\psi_-|^2}{|\psi_+|^2 + |\psi_-|^2}, \\
\tan(2\phi_0) = \frac{\text{Im}(\psi_+^*\psi_-)}{\text{Re}(\psi_+^*\psi_-)}. 
\]

Without loss of generality, we can, however, assume that the pump induces a mean-field state with \( \phi_0 = 0 \), as this simply corresponds to a choice of the reference coordinate system and any other elliptically polarized state can be obtained by rotating the microcavity plane. We then study how the collective excitation spectrum for such spinor polariton fluid, as well as its response to a weak probe beam, change when varying \( \theta_0 \in [0, \pi/4] \)—the other interval \( \theta_0 \in [\pi/4, \pi/2] \) is symmetric for \( \psi_+ \leftrightarrow \psi_- \).

### III. Linear Response to a Weak Probe Beam

In order to probe the spectrum of collective excitations of a resonantly pumped spinor polariton fluid, we introduce an additional weak beam which can be shined at several energies and angles, different from the pump ones. In particular, referring to the schematic setup for the proposed pump-probe experiment in Fig. 2, we consider a homogeneous pump term as in (2) and add to it a homogeneous probe beam with strength \( f_{pb}^b \), shined at a direction \( k_p + k_{pb} \) (with \( k_{pb} \neq 0 \)) and an energy \( \omega_{pb} \) to \( \omega_p \):

\[
\mathcal{F}_p(r,t) = e^{i(k_p \cdot r - \omega_p t)} f_p^b + f_{pb}^b e^{i(k_{pb} \cdot r - \omega_{pb} t)}.
\]

The wave vector \( k_{pb} \) should not be confused with the probe direction; rather, it corresponds to the probe momentum relative to the pump momentum \( k_p \).

We assume that the system is only weakly perturbed by the probe; therefore, we can apply a linear-response approximation, where only two other states are weakly populated aside the mean-field state (3) [26]:

\[
\psi_{\pm}(r,t) = e^{i(k_p \cdot r - \omega_p t)} \left[ \psi_{\pm} + u_{\pm} e^{i(k_{pb} \cdot r - \omega_{pb} t)} + v_{\pm} e^{-i(k_{pb} \cdot r - \omega_{pb} t)} \right].
\]

Note that, although the polariton sample is only excited at two directions, the pump \( k_p \) and the probe one \( k_p + k_{pb} \), transmission must also include an additional signal at \( k_p - k_{pb} \). This is a consequence of polariton interactions which mix the partikelike excitations \( u_{\pm} \), resulting from adding a particle into the mean-field state, with the holelike degrees of freedom \( v_{\pm} \), which are excited by instead removing a particle. Thus, as schematically drawn in Fig. 2, we expect the weak probe to imply a transmission in both directions \( k_p \pm k_{pb} \).

We then study how the collective excitation spectrum for such spinor polariton fluid, as well as its response to a weak probe beam, change when varying \( \theta_0 \in [0, \pi/4] \)—the other interval \( \theta_0 \in [\pi/4, \pi/2] \) is symmetric for \( \psi_+ \leftrightarrow \psi_- \).

The system response to the probe is easily evaluated by substituting (7) into the GPE equation (1) and by expanding at first order in both the probing field strength \( f_{pb}^b \) and the fluctuation terms above mean field, \( u_{\pm} \) and \( v_{\pm} \).

We obtain four coupled equations diagonal in momentum space

\[
(\omega_{pb} \hat{1} - \hat{\mathcal{L}}_{k_p})w = \mathbf{f}^b,
\]

where response and probe have been rearranged into four-component vectors, \( w = (u_+, v_+, u_-, v_-)^T \) and \( \mathbf{f}^b = (f_{pb}^b, 0, f_{pb}^b, 0)^T \). The Bogoliubov operator \( \hat{\mathcal{L}}_\mathbf{k} \) can be written in terms of its polarization components,

\[
\hat{\mathcal{L}}_\mathbf{k} = \begin{pmatrix}
\hat{\mathcal{L}}_{++} & \hat{\mathcal{L}}_{+-}
\hat{\mathcal{L}}_{-+} & \hat{\mathcal{L}}_{--}
\end{pmatrix},
\]

which are given by the expressions (\( i = +, - \))

\[
\hat{\mathcal{L}}_{ii, \mathbf{k}} = \begin{pmatrix}
\tilde{\epsilon}_{i, \mathbf{k}} + \mathbf{k} \cdot \mathbf{v}_p - i \frac{\chi}{2} & \alpha_i \psi_i^2 \\
-\alpha_i \psi_i^2 & -\tilde{\epsilon}_{i, \mathbf{k}} - \mathbf{k} \cdot \mathbf{v}_p - i \frac{\chi}{2}
\end{pmatrix},
\]

\[
\hat{\mathcal{M}}_{++} = \alpha_2 \begin{pmatrix}
\psi_+^* \psi_+ & \psi_+^* \psi_-
-\psi_-^* \psi_+ & -\psi_-^* \psi_-
\end{pmatrix}.
\]

FIG. 2. (Color online) Schematic setup for the pump-probe experiments designed to measure the collective excitation spectrum of a spinor polariton fluid. The cavity [composed of two distributed Bragg mirrors (DBR) with embedded quantum wells (QWs)] is resonantly excited with a pump, with a momentum \( k_p \) and an energy \( \omega_p \) close to the LP dispersion. A second weak probe beam, with momentum \( k_p + k_{pb} \), and energy \( \omega_p + \omega_{pb} \) that can be scanned at different values, is used to probe the system collective excitation spectrum. The transmitted light will have a “partikelike” component emitting at the direction corresponding to \( k_p + k_{pb} \) and a “holelike” component at \( k_p - k_{pb} \).
The parameters appearing in the diagonal components of the Bogoliubov operator are the fluid velocity $\mathbf{v}_p = k_p/m$ and the following energy term:

$$
\hat{\varepsilon}_{\pm, k} = \varepsilon_{\pm, k} + \alpha_1 |\psi_{\pm}|^2, \quad \varepsilon_{\pm, k} = \frac{k^2}{2m} - \Delta_{\pm},
$$

(12)

(13)

$$
\Delta_{\pm} = \omega_p - \left(\frac{k_p^2}{2m} + \alpha_1 |\psi_{\pm}|^2 + \alpha_2 |\psi_{\mp}|^2\right).
$$

(14)

In particular, $\Delta_{\pm}$ can be interpreted as the effective pump detuning, i.e., the energy difference between the laser frequency $\omega_p$ and the LP dispersion at momentum $k_p$ renormalized by the interaction-induced blueshift due to both the intrapolarization coupling $\alpha_1 |\psi_{\pm}|^2$ and the interpolarization coupling $\alpha_2 |\psi_{\mp}|^2$.

Before analyzing the properties of the probe response $\mathbf{w}$ starting from Eq. (8) (Sec. III C), we discuss first the collective excitation spectrum of the spinor polariton fluid and its intrinsic properties (Sec. III A), including its degree of polarization (Sec. III B).

A. Excitation spectrum of the spinor polariton fluid

For a general wave vector $\mathbf{k}$ (as for $k_p$s, here $\mathbf{k}$ is assumed to be measured with respect to the pump wave vector $\mathbf{k}_p$), the different branches of the spectrum of excitations are the eigenvalues of the Bogoliubov operator $\hat{\mathbf{L}}_k$ and thus are evaluated starting from the equation

$$
\det(\hat{\mathbf{L}}_k - \omega \hat{\mathbf{1}}) = 0,
$$

(15)

or equivalently finding the roots of the following polynomial equation:

$$
\prod_{i=\pm,-} \left[ \left( \omega + i \frac{\gamma}{2} - \mathbf{k} \cdot \mathbf{v}_p \right)^2 - E_{\pm, k}^2 \right] = -4\alpha_2^2 \prod_{i=\pm,-} (\varepsilon_{i,k}|\psi_i|^2).
$$

This can be solved exactly, resulting in four branches of the spectrum, which, as explained later, we label with a new index $\alpha = (\uparrow, \downarrow, \uparrow_t, \downarrow)$:

$$
\omega_k^{(\alpha)} = \mathbf{k} \cdot \mathbf{v}_p - i \frac{\gamma}{2} + \eta_{\alpha, v} \left[ \frac{E_{\pm, k}^2 + E_{\mp, k}^2}{2} \right] + \sigma_{\alpha, \pm} \sqrt{\left( \frac{E_{\pm, k}^2 - E_{\mp, k}^2}{2} \right)^2 + 4\alpha_2^2 \prod_{i=\pm,-} (\varepsilon_{i,k}|\psi_i|^2)}^{1/2},
$$

(16)

where $\eta_{\alpha, v} = \pm 1$ for the particlelike and holelike components, respectively, and $\sigma_{\alpha, \pm} = \pm 1$. Here, the energy

$$
E_{\pm, k} = \sqrt{\varepsilon_{\pm, k} (\varepsilon_{\pm, k} + 2\alpha_1 |\psi_{\pm}|^2)}
$$

(17)

determines the excitation spectrum of two independent fluids with opposite circular polarizations, which is given by [5,27]

$$
\lim_{\alpha_2 \to 0} \omega_k^{(\alpha)} = \mathbf{k} \cdot \mathbf{v}_p - i \frac{\gamma}{2} + \eta_{\alpha, v} E_{\pm, k}.
$$

(18)

When setting $\psi_- = 0$ (circular polarization) or $\psi_- = \psi_-$ (linear polarization), one recovers the limiting expressions derived in Ref. [18].

Note that, because of interactions, polarization and particle-hole degrees of freedom do, in general, mix together along the dispersion of each spectrum branch. Yet, the choice of the index $\alpha = (\uparrow_t, \downarrow, \uparrow, \downarrow)$ for labeling the four branches of the excitation spectrum is motivated by the fact that, at large momenta there is no mixing between the particlelike ($\uparrow_t, \downarrow$) and holelike ($\downarrow, \uparrow$) degrees of freedom, while the same does not hold of $+$ and $-$ polarization states that do remain coupled (i.e., as we will see later, the intrinsic polarization of these branches can never be purely circularly $+$ or $-$ polarized even at large momenta, where the energy becomes

$$
\lim_{E \to \pm \infty} \omega_k^{(\alpha=1)} = \mathbf{k} \cdot \mathbf{v}_p - i \frac{\gamma}{2} + \eta_{\alpha, v} \frac{(k + \eta_{\alpha, v} k_p)^2}{2m} \frac{2\alpha_1 |\psi_\uparrow|^2 + \alpha_2 |\psi_\downarrow|^2}{2m}.
$$

(19)

where $\psi_\uparrow, \downarrow \equiv \psi_{\alpha, \pm}$. For this reason, we introduce a new notation $\psi_{\alpha, \pm}$ for the branch index $\alpha = (\uparrow_t, \downarrow, \uparrow, \downarrow)$, indicating that the pure circular polarization degrees of freedom $\pm$ are always coupled.

We now classify all possible different types of excitation spectra (see Fig. 4) and how these evolve from one type to the other by changing the system parameters, as represented in the phase diagram of Fig. 3. Interestingly, for a fixed interaction strength ratio $\alpha_2/\alpha_1 = -0.1$ in the figures), only two dimensionless independent parameters are sufficient in order to classify all possible different types of spectra of a resonantly pumped spinor polariton fluid: 1) the mean-field polarization angle $\theta_0$ (4) and 2) the dimensionless pump energy rescaled by the “self-interaction” energy $\tilde{E} = \alpha_1 |\psi_\uparrow|^2 + |\psi_\downarrow|^2$:

$$
\tilde{\alpha}_p = \frac{\alpha_2}{\alpha_1 (|\psi_\uparrow|^2 + |\psi_\downarrow|^2)}.
$$

(20)

By means of these two parameters only we can fully classify all the allowed spectrum typologies. Note in fact that the value of the pump momentum $k_p$ has the sole effect of tilting the spectrum dispersion; in the quadratic approximation for the LP dispersion considered here, this corresponds to a Galilean transformation [28].

For this reason, we plot, without loss of generality, the spectra of Fig. 4 for a pump in the orthogonal direction to the cavity growth, $k_p = 0$.

In the absence of interpolarization interaction, $\alpha_2 = 0$, and for equal spin populations, $|\psi_\uparrow| = |\psi_\downarrow|$, both sign and value of a single parameter, the rescaled interaction renormalized pump detuning $\Delta = \Delta/|\psi_\uparrow|^2$ determine the four types of possible spectra [5,27]: 1) for $\Delta < 0$ the spectrum is gapped; 2) the gap closes to zero for $\Delta = 0$ and the dispersion is linear at low momenta; while for $\Delta > 0$, particle and hole branches of the spectrum real part touch together in either 3) one ($\Delta \leq 2$) or 4) two ($\Delta > 2$) separate momentum intervals—note that Fig. 4 is a cut at $k_\gamma = 0$, so intervals for those plots in reality correspond to rings in the two-dimensional $k$ space. Both cases 3) and 4) are generally named as diffusive spectra. Note that the linear spectrum is allowed for a single value of the detuning $\Delta$, and thus even if the types of different spectra for $\alpha_2 = 0$ are four in total, the finite interval regions in $\Delta$ displaying different
spectra are only three (and $\Delta = 0$ represents a separating point between two of these regions). For the spinor case, the minimal set of independent dimensionless parameters characterizing the spectrum is instead formed by $\theta_0$ and $\omega_p$. Note that by rescaling the pump detuning $\Delta_p$ (14) by the self-interaction energy $\tilde{E}$ would still lead to a parameter depending on $\theta_0$.

For a coupled spinor fluid, with $\alpha_2 \neq 0$, the classes of different spectra increase from 4 to 18; by counting the parameter finite regions only (and excluding the separating lines), this corresponds to nine regions of different spectra compared to the three of the previous $\alpha_2 \to 0$ limit case. The proliferation of different types of spectra is due to the presence of two nested square roots in Eq. (16). Which of the four branches have a degenerate real part as well as in how many momentum intervals degeneracy occurs, depends on the sign of both square root arguments. All nine possibilities for the spectra are plotted in Fig. 4 and the various phase diagram

FIG. 3. Phase diagram showing the different classes of spectra of a spinor polariton fluid as a function of the two dimensionless parameters $\bar{\omega}_p$ (20) and the mean-field polarization angle $\theta_0$ (4) for $\alpha_2/\alpha_1 = -0.1$—the case of left circular polarization corresponds to $\theta_0 = 0$, while that of linear polarization corresponds to $\theta_0 = \pi/4$. The labels 1–9 correspond to the exact parameter values chosen for the corresponding spectra plotted in Fig. 4. The value of the critical polarization angle $\theta_0^c$ (21) is marked with a dash-dotted line.

The white region (gapped) includes the spectra 1–3 characterized by a gap for the + branches; the clear-gray region (0-diffusive) are the spectra 4–6 displaying diffusive behavior at zero energy, while the dark-gray region ($\omega$-diffusive) are the spectra 7–9 where a diffusive region can also be at finite energy (see text). The striped region is the parametrically unstable region for $\gamma = 1.5\tilde{E}$.

FIG. 4. (Color online) Different types of excitation spectra (real part $\text{Re} \alpha_k^\omega$ in the left panels and imaginary part $\text{Im} \alpha_k^\omega$ in the right panels) allowed for a spinor polariton fluid for $\alpha_2/\alpha_1 = -0.1$ and for a pump wave vector $\mathbf{k}_p = 0$. The energy $\omega$ is measured in units of $\tilde{E} = \alpha_1 (|\psi_+|^2 + |\psi_-|^2)$, while momentum (plots are cuts at $k_y = 0$) is in units of $\sqrt{\tilde{m}}$. Thick (thin) black lines are the $u_1$ ($v_1$) branches, while thick (thin) red-gray lines are the $u_2$ ($v_2$) branches. The labels 1–9 correspond to the very same parameters ($\bar{\omega}_p, \theta_0$) shown in the phase diagram of Fig. 3, where the corresponding labels appear. The polariton decay rate is fixed to $\gamma = 1.5\tilde{E}$ for the spectra 1–8 and to $\gamma = 2\tilde{E}$ for spectrum 9.
regions in the \((\vartheta_p, \theta_0)\) parameter space where such spectra are allowed are plotted in Fig. 3.

For both negative as well as small positive values of the renormalized dimensionless pump energy detuning, \(\vartheta_p\), the spectrum is fully gapped, i.e., none of the four branches mix together [panel 1 in Fig. 4]. By increasing the value of \(\vartheta_p\) at fixed \(\theta_0\), the \(\uparrow\) branches (black lines) are still gapped, while the real part of the particliform \(v_+\) (thick red-gray) and holelike \(v_-\) (thin red-gray) branches undergo the same changes as previously described for a single-component fluid: They touch each other first in a single momentum interval (panel 2) and then in two separate momentum intervals (panel 3).

Even if for panels 2 and 3 the \(\downarrow\) branches have a diffusivelike character, with an imaginary part deviating from the polariton lifetime, \(\text{Im} \omega \neq -\gamma / 2\), in all three regimes 1–3 described, the \(\uparrow\) and \(\downarrow\) real part branches are never degenerate and hence do not mix one with the other, so that each maintains its own character: We group the three cases as gapped spectra (white region of Fig. 3).

When we further increase the value of \(\vartheta_p\), the opposite polarization branches can, however, mix together and the spectrum evolves differently depending on the value of the mean-field polar angle \(\theta_0\). In particular, it either changes from the type 3 to 7 if \(\theta_0 < \theta_0^c\), or from 2 to 4 if \(\theta_0 > \theta_0^c\), where the critical angle \(\theta_0^c\) is given by

\[
\cos (2\theta_0^c) = \frac{1}{2} \sqrt{1 + \frac{\alpha_2}{\alpha_1}}. \tag{21}
\]

For \(\alpha_2/\alpha_1 = -0.1\), this critical value of the polarization angle is given by \(\theta_0^c = 0.54\). The dash-dotted line in Fig. 3 indicates where \(\theta_0^c\) determines the boundary between these two cases. The difference between the two is how the \(\uparrow\) and \(\downarrow\) branches mix together and at which energy the mixing happens. For \(\theta_0 > \theta_0^c\), there is only mixing of the branches at zero energy, as for the three spectra 4–6, which we group under the naming 0-diffusive and \(\uparrow\downarrow\) real parts are degenerate at \(\theta_0 = 0\), where the upper indices \((\uparrow)\) refer to the collective spectrum, while the lower indices \((\downarrow)\) refer to the eigenvalues. For each branch labeled by \(\eta\) and for a given direction \(\mathbf{k}\) (measured with respect to the pump wave vector \(\mathbf{k}_p\)), we might expect that, out of a four-component complex vector \(\mathbf{x}_k^{(\eta)}\), the degree of polarization will be characterized by two polar angles \(\omega_p, \theta\). In particular, it either changes from \(\uparrow\downarrow\) to \(\uparrow\uparrow\downarrow\) or from \(\uparrow\downarrow\) to \(\uparrow\uparrow\downarrow\). Thanks to this degeneracy at zero energy is, however, still possible for the \(\downarrow\) branch, as seen in panels 8 and 9.

The spectral phases of a circularly (linearly) polarized fluid are the ones found on the line \(\theta_0 = 0\) (\(\theta_0 = \pi / 4\)) and coincide with the results discussed in Ref. [18]. After having classified completely all possible excitation spectra, we now discuss in the next section their intrinsic polarization properties.

### B. Degree of polarization of collective excitations

Each mode of the collective spectrum does emit with an intrinsic degree of polarization along the dispersion. This can be determined by starting from the eigenvalue equations,

\[
\lambda_k \mathbf{x}_k^{(\eta)} = \omega_p^{(\eta)} \mathbf{x}_k^{(\eta)}, \tag{22}
\]

where \(\omega_p^{(\eta)}\) are the four branches (16) labeled by the index \(\eta = \uparrow, \downarrow, v_+, v_-\) and \(\mathbf{x}_k^{(\eta)}\) are the four-component eigenvectors of the Bogoliubov matrix \(\Lambda_k\).

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\]

where \(\omega_p^{(\eta)}\) are the four branches (16) labeled by the index \(\eta = \uparrow, \downarrow, v_+, v_-\) and \(\mathbf{x}_k^{(\eta)}\) are the four-component eigenvectors of the Bogoliubov matrix \(\Lambda_k\).

For each branch labeled by \(\eta\) and for a given direction \(\mathbf{k}\) (measured with respect to the pump wave vector \(\mathbf{k}_p\)), we might expect that, out of a four-component complex vector \(\mathbf{x}_k^{(\eta)}\), the degree of polarization will be characterized by two polar angles \(\omega_p, \theta\). In particular, it either changes from \(\uparrow\downarrow\) to \(\uparrow\uparrow\downarrow\) or from \(\uparrow\downarrow\) to \(\uparrow\uparrow\downarrow\). Thanks to this degeneracy at zero energy is, however, still possible for the \(\downarrow\) branch, as seen in panels 8 and 9.

The spectral phases of a circularly (linearly) polarized fluid are the ones found on the line \(\theta_0 = 0\) (\(\theta_0 = \pi / 4\)) and coincide with the results discussed in Ref. [18]. After having classified completely all possible excitation spectra, we now discuss in the next section their intrinsic polarization properties.
Mixing of the $\uparrow$ and $\downarrow$ branches causes instead sudden changes of both normal and antinormal angles along each dispersion. This is the case for both the $0$-diffusive spectrum shown in panel (b) of Fig. 5 (corresponding to panel 5 of Fig. 4) as well as the $\omega$-diffusive spectrum shown in panel (c) (and corresponding to the spectrum 7 of Fig. 4), even if the mixing happens in different ways. In particular, for the $0$-diffusive spectrum, different branches touch each other in two separated regions in $k$ space. The branches $u_1$ and $v_1$ mix together for $k \lesssim 1.5 \sqrt{\Delta E}$, where the angles $\theta_{1,k}$ and $\eta_{1,k}$ coincide. Inside this region, there is another region where also the $u_1$ and $v_1$ mix together. Here, also the angles $\theta_{1,k}$ and $\eta_{1,k}$ will coincide. In between these mixing regions, the values of the angles undergo a sudden change in value from almost a purely left-circularly polarized degree to a right-circularly polarized degree.

The last case we analyze is the $\omega$-diffusive spectrum shown in panel (c) of Fig. 5. The difference with the case previously considered of a $0$-diffusive spectrum, lies in the fact that now all four real part branches do not become degenerate in the same momentum region and that the degeneracy of $\uparrow$ and $\downarrow$ branches is allowed at finite energy. Aside from these differences in how $\uparrow$ and $\downarrow$ branches do mix with each other, we observe in panel (f) a similar sudden flip of both normal and antinormal polar angles along the dispersion that we also observed for the $0$-diffusive spectrum in panel (e).

### C. Probe response

After having discussed the intrinsic properties of the collective spectrum, including the emission degree of polarization along the dispersion of each branch, we derive now the response of the spinor fluid to an additional probe beam and how this is related to the intrinsic spectral properties discussed so far. To this end, we go back to the system linear response (8) to a weak external probe shined at a direction $k_p + \hat{k}_p b$ and an energy $\omega_p + \omega_{pb}$. Equation (8) can be easily inverted to give the system response $\mathbf{w} = (u_+, v_+, u_-, v_-)^T$ in terms of the probe vector $\mathbf{f}_{pb} = (f^+_{pb}, 0, f^0_{pb}, 0)^T$:

$$\mathbf{w} = (\omega_p \hat{k} - \hat{\Delta}_{pb})^{-1} \mathbf{f}_{pb}. \tag{26}$$

As explained previously, and also illustrated in the schematic setup of Fig. 2, the intensity of the response in the direction $k_p + \hat{k}_p b$ will be given by the particielike component $|u_+|^2$, while the one at the direction $k_p - \hat{k}_p b$ will be given by the holelike component $|v_+|^2$—from now onwards, we will consider the particular case of a pump shined orthogonally to the cavity plane, $k_p = 0$. From Eq. (26), we expect that the response will be enhanced when $(\omega_p, k_{pb})$ is scanned close to one of the branches of the collective excitation spectrum, i.e., the eigenvalues of the Bogoliubov matrix $\hat{\Delta}_{pb}$. Because the spectrum is complex, we expect a broadened enhanced emission—typically of the order of the polariton linewidth $\gamma$, with variations in the diffusive regions in momentum space where different branches touch each other and the imaginary part of the spectrum deviates from the polariton lifetime $\gamma$. While the resolution in energy is limited by the imaginary part of the spectrum, the resolution in momentum space can only suffer experimental limitations, such as the angular resolution.
of the detection device and the intrinsic angular width of the probe beam. The probe beam resolution in momentum can be improved by considering a large enough homogeneous profile [29].

We express the probe vector \( f^{pb} \) in terms of the polar \( \theta_{pb} \) and azimuthal \( \phi_{pb} \) angles, quantifying the polarization degree of the probe:

\[
f^{pb} = |f^{pb}|(\cos 2\theta_{pb}, 0, e^{i\phi_{pb}} \sin 2\theta_{pb}, 0)^T,
\]

where \( |f^{pb}|^2 \) is the probe beam intensity. We set \( \phi_{pb} = 0 \).

Similarly, the response \( w \) can be conveniently parametrized in terms of the particlelike \( I_u = |u_+|^2 + |u_-|^2 \) and holelike \( I_v = |v_+|^2 + |v_-|^2 \) intensities, as well as the polar angles \( \theta_{u,v} \) along the two directions \( \pm k_{pb} \):

\[
I_u \cos(2\theta_u) = |u_+|^2 - |u_-|^2, \\
I_v \cos(2\theta_v) = |v_+|^2 - |v_-|^2.
\]

Finally, as already discussed, we assume that the pump induces a mean-field state with \( \phi_0 = 0 \) and \( \theta_0 \in [0, \pi/4] \) (4).

We plot in Figs. 6 and 7 the two-dimensional maps for both intensities \( I_{u,v} \) and polar angles \( \theta_{u,v} \) for the response to a probe beam by scanning different values of the probe energy \( \omega_{pb} \) and momentum \( k_{pb} \). We choose the particular case of a gapped spectrum (Fig. 6), corresponding to the same conditions as panels (a) and (d) of Fig. 5 and the case of a 0-diffusive spectrum (Fig. 7), corresponding to the same conditions as panels (b) and (e) of Fig. 5. We first note that, as expected, the probe beam (27) has only a finite strength in the particlelike channels and cannot directly excite holelike quasiparticles. Nevertheless, the response has a finite emission intensity also for the holelike branches because of the interactions mixing together particle and hole degrees of freedom. As previously illustrated in Eq. (19), particle and hole modes asymptotically decouple one from the other at large momenta. We thus expect that the response emission intensity quickly drops to zero for all holelike branches. In panel (b) of Fig. 6 this behavior is clearly visible, as the emission intensity of the hole branches is strongly reduced with respect to that of the particle branches.

In the previous section we have seen that each branch of the collective excitation spectrum is characterized by an intrinsic degrees of polarization, quantified by the normal (\( \theta_{\uparrow,\downarrow} \)) and antinormal (\( \eta_{\downarrow,\uparrow} \)) polar angles. In Fig. 6, the gapped spectrum is probed with a linearly polarized beam, \( \theta_{pb} = \pi/4 \). Here, we obtained an enhanced emission when the probe is in resonance with either the \( u_\uparrow \) or the \( u_\downarrow \) branch. We remind one that for a gapped spectrum there is no mixing between the \( \uparrow \) and \( \downarrow \) degrees of freedom, and thus the \( \theta_{\downarrow,\uparrow} \) angles only weakly deviate from their asymptotic values at large \( k \) (see Fig. 5), in this particular case almost purely circularly left or right polarized. Hence, one expects that the emission intensity for the \( u_\uparrow \) and \( u_\downarrow \) branches is comparable, since

FIG. 6. (Color online) Response to a probe beam for a gapped spectrum of excitations. Two-dimensional maps of the particlelike \( I_u \) [panel (a)] and holelike \( I_v \) [panel (b)] intensities, and of the polar angles along the \( k_{pb} \) direction \( \theta_u \) [panel (c)] and the \( -k_{pb} \) direction \( \theta_v \) [panel (d)] as a function of both the probe momentum \( k_{pb} \) and energy \( \omega_{pb} \). The system parameters are the same ones fixed in panels (a) and (d) of Fig. 5, the probe is linearly polarized (\( \theta_{pb} = \pi/4 \)) and the polariton decay rate is set to \( \gamma = 1.5E \). White dashed lines are the real part of the excitation spectrum \( \text{Re} \omega^{(a)} \). Note that the intensity of the holelike signal has been multiplied by a factor 20 with respect to the particlelike to obtain a clearer contrast.

FIG. 7. (Color online) Response to a probe beam for a 0-diffusive spectrum of excitations. Two-dimensional maps of the particlelike \( I_u \) [panel (a)] and holelike \( I_v \) [panel (b)] intensities, and of the polar angles along the \( k_{pb} \) direction \( \theta_u \) [panel (c)] and the \( -k_{pb} \) direction \( \theta_v \) [panel (d)] as a function of both the probe momentum \( k_{pb} \) and energy \( \omega_{pb} \). The system parameters are the same ones fixed in panels (b) and (e) of Fig. 5, the probe purely circularly right-polarized (\( \theta_{pb} = \pi/2 \)) and the polariton decay rate is set to \( \gamma = 1.5E \). White dashed lines are the real part of the excitation spectrum \( \text{Re} \omega^{(a)} \).
the linearly polarized probe couples identically to left- and right-polarized modes. In panel (c) of Fig. 6 we observe the polarization to undergo a rotation when in resonance with one of the spectral branches. As expected, probing at resonance with the $u_1$ branch results in a largely left-polarized emission, while the $u_1$ branch induces a right-circular polarization.

Interesting effects are observed for the case of diffusive spectra, and we show, in particular, the case of a 0-diffusive spectrum in Fig. 7. For this case we consider the case of a right-circularly polarized probe beam ($\theta_{pb} = \pi/2$) so that we have a $u_1$ branch emitting much stronger than the $u_1$ branch [see panel (a) of Fig. 7]. However, around $k_p = 0$, where the spectral branches mixed one with the other in a diffusive region, we see that the resonant branch is not $u_1$, rather $v_1$. Here, the strongest emission intensity comes from the $u_1$ branch at large momenta and from the $v_1$ branch inside the diffusive region at small $k$. Because of the mixing between $\uparrow$ and $\downarrow$ degrees of freedom, we have observed a sudden spin flip for the degree of polarization of the eigenvectors around $|k| = 1.2\sqrt{m\bar{E}}$. This is the reason for the resonant emission transfers from $u_1$ to $v_1$ at small $k$. From panel (b) of Fig. 7 it is clear that, in the beam transferred in the $-k_p$, direction, the emission is the strongest in the diffusive momentum ring at $\omega_p = 0$ where $u_1$ and $v_1$ are mixed. The stronger intensity for these modes can be explained in terms of an onset of parametric amplification. In fact, in this diffusive region the imaginary part of the spectrum deviates from the value of the constant decay rate and there is an additional contribution from the negative argument of the outer root in Eq. (16) [see panel 5 in Fig. 4]. For this reason, these modes acquire a longer lifetime, and therefore undergo enhanced scattering. Similarly, in panel (a) of Fig. 7, one can appreciate an increase in transmission in the same diffusive region. In addition, there is also an increased emission on the diffusive region at $\omega_p = 0$ where the $u_1$ and $v_1$ branches are degenerate. Although, for these values of the momentum, these branches are circularly left polarized, the interspin interaction coupling the $\uparrow$ and $\downarrow$ modes, combined with the strong parametric amplification, still leads to a strong emission. It is interesting to note that in previous work [12], the proximity to an instability to a parametric decay rate and there is an additional contribution from the negative argument of the outer root in Eq. (16) [see panel 5 in Fig. 4]. For this reason, these modes acquire a longer lifetime, and therefore undergo enhanced scattering. Similarly, in panel (a) of Fig. 7, one can appreciate an increase in transmission in the same diffusive region. In addition, there is also an increased emission on the diffusive region at $\omega_p = 0$ where the $u_1$ and $v_1$ branches are degenerate. Although, for these values of the momentum, these branches are circularly left polarized, the interspin interaction coupling the $\uparrow$ and $\downarrow$ modes, combined with the strong parametric amplification, still leads to a strong emission.

In panel (c) [(d)] of Fig. 7 we plot the azimuthal polarization angle for a particlelike (holelike) transmitted signal at $k_p$ ($-k_p$). The resonant branch is $u_1$ at large $k$ and $v_1$ for small values of the momentum and strongly emits circularly right-polarized light—$\theta_{pb} \approx \theta_{pb} \approx \pi/2$. Yet the parametrically amplified region around $k_p = 0$ emits both in the transmitted $k_p$ and $-k_p$ directions an almost purely circularly left-polarized light. Hence, the parametric amplification of the mixed $u_1$ and $v_1$ modes causes the left-polarized incoming probe light to undergo a spin flip when interacting with the polariton sample. We show in Fig. 8 that $\theta_{pb} \approx 0$ to 0 (i.e., almost pure left polarization), when the parametric amplified mode $k_p = 0.5\sqrt{m\bar{E}}$, probed with $\omega_{pb} = 0$, is brought closer to resonance by varying the polariton decay rate $\gamma \rightarrow \gamma_c$—here $\gamma_c$ is the minimum value of the polariton decay rate required for the system stability, i.e., $\text{Im} \omega < 0$. The emission intensity [panel (a) of Fig. 8] diverges as 1/($\gamma - \gamma_c$) for both particllecile and holelike signals when they are brought close to resonance. This can be understood from the expression (26), where we see that the right-hand side becomes singular if $\omega_{pb}$ equals $\text{Re} \omega_{pb}$ and at the same time $\text{Im} \omega_{pb} \rightarrow 0^-$. In this limit, the response $\omega_p \rightarrow 0^-$. As can be seen in panel (b) of Fig. 8. Note that $\gamma = \eta_k$ exactly in the diffusive disk around $k = 0$, where $u_1$ and $v_1$ coincide. In Fig. 8, we have measured the decay rate $\gamma$ in units of self-interaction energy $\bar{E}$.

Experimentally, however, $\gamma$ is fixed for a given microcavity, but the ratio $\gamma/\bar{E}$ can be tuned by varying the laser pump power, $|f|^2$ in Eq. (2). One should then calculate the mean-field expressions (3) to derive the polariton spin densities $|\psi_\pm|^2$ and thus the self-interaction energy $\bar{E}$. By varying the tilting angle of the probe beam with respect to the pump, one can scan through $k$ space. As depicted on Fig. 2, also the detectors should be placed accordingly: one at the same angle as the probe and the other at the mirrored angle, to detect the particle and the holelike signal, respectively. At each position of the probe, a vertical slice on the response figures can be reconstructed by changing the probe frequency $\omega_{pb}$ and measuring the intensity and polarization at both detectors.

IV. CONCLUSIONS

We have analytically derived the spectrum of elementary excitations for a spinor polariton fluid in the linear response approximation scheme. For fixed interaction strength, the spectra can be classified in terms of two dimensionless parameters only: mean-field polarization angle and the renormalized pump detuning. Even though there is a large variety of possible spectra, we identify three major classes—gapped, 0-diffusive, and $\omega$-diffusive—depending how the opposite polarization spectral branches mix together and at which energy. For
0-diffusive the mixing happens at zero energies; for $\omega$-diffusive it happens at finite energy. Interestingly, only the mean-field polarization is sufficient to distinguish between these two different diffusivelike spectra. We show that the mixing of $\uparrow$ and $\downarrow$ branches characterizes sudden spin flips of the intrinsic degree of polarization along the branches for both diffusivelike spectra. We have characterized the response of the system to an external probe in terms of the spectral intrinsic properties. In particular, we have shown that the intrinsic polarization of an elementary excitation is reflected in the transmitted signal of a probe beam experiment. For gapped spectra the degree of polarization along the branches for both diffusivelike and 0-diffusive spectra is to a parametric instability, the larger the amount of spin mixing between opposite polarization branches at small energies. In contrast, for both 0-diffusive and $\omega$-diffusive spectra, the strong mixing between opposite polarization branches at small momenta leads to a spin flip of the transmitted degree of polarization along the branch. The closer the polariton spinor fluid is to a parametric instability, the larger the amount of spin flip, independently of the degree of polarization of the probing beam.

Recently, numerous fascinating results have been achieved in the study of the response of a spinor polariton fluid to a magnetic field, such as the spin Meissner effect [24] and effective magnetic monopoles [23]. As a future perspective, it could be interesting to include the Zeeman-splitting terms in our model and study their influence on the spectrum of excitations. In addition, also effects of disorder [30,31] and TE-TM splitting could be incorporated.

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