DEPARTMENT OF ECONOMICS

Informality and Macroeconomic Fluctuations: A Small Open Economy New Keynesian DSGE Model with Dual Labour Markets

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Informality and Macroeconomic Fluctuations: A Small Open Economy New Keynesian DSGE Model with Dual Labour Markets*

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Abstract

How do key macroeconomic variables of a small open economy with segmented labour markets behave in response to domestic and external shocks? In this paper we attempt to address this question by modeling the coexistence of a formal labour market with higher wage rates and search frictions, and an informal labour market with the opposite attributes in the standard multi-sector small open economy New Keynesian DSGE model. The model is calibrated for a typical Sub-Saharan African economy and the behaviour of key macroeconomic variables in response to domestic and external shocks is analysed. The results show that almost all the impulse response functions of our model are consistent with what theory predicts and what other empirical works show about the responses of low income countries to the shocks we consider. However, our results do not seem to corroborate the widely held wisdom that the existence of an informal sector plays a stabilizing role in the event of shocks.

JEL Classification: E24, E26, E32, E41, O55

Keywords: Dual labor markets, Informal sector, Open economy, New Keynesian DSGE, Low-income countries, Sub-Saharan Africa

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> Some branches of economics work with models that assume that everybody who works participates in a single, undifferentiated labour market. I regard such models as grossly unrealistic. A better description, I would maintain, is that jobs differ in quality, these different groups being called "segments" or "sectors". Thus, labour market segmentation is said to exist if 1) Jobs for individuals of a given skill level differ in terms of their pay or other characteristics, and 2) Access to the more attractive jobs is limited in that not all who want the better jobs can get them. (Fields (2009:1))

1 Introduction

How do key macroeconomic variables of a small open economy with labour markets segmented into formal and informal, and where the informal sector employs a large proportion of the labour force, behave in response to domestic and external shocks? This question is pertinent to emerging and low-income economies since market segmentation into formal and informal (of goods markets, financial markets, and labour markets) is widespread and a significant proportion of their GDP and employment is accounted for by the informal sector. The segmentation also implies that the response of the economy to various shocks depends on how the formal and informal sectors respond to the shocks and how they interact among themselves. But the conventional models that are currently popular tools for macroeconomic analysis have not given due attention to this issue.

The New Keynesian dynamic stochastic general equilibrium (DSGE) model has become the workhorse of macroeconomic research and is now a common tool in academics, central banks, and other policy institutions. However, until recently this model did not capture labour market frictions and involuntary unemployment, let alone labour market segmentation, a weakness that is acknowledged even by prominent economists in this school (see, among others, Blanchard (2009), Blanchard and Gali (2010), Gali (2011), and Gali et al. (2011)). For instance, Blanchard (2009: 216) describes this weakness as a "striking (and unpleasant) characteristic" of the standard New Keynesian DSGE model.

Recent works in DSGE modeling, in part in response to these criticisms, have come up with many ways of introducing labour market frictions and involuntary unemployment

into the New Keynesian model (see, for instance, Blanchard and Gali (2010), Castillo and Montoro (2012), Christiano et al. (2010 and 2011), Gertler and Trigari (2009), Gertler et al. (2008), Mattesini and Rossi (2009), Sala et al. (2008), and Trigari (2007)). Some of these studies incorporate different variants of the search and matching labour market models in the tradition of Diamond-Mortensen-Pissarides into the DSGE framework. By modeling the adjustment of labour both at the extensive and intensive margins and capturing the existence of involuntary unemployment at equilibrium, these studies address one of the serious weaknesses of the New Keynesian DSGE model. Furthermore, given that the estimated versions of this model are also promising in fitting the data, the works in this direction seem to be significant advances in the field (see Christiano et al. (2010 and 2011), Gali et al. (2011), and Gertler et al. (2008)).

Although they recognize and model the existence of labour market frictions, most of the works mentioned above assume that the labour market is homogenous; that is, all workers and firms in an economy interact in a single market. As it is well stated in the opening quote from Fields (2009), the assumption of a homogenous labour market is far from being realistic. There is considerable heterogeneity in the labour market emanating both from the characteristics of the job, and from those of the workers. At the same time, it is also understandable that accounting for the whole range of heterogeneity is analytically intractable. However, modeling this heterogeneity via the assumption of duality in the labour market is an old tradition in the literature, particularly in development economics. This approach has recently made its way into the New Keynesian DSGE model. For instance, Castillo and Montoro (2012) and Mattesini and Rossi (2009) recognize the existence of duality in the labour market, though they differ on the type of duality they deal with. In Mattesini and Rossi (2009) the duality arises from the coexistence of a labour market characterized by perfectly flexible wages with a unionized labour market characterized by rigid real wages. In Castillo and Montoro (2012), by contrast, the existence of duality is due to the coexistence of formal and informal labour markets in an economy where both formal and informal labour markets are characterized by some frictions. Both of these works deal with a closed economy New Keynesian DSGE model. To the best of our knowledge there is no work that addresses the labour market segmentation in an open economy New Keynesian DSGE framework.

In this study we extend the closed economy New Keynesian DSGE model with labour market frictions into a multi-sector open economy setup with dual labour markets. The model is then used to assess the implications of the duality of the labour market for the dynamics of key macroeconomic variables in response to various domestic and external shocks.

The rest of the paper is organized as follows. In section 2 we briefly discuss the nature and importance of labour market segmentation in Sub-Saharan Africa (SSA). We also highlight various issues associated with modeling labour market segmentation and the contribution of our study relative to the literature on this subject. We outline the model in section 3. In section 4 we calibrate and simulate the model for a typical SSA economy and assess the behaviour of selected macroeconomic variables of the model in response to various domestic and external shocks. In section 5 we discuss the sensitivity of the results and in section 6 we conclude.

2 Labour Market Segmentation in SSA

Labour market segmentation is one of the defining features of low-income countries similar to those in SSA. This implies that understanding how the economy responds to domestic and external shocks requires knowledge about how these segmented markets respond to the shocks and how they interact among themselves.

Literature shows that the labour market segmentation in SSA fits more to the formal/informal dichotomy than to the unionized/non-unionized classification. For instance, Kingdon et al. (2006) identify and assess three attributes of labour market flexibility, namely downward flexibility of real wages over time, the tendency for wages to respond to unemployment rates, and the extent of wage differentials across sectors and firms within the context of African economies. Their findings show that African labour markets could be seen as flexible in terms of downward flexibility of wages and responsiveness of wage rates to unemployment rates. They also report that there exists strong evidence to conclude that labour markets in Africa are rigid in terms of wage differentials between sectors and/or firms. That is, in the same economy, there is a high wage paying sector (formal sector), on the one hand, and low wage paying sector (informal sector), on the

other hand. The coexistence of formal and informal labour markets is common in almost all countries of the world, both developed and developing countries alike. What makes the labour markets in SSA peculiar is that the informal sector employs the largest proportion of the labour force; in the developed countries, by contrast, the informal labour markets are usually too small to be of concern. For instance, the share of this sector as a percent of non-agricultural employment in SSA is the largest of all regions of the world. This share varies across countries of the region, with a regional average of about 77 percent (Charmes (2000)). See also Blunch et al. (2001), ILO (2002), and Jutting and de Laiglesia (2009). Furthermore, the literature shows that the proportion of informal sector employment has shown a tendency to increase rather than to decrease (ILO (2002) and Schneider (2005)).

Kingdon et al. (2006) also assert that the duality of the labour market – the formal sector with relatively high wages and the informal sector with relatively low wages serving as an employer of last resort[1] - is a feature that is common to all labour markets in Africa. Moreover, the wage differential between the two sectors is significant. Hence, in order to understand the effects of various domestic and external shocks on the macroeconomic performance of the countries in the region, this feature of the labour market needs to be systematically modeled. We believe that incorporating labour market segmentation into the open economy New Keynesian DSGE model fills a gap in the literature on labour market frictions. More importantly, since it captures the specificities of low-income countries like those in SSA, such a model is of significant help to understand how these economies respond to various domestic and external shocks.

Accordingly, as discussed in the previous section, in this study we extend the closed economy New Keynesian DSGE model with labour market frictions, specifically the work of Blanchard and Gali (2010), into a multi-sector open economy setting with dual labour markets. Therefore, our study is similar to Castillo and Montoro (2012) who introduced the dual labour market assumption into the Blanchard and Gali (2010) model while maintaining the closed economy setting.

[1]However, there is a long-standing debate on whether the informal sector is the employer of last resort or a desirable sector that workers/households join as a matter of preference (see, for example, Maloney (2004); Fiess et al. (2010); and Hart (1973)).

However, it is worth emphasizing that though our model shares many of the features of that of Castillo and Montoro (2012), since both works try to introduce duality of the labour market into the same model (Blanchard and Gali (2010)), the two works differ in a number of important respects.

First, we assume a multi-sector open economy setting where the economy produces two composite goods - tradable and non-tradable goods (as in Lubik (2003), Santacreu (2005), and Matteson (2010), among others) while Castillo and Montoro (2012) deal with a closed economy that produces a single final good.

Second, unlike Castillo and Montoro (2012) who assume that all firms have both formal and informal employment arrangements, we assume that some firms operate in the formal sector while others operate in the informal sector. In our model tradable goods are produced by both types of firms while non-tradable goods are produced by firms operating in the informal sector[2]. This is important since the informality assumed in Castillo and Montoro (2012) is not the nature of the economy as a whole but the result of the employment decision of firms (to maximize their profit by striking the right mix or balance of the two types of employment). This is clear from their assumption that within the same firm there are workers who have a formal employment contract and others who have no such contract. This type of duality, indeed, exists in low-income countries and even in developed countries where firms keep a certain level of workers with formal employment contracts and hence pay the associated benefits and at the same time resort to informal employment arrangements in response to temporary demand shocks. This is not the type of informality we are dealing with in this study. In our study, the duality is the nature of the economy as a whole as in the framework of the dual economy literature of Lewis (1954) and others.

Third, the wage determination process in our model is different from the works mentioned above. For example, Castillo and Montoro (2012) assume that the wage rate in both the formal and the informal labour markets is determined via a bargain between firms and workers; that is, wages are Nash bargained. This assumption is not consistent

[2]It is important to note that, in the real world, some non-tradable goods are also produced by firms that operate in the formal sector. Therefore, a more realistic model should consider this fact which leads to four types of production processes instead of the three types we assumed here.

with the characteristics of the informal sector discussed in most literature and untenable for the type of economies modeled in this study. In SSA the largest proportion of the population (about 2/3) lives in rural areas (United Nations (2009)) where agriculture and related activities are the main source of livelihood. Most of these activities are informal self-employment, the primary occupation being self-employed smallholder farming (Eicher and Baker (1992)). This implies that the assumption of wage determination via bargaining between workers and firms in both sectors does not make much sense.

The literature on segmented labour markets provides various models of wage determination and of the interaction between the wage rates in the formal and the informal labour markets. Fields (2009) identifies three types: models that assume an integrated labour market with wage equalization and no unemployment, models with wage differentials but no open unemployment (Lewis model and various extensions), and models with wage differentials and unemployment (Harris-Todaro model and extensions). Of these models, the Harris-Todaro model deserves further discussion as its extended version is commonly used to study the topic of this paper in the literature outside the DSGE framework.

The original Harris-Todaro model (Harris and Todaro (1970)) is not about the interaction between formal and informal sectors, but is meant to explain why rural-urban migration of labour continues though there is high urban unemployment. The main idea in the Harris-Todaro model is that the urban real wage is politically fixed at a rate that is higher than the real return from agricultural activities in rural areas. Hence, "rural-urban migration will continue so long as the expected urban real income at the margin exceeds real agricultural product - i.e., prospective rural migrants behave as maximizers of expected utility" (Harris and Todaro (1970: 127)). That is, migrant workers make the decision to migrate knowing that they could remain unemployed for sometime while searching for job. Harris and Todaro argue that there will be equilibrium unemployment resulting from the wage differential between the rural and urban sectors. In their words:

> ....in many developing nations the existence of an institutionally determined urban minimum wage at levels substantially higher than that which the free market would allow can, and usually does, lead to an equilibrium with considerable urban unemployment (Harris and Todaro (1970:129)).

This original Harris-Todaro model has been extended by many to study various aspects of inter-sectoral flows of labour and migration. One such extension is used to study the interaction between the formal and informal sectors. In this extended model, it is due to the existence of an institutionally determined formal sector wage at a level that is substantially higher than the economy wide market clearing wage rate that the economy will have an equilibrium with significant formal sector unemployment.

In view of the characteristics of the labour market in Africa that we discussed above (Kingdon et al. (2006)), the Harris-Todaro type model seems to be the most appropriate to model labour market segmentation in SSA. Accordingly, in this study we employ the original Harris-Todaro type wage determination where the wage rate in the formal sector is set above the wage rate in the informal sector due to various institutional factors such as labour unions, legal codes, factors associated with the efficiency wage hypothesis, etc. However, unlike the Harris-Todaro model where the wage rate is exogenously given, we assume that the wage rate in this sector is determined via bargaining between the employers and the workers. By contrast, the informal labour market is assumed to be perfectly competitive and wage rates are equal to the value of the marginal products of labour. Hence, anyone who is willing to accept this wage rate can obtain a job; that is, the wage rate in the informal sector is the market clearing wage. However, this assumption also abstracts from modeling the existence of some form of income sharing arrangement that seems to be the most appropriate approach for the informal sector[3].

As a result of the attributes of the formal sector discussed above, all household members prefer to be employed in this sector but only few end up being employed in that sector with the residual going to the informal sector or remain unemployed while searching for a job in the formal sector. This implies that household members do not know *a priori* whether they will be employed or not and, even if employed, they do not know *a*

[3]The literature in development economics indicates that in some production processes of the low-income countries the marginal product of labour is very low which makes the assumption of equality between the wage rate and marginal productivity problematic since a real wage rate equal to the marginal product of labour is not enough for the worker (and her family) to survive on. Instead, the literature reports that there are some moral and social norms that determine the wage rate in such economies. This implies that there are some forms of income sharing mechanisms in place (see, for example, Ranis (2006) for more discussion on this subject).

priori whether they will be employed in the formal or informal sector. Like other works in the area, we assume that there is complete risk sharing at the household level against unemployment and fluctuation in income due to possible switches in sectors. This has very important implications: becoming unemployed or moving from the formal to the informal sector will not change the utility of the household member only, but it changes the utility of the household as a whole. This assumption helps to maintain the tractability of the representative agent model by avoiding the heterogeneity that comes in otherwise (see, among others, Blanchard and Gali (2010), Gali (2011), Christiano et al. (2010 and 2011)).

At any given time, an unemployed household member (who is actively searching for job by the assumption of full participation) will either obtain a job in the formal or informal market, or remain unemployed. Therefore, unemployment in this model exists due to workers searching for a job in the formal sector and deciding not to take one in the informal sector (this corresponds to the migrant workers who remain unemployed while searching for job in urban areas in the original Harris-Todaro model as stated in the quotation above).

A related literature to our study, though not in the DSGE framework and addressing quite a different issue, is Zenou (2008). In his paper, Zenou develops a labour market model for a developing economy that has the characteristics discussed above, namely; firms and workers in the formal sector face labour market frictions while the informal sector has a perfectly competitive labour market. Zenou (2008) is based on the extended Harris-Todaro model that he further extended into the conventional search-matching approach to labour market analysis of Diamond-Mortensen-Pissarides. According to his model, the formal sector is characterized by search-matching frictions and wages are determined through bargaining between workers and firms. The informal sector, by contrast, is perfectly competitive and wages are determined by market forces. The model in Zenou (2008) captures many interesting features of the labour market of a typical low-income country. As it will be seen from discussions in the sections that follow, though our frameworks differ, we borrow many underlying ideas about the features of the labour markets in low income countries from his model.

3 The Model

The model in this paper builds on the works of Gali and Monacelli (2005) and Blanchard and Gali (2010) where the small open economy New Keynesian DSGE elements are from the former while the labour market components follow the latter. Our work also borrows the multi-sector production (i.e., distinction between tradable and non-tradable goods production) feature from Lubik (2003), Santacreu (2005), and Matheson (2010)[4].

3.1 Preferences

We assume that the economy is populated by a large number of identical and infinitely lived households that can be represented by a household that is made up of a continuum of members that can be represented by the unit interval. The household obtains utility from consumption of both domestically produced and imported goods and leisure. We assume that there is full participation rate, therefore, at a given point in time, a household member is either employed or unemployed. Those employed are working either in the formal or the informal sector.

The household maximizes the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left(C_t, L_t \right) \tag{3.1}$$

where E is the expectation operator, β is the subjective discount factor of the household, C_t represents the household's consumption of goods, and L_t is an index of household's aggregate labour supply. We assume that the representative household has the following additively separable instantaneous utility function

$$\Lambda_t = \frac{(C_t - bC_{t-1})^{1-\sigma}}{1-\sigma} - \eta \frac{(L_t)^{1+\varphi}}{1+\varphi} \tag{3.2}$$

where σ is the inverse of elasticity of intertemporal substitution in consumption, φ is the inverse of Frisch elasticity of labour supply, and b represents the degree of habit persistence in consumption preference. η captures the marginal disutility from working.

[4] Some of the model components are taken directly from Senbeta (2011) which is, in turn, developed based on the literature mentioned in this section.

Consumption C_t is an index of the quantities of tradable and non-tradable goods consumed by the household that can be given by the following CES aggregator:

$$C_t = \left[(1-\gamma_1)^{\frac{1}{\theta_1}} C_{T,t}^{\frac{(\theta_1-1)}{\theta_1}} + \gamma_1^{\frac{1}{\theta_1}} C_{N,t}^{\frac{(\theta_1-1)}{\theta_1}} \right]^{\theta_1/(\theta_1-1)} \tag{3.3}$$

where $C_{T,t}$, $C_{N,t}$ denote consumption of tradable and non-tradable goods, respectively. The parameter θ_1 measures the elasticity of intratemporal substitution of consumption between tradable and non-tradable goods. $\gamma_1 \, \epsilon \, (0,1)$ measures the proportion of non-tradable goods in the consumption of households. The representative household aims at maximizing the utility from consumption of both tradable and non-tradable goods by minimizing the expenditure on these two varieties while maintaining a certain target level of consumption. Solving this problem of optimal allocation of expenditure on tradable and non-tradable goods yields the following demand functions for these goods:

$$C_{T,t} = (1-\gamma_1) \left(\frac{P_{T,t}}{P_t} \right)^{-\theta_1} C_t \tag{3.4}$$

$$C_{N,t} = \gamma_1 \left(\frac{P_{N,t}}{P_t} \right)^{-\theta_1} C_t \tag{3.5}$$

where $P_{T,t}$, $P_{N,t}$, P_t are the price indices of tradable, non-tradable and overall consumer goods, respectively. Both tradable and non-tradable goods are composite indices that are bundles of differentiated products. The overall consumer price index is given by

$$P_t = \left[(1-\gamma_1) \left(P_{T,t} \right)^{1-\theta_1} + \gamma_1 \left(P_{N,t} \right)^{1-\theta_1} \right]^{1/(1-\theta_1)} \tag{3.6}$$

The tradable goods consumed domestically are either domestically produced or imported from the rest of the world. Hence, the consumption of tradable goods is determined as a CES index composed of home produced tradable goods and imports as follows:

$$C_{T,t} = \left[(1-\gamma_2)^{\frac{1}{\theta_2}} \left(C_{H,t} \right)^{\frac{(\theta_2-1)}{\theta_2}} + (\gamma_2)^{\frac{1}{\theta_2}} \left(C_{M,t} \right)^{\frac{(\theta_2-1)}{\theta_2}} \right]^{\theta_2/(\theta_2-1)} \tag{3.7}$$

The parameter θ_2 measures the elasticity of intratemporal substitution of consumption between domestically produced tradable goods $C_{H,t}$ and imported goods $C_{M,t}$. $\gamma_2 \, \epsilon \, (0,1)$ denotes the share of imported goods in the total consumption of tradable goods consumed domestically. It is also interpreted as a measure of openness of the economy. As with the case of total consumption, expenditure minimization on the tradable goods yields

the demand functions for domestically produced tradable goods and imported tradable goods as in the following equations.

$$C_{H,t} = (1-\gamma_2)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta_2} C_{T,t} \tag{3.8}$$

$$C_{M,t} = \gamma_2\left(\frac{P_{M,t}}{P_{T,t}}\right)^{-\theta_2} C_{T,t} \tag{3.9}$$

where $P_{H,t}$, $P_{M,t}$ are, respectively, price of domestically produced tradable goods and domestic currency price of imported goods. The tradable goods price index is given by

$$P_{T,t} = \left[(1-\gamma_2)\left(P_{H,t}\right)^{1-\theta_2} + \gamma_2\left(P_{M,t}\right)^{1-\theta_2}\right]^{\frac{1}{(1-\theta_2)}} \tag{3.10}$$

Total consumption expenditure by households is given by the sum of the expenditures on tradable and non-tradable goods they consume

$$P_t C_t = P_{T,t}C_{T,t} + P_{N,t}C_{N,t} = P_{H,t}C_{H,t} + P_{M,t}C_{M,t} + P_{N,t}C_{N,t} \tag{3.11}$$

On the other hand, as indicated above, the household's aggregate labour supply index, L_t, is composed of the fractions of household members supplied to the formal sector, $L_{F,t}$, and the informal sector, $L_{I,t}$, and is given by the following CES function

$$L_t = \left[\tau_L^{-\frac{1}{\theta_L}}\left(L_{F,t}\right)^{\frac{1+\theta_L}{\theta_L}} + (1-\tau_L)^{-\frac{1}{\theta_L}}\left(L_{I,t}\right)^{\frac{1+\theta_L}{\theta_L}}\right]^{\frac{\theta_L}{(1+\theta_L)}} \tag{3.12}$$

where τ_L is a share parameter that can also be interpreted as the probability that a household member is employed in the formal sector. $\theta_L \ \epsilon\,(0,\infty)$ is the elasticity of substitution of the supply of labour between the two sectors (for a similar approach of defining the representative household's labour supply, see Berg et al. (2010), Bouakez et al. (2009), Dagher et al. (2012), and Mattesini and Rossi (2009)). The justification for such a formulation of a representative household's labour supply to different sectors, according to Bouakez et al. (2009: 1246), is to account for the limited mobility of workers across sectors in a multisector economy models. This argument is more appealing in the case of the current study since it is consistent with the assumption of labour market segmentation. The labour market segmentation in this study (the formal-informal duality) exists due mainly to limited mobility of labour between the two sectors. Hence, the aggregator in (3.12) captures the idea that labour mobility between the formal and

informal sectors is imperfect as a result of which wage rates and the fraction of household members working in the two sectors are different. In this setting, θ_L measures the responsiveness of the household to change the ratio of its labour supply to the two sectors as a result of a change in the ratio of the wage rates in the two sectors. In the extreme case, as θ_L approaches ∞ the expression (3.12) becomes linear aggregation technology, $L_t = L_{F,t} + L_{I,t}$, which implies perfect mobility between the two sectors and, therefore, equal wage rates[5].

Furthermore, following the original Harris-Todaro model and discussions in Zenou (2008), we introduce the following two assumptions for the tractability of the model. First, we assume that workers in the formal sector prefer to stay unemployed and search for the formal sector jobs to taking the informal sector job when they are separated from job. Second, the workers in the informal sector cannot search for formal sector job while they are working in the informal sector. This implies that workers in the informal sector must first separate from informal sector job and move to unemployment status to search for a job in the formal sector. Accordingly, at any time t certain fraction δ_I of workers in the informal sector decide to leave their jobs. This is what we refer to as rate of separation from job in the informal sector. Given these assumptions, the fractions of household members that are employed at time t, $L_{F,t}$ and $L_{I,t}$, are given by the following equations of motions:

$$L_{F,t} = (1 - \delta_F)L_{F,t-1} + A_{F,t} \tag{3.13}$$

and

$$L_{I,t} = (1 - \delta_I)L_{I,t-1} \tag{3.14}$$

where $A_{F,t}$ represents newly created jobs in the formal sector and δ_F is the rate of separation from job in the formal sector. For simplicity, we assume that $\delta_F = \delta_I = \delta$. It is important to emphasize the implications of the two assumptions introduced above as they

[5]The intuition behind the aggregator in (3.12) is that the household prefers its members distributed between the two sectors to supplying all members to any one of the sectors. The reason is that though all household members are the same (homogenous labour), from the point of view of the household, the supplies of labour to the different sectors are not perfect substitutes. Because, depending on the characteristics of jobs, households obtain different levels of leisure from working in the two sectors. It is analogous to the love of variety that lies behind the CES aggregator for the consumption of varieties of goods and services.

are reflected in (3.13) and (3.14). That is, it is assumed that at the household level (and the economy level) there is a certain constant quantity of labour force that is distributed between formal sector employment, informal sector employment, and unemployment. There is a flow of labour from both sectors into the pool of unemployed workers but there is only one direction of flow out of unemployment to employment which is the new hire into the formal sector, $A_{F,t}$.

Therefore, according to (3.13) and (3.14), each period there are separations from jobs that decrease (increase) employment (unemployment) given by $\delta L_{F,t-1}$ and $\delta L_{I,t-1}$, and creation of new jobs $A_{F,t}$ that increases (decreases) employment (unemployment). This implies that the labour supply in this model adjusts at an extensive margin unlike the traditional representation in which $L_{F,t}$ and $L_{I,t}$ represent not the fraction of household members but the number of labour hours that the representative household supplies in which case the change is interpreted as a change in the number of hours supplied (the intensive margin).

Given our assumption of full participation rate, the fraction of the household members that are unemployed at the beginning of any period t is given by

$$U_t = 1 - (1 - \delta)\left(L_{F,t-1} + L_{I,t-1}\right) \tag{3.15}$$

and the end of period unemployment rate is given by

$$u_t = 1 - L_{F,t} - L_{I,t} \tag{3.16}$$

Employment in the formal and informal sectors commands a nominal wage rate of $W_{F,t}$ and $W_{I,t}$, respectively. In principle, following Pissarides (2000), we can also assume that the unemployed worker enjoys some constant real return, such as unemployment benefit, the imputed value of leisure time, home production, etc. But that will not add much to the dynamics of the model in this paper due, in part, to the focus of our study. The primary focus of this study, as discussed in the preceding sections, is on the response of the economy to various domestic and external shocks, other than labour market policies. Had the study intended to analyze the effects of labour market policies, modeling the returns to unemployment would have been imperative as changes in the unemployment benefit are among the most important instruments available to policy makers.

Given our characterization of the labour market, the following conditions must hold:

$$L_{F,t} \geq 0,\ L_{I,t} \geq 0, u_t \geq 0,\ W_{F,t} > W_{I,t},\ L_{F,t} + L_{I,t} + u_t = 1 \tag{3.17}$$

The households in the model economy own the firms and hence earn dividends. They also earn wage income from the supply of labour services. For the sake of simplicity, we ignore the financial intermediaries; like most authors in this field, we assume that households directly lend to the public sector. Furthermore, there is no investment and, therefore, no capital stock in this model[6]. Therefore, the representative household tries to maximize its lifetime utility subject to a sequence of budget constraints of the form:

$$P_t C_t + B_t \leq W_{F,t} L_{F,t} + W_{I,t} L_{I,t} + D_t + R_{t-1} B_{t-1} \tag{3.18}$$

where R_{t-1} is the gross nominal return on bonds. This budget constraint implies that the household's expenditure, as given by the left hand-side, consists of expenditure on consumption, C_t, and purchase of public bonds, B_t. The flow of income, as given by the right-hand-side of the budget constraint, is composed of wage income from labour services, dividends, D_t, and the income from previous holdings of bonds, B_{t-1}.

The problem faced by the representative household can now be summarized by the Lagrange function given below. Note that the representative household solves the maximization problem taking $L_{F,t}$ as given. This stems from the fact that the supply of $L_{F,t}$ is the outcome of the bargaining process between firms and workers to set the wage rate of the formal sector as discussed in section 3.4 of this paper (for a similar approach see Mattesini and Rossi (2009: 1489))[7].

$$\underset{C_t, B_t, L_{I,t}}{\text{Max}} \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{(C_t - bC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\varphi} \left[\tau_L^{-\frac{1}{\theta_L}} \left(L_{F,t}\right)^{\frac{1+\theta_L}{\theta_L}} + (1-\tau_L)^{-\frac{1}{\theta_L}} \left(L_{I,t}\right)^{\frac{1+\theta_L}{\theta_L}} \right]^{\frac{\theta_L(1+\varphi)}{(1+\theta_L)}} \\ -\lambda_t \left[P_t C_t + B_t - W_{F,t} L_{F,t} - W_{I,t} L_{I,t} - D_t - R_{t-1} B_{t-1} \right] \end{array} \right\} \tag{3.19}$$

[6] See Walsh (2010: 330) why variation in capital stock are ignored in the basic New Keynesian model.

[7] Mattesini and Rossi (2009) model the coexistence of the Walrasian (competitive) labour markets with unionized labour markets in a closed economy New Keynesian DSGE framework. In their model total labour supply is a composite of labour supplies to unionized and competitive sectors. Labour supply to the unionized market is derived from the unions' maximization problem and the supply to the competitive market is given by the household's marginal rate of substitution between consumption and labour.

The first order necessary conditions of the optimization problem of this household are given as

$$C_t : \qquad (C_t - bC_{t-1})^{-\sigma} = \lambda_t P_t \tag{3.20}$$

$$B_t : \qquad \beta E_t \lambda_{t+1} R_t = \lambda_t \tag{3.21}$$

Combining the above two equations, we can derive the usual Euler equation of consumption

$$\beta E_t \frac{(C_{t+1} - bC_t)^{-\sigma}}{(C_t - bC_{t-1})^{-\sigma}} \frac{P_t}{P_{t+1}} = \frac{1}{R_t} \tag{3.22}$$

$$L_{I,t} : \quad \eta \left[\tau_L^{-\frac{1}{\theta_L}} (L_{F,t})^{\frac{1+\theta_L}{\theta_L}} + (1-\tau_L)^{-\frac{1}{\theta_L}} (L_{I,t})^{\frac{1+\theta_L}{\theta_L}} \right]^{\frac{(\theta_L \varphi - 1)}{(1+\theta_L)}} \left(\frac{L_{I,t}}{1-\tau_L} \right)^{\frac{1}{\theta_L}} = \lambda_t W_{I,t} \tag{3.23}$$

Combining (3.20) and (3.23), we can derive the usual marginal rate of substitution between labour and consumption

$$\eta \left[\tau_L^{-\frac{1}{\theta_L}} (L_{F,t})^{\frac{1+\theta_L}{\theta_L}} + (1-\tau_L)^{-\frac{1}{\theta_L}} (L_{I,t})^{\frac{1+\theta_L}{\theta_L}} \right]^{\frac{(\theta_L \varphi - 1)}{(1+\theta_L)}} \left(\frac{L_{I,t}}{1-\tau_L} \right)^{\frac{1}{\theta_L}} (C_t - bC_{t-1})^{\sigma} = \frac{W_{I,t}}{P_t} \tag{3.24}$$

The equality between the real wage and the marginal rate of substitution of consumption for leisure holds only for the informal sector. As indicated above, the supply of labour to the formal sector is determined via the negotiation between workers and firms. The argument is that the formal sector is characterized by search friction and established employment relationships in the sector generate economic rents. The rents are distributed between workers and employers according to the Nash bargaining mechanism. This surplus means the marginal rate of substitution is always less than the real wage in the formal sector. There are debates on which of the approaches to model labour market imperfections can best capture this difference between marginal rate of substitution of consumption for leisure and the real wage rate - what is also referred to as the labour wedge. But there is almost a consensus on its existence which is also strongly supported by data (see, for example, Shimer (2009 and 2010) for a detailed discussion of the labour wedge).

3.2 The real exchange rate, the terms of trade, and incomplete pass-through

The domestic market for imported goods is characterized by monopolistic competition where firms have some power on the prices of goods they import and distribute. This market power together with the "pricing-to-market" - that is imports are priced in terms of domestic currency - leads to a difference between the domestic and foreign prices of imported goods when expressed in terms of the same currency. That is, it creates deviation from the Law of One Price. It is assumed that the Law of One Price holds at the border and the distortion comes in as the importing firms try to exercise their power to set their profit maximizing price. This distortion is referred to as the Law Of One Price Gap (LOOPG) (Monacelli (2005: 1051)), and is given by the ratio of the foreign price index in terms of domestic currency to the domestic currency price of imports

$$\Psi_t = \frac{\varepsilon_t P_t^*}{P_{M,t}} \tag{3.25}$$

where ε_t and P_t^* are the nominal exchange rate and the price index of the rest of the world, respectively. The nominal exchange rate is defined as the domestic currency price of a unit of foreign currency. $P_{M,t}$ is the average price of imported goods in terms of domestic currency. Note that if the law of one price holds, Ψ_t is identically equal to unity.

The real exchange rate is given as the ratio of the price index of the rest of the world (in terms of domestic currency) to the domestic price index:

$$Q_t = \frac{\varepsilon_t P_t^*}{P_t} \tag{3.26}$$

Another important relationship is the terms of trade of the domestic economy which measures the competitiveness of the economy. The terms of trade of the domestic economy is defined as the export price (price of domestically produced tradable goods) relative to the domestic currency price of imports.

$$V_t = \frac{P_{H,t}}{P_{M,t}} \tag{3.27}$$

Hence, increasing V_t indicates improvement of the terms of trade of the economy in the international market.

3.3 International risk sharing and the uncovered interest parity condition

The assumption of international risk sharing links domestic consumption with the consumption level of the rest of the world. This link between domestic consumption and that of the rest of the world can be derived using the consumption Euler equation for the domestic households that can be obtained by combining the first order conditions of the representative household with respect to consumption and bond holding, and can be rewritten as

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{R_t} \quad \text{implies that} \quad \beta E_t \frac{(C_{t+1} - bC_t)^{-\sigma}}{(C_t - bC_{t-1})^{-\sigma}} \frac{P_t}{P_{t+1}} = \frac{1}{R_t}$$

Since agents in the rest of the world have access to the same set of bonds, their Euler equation can also be given by the following equation (assuming that agents in the domestic economy and the rest of the world have the same preferences)

$$\beta E_t \frac{\left(C^*_{t+1} - bC^*_t\right)^{-\sigma}}{\left(C^*_t - bC^*_{t-1}\right)^{-\sigma}} \frac{\varepsilon_t P^*_t}{\varepsilon_{t+1} P^*_{t+1}} = \frac{1}{R_t} \tag{3.28}$$

This implies that

$$\beta E_t \frac{(C_{t+1} - bC_t)^{-\sigma}}{(C_t - bC_{t-1})^{-\sigma}} \frac{P_t}{P_{t+1}} = \beta E_t \frac{\left(C^*_{t+1} - bC^*_t\right)^{-\sigma}}{\left(C^*_t - bC^*_{t-1}\right)^{-\sigma}} \frac{\varepsilon_t P^*_t}{\varepsilon_{t+1} P^*_{t+1}}$$

or

$$(C_t - bC_{t-1}) = E_t \frac{(C_{t+1} - bC_t)}{Q^{\frac{1}{\sigma}}_{t+1} \left(C^*_{t+1} - bC^*_t\right)} Q^{\frac{1}{\sigma}}_t \left(C^*_t - bC^*_{t-1}\right) \tag{3.29}$$

In equilibrium, according to Gali and Monacelli (2005), the following must hold

$$(C_t - bC_{t-1}) = \chi Q^{\frac{1}{\sigma}}_t \left(C^*_t - bC^*_{t-1}\right) \tag{3.30}$$

for all t. χ is a constant that is determined by the relative initial conditions in asset holdings.

The assumption of complete asset markets allows to derive the link between the domestic and foreign interest rates through the uncovered interest parity condition. Assuming, as before, that domestic and foreign economic agents have the same preferences, the consumption Euler equation of the rest of the world can be given by

$$\beta E_t \frac{\left(C^*_{t+1} - bC^*_t\right)^{-\sigma}}{\left(C^*_t - bC^*_{t-1}\right)^{-\sigma}} \frac{P^*_t}{P^*_{t+1}} = \frac{1}{R^*_t} \tag{3.31}$$

Log-linearizing (3.22) and (3.31) around a steady-state, using the definition of real exchange rate, and after some algebra we obtain[8]

$$E_t e_{t+1} = e_t + r_t - r_t^* \tag{3.32}$$

This equation shows that expected rate of appreciation/depreciation of the domestic currency is determined by the difference between the nominal interest rates of domestic economy and that of the rest of the world. With this we turn to the production side of the economy.

3.4 Domestic goods producing firms

3.4.1 Intermediate tradable and final non-tradable goods

The economy produces two types of goods: tradable and non-tradable goods. The tradable goods production is modeled as a two stage production process of the intermediate-final goods production that is common in the New Keynesian DSGE models. The primary reason behind employing this modeling approach in this paper, as it is the case in other works that introduced labour market friction into New Keynesian models (see, for example, Blanchard and Gali (2010)), is to separate wage bargaining from price setting since having both for a firm makes the model intractable and the analysis more difficult. Hence, we assume that there are a continuum of firms that produce the intermediate tradable goods. These intermediate tradable goods are sold on a competitive market to a continuum of monopolistically competitive firms that package these goods into final tradable goods and supply to both domestic and foreign markets.

There are two types of intermediate tradable goods both of which are produced by a continuum of firms. The first type of these goods, $Y_{HF,t}$, are produced by firms that operate in the formal sector and the second type, $Y_{HI,t}$, are produced by firms operating in the informal sector. These two types of intermediate tradable goods are different and sold in a competitive market for different prices, $P_{HF,t}$ and $P_{HI,t}$, respectively. As indicated above, these goods are used as inputs by firms that produce final tradable goods, $Y_{H,t}$.

[8] Detail derivation of this and other log-linearized model equations is given in the technical appendix to this paper.

The non-tradable goods are entirely produced by firms operating in the informal sector. Following the literature on dual labour markets, we assume that the informal sector is characterized by low productivity of labour compared to the formal sector[9]. We also assume that the technology in all sectors is given by a constant returns to scale production function.

Accordingly, the production functions of the two types of intermediate tradable goods producers are given as

$$Y_{HF,t} = Z_{H,t}L_{F,t} \tag{3.33}$$

and

$$Y_{HI,t} = \omega Z_{H,t}L_{HI,t} \tag{3.34}$$

where $\omega < 1$ captures the assumption that the productivity of workers in the informal sector is less than those of working in the formal sector. $L_{HI,t}$ is the fraction of informal sector workers working in the firms producing intermediate tradable goods. $Z_{H,t}$ captures productivity in the tradable goods sector, it is determined exogenously and is assumed to follow a first order autoregressive process in its logarithm

$$\ln Z_{H,t} = \rho_H \ln Z_{H,t-1} + \epsilon_{H,t}, \quad 0 < \rho_H < 1 \tag{3.35}$$

where $\epsilon_{H,t}$ is independently and identically distributed normal error term with zero mean and a standard deviation of $\sigma_{\epsilon H}$.

As discussed above, non-tradable goods are produced entirely by the informal sector and the technology is given by

$$Y_{N,t} = Z_{N,t}L_{NI,t} \tag{3.36}$$

where $L_{NI,t}$ is the fraction of informal sector workers working in the firms producing non-tradable goods. $Z_{N,t}$ captures productivity in the non-tradable goods sector. As in

[9] There is ample empirical evidence that shows that labour productivity in the informal sector is lower than it is in the formal sector. Some assert that informality by its very nature keeps the firms in the sector from accessing facilities and infrastructure that enhance productivity to which the firms of the formal sector do have access (See Zenou (2008) and references in it). One additional assumption that might describe the economies that our model tries to characterize much better is that the informal sector is labour intensive compared to the formal sector (see Batini et al. (2011)) which can partly explain why productivity of workers in the former is lower than those of workers in the latter.

the case of tradable goods sector, we assume that $Z_{N,t}$ follows a first order autoregressive process in its logarithm

$$\ln Z_{N,t} = \rho_N \ln Z_{N,t-1} + \epsilon_{N,t}, \quad 0 < \rho_N < 1 \tag{3.37}$$

where $\epsilon_{N,t}$ is an independently and identically distributed normal error term with zero mean and a standard deviation of $\sigma_{\epsilon N}$.

Labour Market Frictions As discussed in the previous section, in this paper labour market friction is captured by assuming that firms and workers in the formal sector face search frictions[10]. This labour market friction in the formal sector implies that firms incur cost in the process of posting vacancies, screening and employing workers. Similarly, workers need to find firms with vacancies and send in applications, show up for an interview, etc., a process that entails both time and financial cost. The cost from this search friction is a function of labour market tightness, which in the original labour market search and matching models such as Pissarides (2000), is given by the ratio of vacancies to the unemployed workers. This cost is increasing in the vacancies to unemployment ratio for firms and decreasing in the same ratio for workers searching for jobs in this sector. That is, the larger the vacancy to unemployment ratio the easier for workers to get a job while the more costly for firms to find workers and fill their vacancies. An alternative index of labour market tightness is the one introduced by Blanchard and Gali (2010), who defined labour market tightness as a ratio of aggregate hire to unemployment which they also refer to as the job finding rate. In our setting this index is given by

$$X_{F,t} = \frac{A_{F,t}}{U_t} \tag{3.38}$$

where, as discussed earlier, $A_{F,t}$ and U_t represent the new hires to the formal sector and beginning of period unemployment rate, respectively. According to Blanchard and Gali (2010), this specification is preferable to the original vacancy to unemployment ratio since it simplifies the link between hiring costs and labour market tightness. The hiring

[10] Zenou (2008) gives excellent intuition and empirical evidence that explains why the formal sector firms and workers face search frictions and why those in the informal sector do not.

cost faced by each firm is given as

$$G_{F,t} = Z_{H,t}\Theta X_{F,t}^{\alpha} \qquad \alpha \geq 0. \tag{3.39}$$

where $G_{F,t}$ is the hiring cost faced by a firm in the formal sector, measured in terms of composite goods, and Θ is a positive constant. It is worth mentioning that this cost is exogenously given for a firm since it is a function of aggregate vacancies and aggregate beginning of period unemployment.[11]

Firms in both sectors try to maximize an expected discounted profit by making choice on the level of employment of labour given the wage rate and the cost of hiring (in the case of the formal sector)

$$E_t\left(\sum_{j=0}^{\infty}\Xi_{t,t+j}\Pi_{t+j}(i,s)\right) \tag{3.40}$$

where $i = F$ or I denotes whether the firm is operating in the formal or informal sector while $s = H$ or N denotes tradable and non-tradable goods, respectively. $\Xi_{t,t+j} = \beta^j \frac{U'(C_{t+j})}{U'(C_t)}$ is the stochastic discount factor over the interval $[t, t+j]$ and Π stands for the instantaneous profit function of the specific firm that can be given by the following equations for the three types of firms:

$$\Pi_t(F,H) = \frac{P_{HF,t}}{P_t}Z_{H,t}L_{F,t} - \frac{1}{P_t}\left[W_{F,t}L_{F,t} + P_t G_{F,t}A_{F,t}\right] \tag{3.41}$$

$$\Pi_t(I,H) = \frac{P_{HI,t}}{P_t}\omega Z_{H,t}L_{HI,t} - \frac{1}{P_t}W_{I,t}L_{HI,t} \tag{3.42}$$

and

$$\Pi_t(I,N) = \frac{P_{N,t}}{P_t}Z_{N,t}L_{NI,t} - \frac{1}{P_t}W_{I,t}L_{NI,t} \tag{3.43}$$

For the intermediate tradable goods producing firms that operate in the formal sector, the first order conditions are given by the following equation:

$$\begin{bmatrix} \frac{P_{HF,t}}{P_t}Z_{H,t} \\ -\frac{1}{P_t}\left[W_{F,t} + P_t G_{F,t} - (1-\delta)\beta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}P_{t+1}G_{F,t+1}\right)\right] \end{bmatrix} = 0 \tag{3.44}$$

This indicates that the intermediate tradable goods producing firms that operate in the formal sector employ labour up to the point where the marginal benefit from the

[11] According to Blanchard and Gali (2010), $Z_{H,t}$ enters the hiring cost function to prevent the productivity shock from influencing hiring cost relative to production cost. We followed the same procedure.

additional labour equals the marginal cost due to employing that additional worker. In the competitive labour market setting, the marginal benefit to the firm is the value of the marginal product of labour and the marginal cost is the wage rate (given that labour is the only input). However, when there is labour market friction due to the presence of search, there will be additional costs and benefits. In such market setting the hiring cost incurred in employing a worker at time t is part of the marginal cost of the firm while the discounted future saving on hiring costs accruing to the firm from maintaining the same worker adds to the benefit of the firm. Hence, firms take both quantities into account, in addition to the marginal product and the wage rate, when making the employment decision.

For the firms that produce intermediate tradable goods but operate in the informal sector the first order conditions are given as

$$\frac{P_{HI,t}}{P_t}\omega Z_{H,t} - \frac{1}{P_t}W_{I,t} = 0 \tag{3.45}$$

Since there is no labour market friction for firms and workers operating in the informal sector, the wage rate is equal to the value of the marginal product of labour. The conditions for the non-tradable goods producing firms are the same.

$$\frac{P_{N,t}}{P_t}Z_{N,t} - \frac{1}{P_t}W_{I,t} = 0 \tag{3.46}$$

The first order necessary conditions indicate that profit maximizing firms equate their marginal revenue to their marginal costs at the optimal level of employment. Accordingly, the firm that produces intermediate tradable goods in the formal sector equates its marginal revenue product with the sum of the wage rate and the hiring cost net of future savings on the hiring cost. For the other two types of firms, the condition is the usual classical case where the firm equates the marginal revenue product with the wage rate.

Wage Determination As discussed in the previous sections the employment process in the formal sector is characterized by labour market frictions where firms and workers face searching costs. The literature in the search-matching framework (see, for example, Pissarides (2000)) asserts that the employment relation established between the firm and the worker through the search-matching process yields pure economic rent that is shared

between the two parties. The size of the share to each party depends on the bargaining power held. Following Pissarides (2000) and other works on the subject, we assume that the economic rent created by the employment relationship is shared according to the Nash solution to a bargaining problem where the negotiated wage rate is the one that maximizes the Nash product (i.e., the product of the value of the job to the firm and the value of the same job to the worker). The value of the job to the firm equals the price of the product minus the wage rate and the hiring cost while the value of the same job for the worker is wage rate minus what the worker gives up.

Let $V_{F,t}$, represents the value to the household of a member who is employed in the formal sector and $V_{u,t}$ denotes the value to the household of unemployed member. Furthermore, let $w_{F,t}$ and $w_{I,t}$ are, respectively, the real wages in the formal and informal sectors. The value of the job to a household whose member is employed in the formal sector can be given by

$$V_{F,t} = \left\{ \begin{array}{c} w_{F,t} - \eta \left(L_t\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{L_{F,t}}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} \\ +E_t \left\{\Xi_{t,t+1} \left[(1-\delta+\delta X_{F,t+1})V_{F,t+1} + \delta\left(1-X_{F,t+1}\right)V_{u,t+1}\right]\right\} \end{array} \right\} \tag{3.47}$$

This expression shows that the value of a job in the formal sector to the household is wage rate in the formal sector net of the marginal rate of substitution between consumption and labour plus the discounted future value of the following three states: maintaining the job, separated from the job but reemployed in the formal sector, and separated from the job and remained unemployed while searching for job in the formal sector[12].

On the other hand, the value to a household of unemployed member is given by

$$V_{u,t} = E_t \left\{\Xi_{t,t+1} \left[X_{F,t+1}V_{F,t+1} + \left(1-X_{F,t+1}\right)V_{u,t+1}\right]\right\} \tag{3.48}$$

That is, the value to the household of unemployed member is the discounted return from future employment in the formal sector, or the imputed real return from remaining unemployed.

Assuming that the bargaining power of the workers in the formal sector is given by $\xi \, \epsilon \, [0,1]$ and the fact that the economic rent created due to the established employment

[12] In a more general dual labour market model where workers in the formal sector are allowed to take informal sector job when they lose job in the formal sector and where on-the-job search is allowed in the informal sector, the value function of a job in any of the sectors to a household is more richer than the one in our model.

relationship is the cost of hiring new worker (since the firm can readily employ a new worker if it is willing to incur the hiring cost), the share of this surplus to the worker must satisfy the following condition

$$V_{F,t} - V_{u,t} = \xi G_{F,t} \tag{3.49}$$

That is,

$$\xi G_{F,t} = \left[\begin{array}{c} w_{F,t} - \eta \left(L_t\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{L_{F,t}}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} \\ + \left(1-\delta\right) E_t \left\{\Xi_{t,t+1} \left[\xi \left(1 - X_{F,t+1}\right) G_{F,t+1}\right]\right\} \end{array} \right] \tag{3.50}$$

Substituting for $G_{F,t}$ and $G_{F,t+1}$ we obtain the following wage equation for workers employed in the formal sector[13]

$$w_{F,t} = \left\{ \begin{array}{c} \xi Z_{H,t} \Theta X_{F,t}^{\alpha} + \eta \left(L_t\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{L_{F,t}}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} \\ + \left(1-\delta\right) E_t \frac{\left(C_t - bC_{t-1}\right)^{\sigma}}{\left(C_{t+1} - bC_t\right)^{\sigma}} \xi \left(1 - X_{F,t+1}\right) Z_{H,t+1} \Theta X_{F,t+1}^{\alpha} \end{array} \right\} \tag{3.51}$$

The wage rate in the informal sector is determined by the market forces by the equality of the firms' optimization condition where marginal product equals real wage and the households' optimality condition where marginal rate of substitution of consumption for leisure equals real wage.

Labour Market Equilibrium Conditions Labour market equilibrium in the formal sector requires the equality of the demand for and supply of labour to the sector. The demand for labour in the formal sector is derived from the profit maximization problem of the intermediate tradable goods producing firms that operate in the formal sector and is given by

$$w_{F,t} = \frac{P_{HF,t}}{P_t} Z_{H,t} - Z_{H,t} \Theta X_{F,t}^{\alpha} + \left(1-\delta\right) \beta E_t \frac{\left(C_t - bC_{t-1}\right)^{\sigma}}{\left(C_{t+1} - bC_t\right)^{\sigma}} \frac{P_{t+1}}{P_t} Z_{H,t+1} \Theta X_{F,t+1}^{\alpha} \tag{3.52}$$

$$L_{F,t} = \frac{P_{HF,t}}{P_t} Y_{HF,t} \left[\frac{1}{w_{F,t} + Z_{H,t} \Theta X_{F,t}^{\alpha} + \left(1-\delta\right) \beta E_t \frac{\left(C_t - bC_{t-1}\right)^{\sigma}}{\left(C_{t+1} - bC_t\right)^{\sigma}} \frac{P_{t+1}}{P_t} Z_{H,t+1} \Theta X_{F,t+1}^{\alpha}} \right]$$

[13] This equation can be written as $w_{F,t} - \eta \left(L_t\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{L_{F,t}}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} = \xi Z_{H,t} \Theta X_{F,t}^{\alpha} + \left(1-\delta\right) E_t \frac{\left(C_t - bC_{t-1}\right)^{\sigma}}{\left(C_{t+1} - bC_t\right)^{\sigma}} \xi \left(1 - X_{F,t+1}\right) Z_{H,t+1} \Theta X_{F,t+1}^{\alpha}$. Note that in the competitive labour market the left-hand-side, the difference between the real wage and the marginal rate of substitution of consumption for leisure or the labour wedge, is zero. In other words, since in a competitive labour market $X_{F,t} = 0$ for all t, the right-hand-side is always zero establishing that the optimality condition for the worker household is to equate the marginal rate of substitution of consumption for leisure to the real wage.

On the other hand, the supply of labour to the formal sector is given by the wage schedule that is obtained from the value that a household attaches to the job combined with the Nash bargaining process

$$w_{F,t} = \left\{ \begin{array}{c} \xi Z_{H,t}\Theta X_{F,t}^{\alpha} + \eta \left(L_t\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{L_{F,t}}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} \\ + \left(1-\delta\right)\beta E_t \frac{\left(C_t - hC_{t-1}\right)^{\sigma}}{\left(C_{t+1} - hC_t\right)^{\sigma}} \xi \left(1 - X_{F,t+1}\right) Z_{H,t+1}\Theta X_{F,t+1}^{\alpha} \end{array} \right\} \tag{3.53}$$

Similarly, the equilibrium in the informal labour market is determined by the equality of the demand for and the supply of labour to the sector. Accordingly, the demand for labour of the intermediate tradable goods producing firms and the non-tradable goods producing firms are given, respectively, as

$$L_{HI,t} = \frac{P_{HI,t}}{W_{I,t}} Y_{HI,t} \tag{3.54}$$

$$L_{NI,t} = \frac{P_{NI,t}}{W_{I,t}} Y_{NI,t} \tag{3.55}$$

Hence, the market demand for labour in the informal sector is given by the sum of the demand for labour by the two types of firms

$$L_{I,t} = L_{HI,t} + L_{NI,t} = \frac{P_{HI,t}}{W_{I,t}} Y_{HI,t} + \frac{P_{NI,t}}{W_{I,t}} Y_{NI,t} \tag{3.56}$$

On the other hand, the supply of labour to the informal sector is derived from the optimality condition of the household's intertemporal utility maximization. The optimality condition dictates that the household supplies labour up to the point where its marginal rate of substitution between consumption and leisure equals the real wage. For reasons discussed in the household optimization section, this optimality condition holds only for the informal sector.

$$\eta \left[\tau_L^{-\frac{1}{\theta_L}} \left(L_{F,t}\right)^{\frac{1+\theta_L}{\theta_L}} + \left(1-\tau_L\right)^{-\frac{1}{\theta_L}} \left(L_{I,t}\right)^{\frac{1+\theta_L}{\theta_L}} \right]^{\frac{\left(\theta_L\varphi-1\right)}{\left(1+\theta_L\right)}} \left(\frac{L_{I,t}}{1-\tau_L}\right)^{\frac{1}{\theta_L}} \left(C_t - bC_{t-1}\right)^{\sigma} = \frac{W_{I,t}}{P_t} \tag{3.57}$$

$$L_{I,t} = \left(1-\tau_L\right) \left[\frac{W_{I,t}}{P_t} \frac{1}{\eta} \left(L_t\right)^{\left(\frac{1-\varphi-\varphi\theta_L}{1+\theta_L}\right)} \left(C_t - bC_{t-1}\right)^{-\sigma} \right]^{\theta_L}$$

The above expression shows that the value of the marginal products of labour in the two types of firms operating in the informal sector must be equal to the same wage rate.

Price setting behaviour of non-tradable goods producers As highlighted above, markets for both types of intermediate tradable goods are perfectly competitive. Therefore, firms are price takers. The non-tradable goods producing firms are monopolistically competitive firms and they face price setting frictions that is modeled according to Calvo staggered price setting mechanism *à la* Calvo (1983). Accordingly, at a given point in time a random fraction ϵ_N of firms cannot adjust their prices while the remaining $1-\epsilon_N$ can do. Furthermore, we also assume that those firms who can reset their prices are of two types - in the literature referred to as "forward-looking" and "backward - looking" firms. Suppose random fractions ς_N of firms in the non-tradable goods sector set their prices based on rules of thumb using their knowledge of the historical development of price levels (hence, backward looking). Then, fractions $(1-\varsigma_N)$ of firms in the non-tradable goods sector are "forward-looking" and set their prices according to the Calvo price setting mechanism. Combining this assumptions with the optimal prices of firms who can and cannot reset their prices, after some algebraic manipulations, yields the hybrid New Keynesian Phillips Curve developed by Gali and Gertler (1999)[14]. Let $\pi_t = \ln P_t - \ln P_{t-1}$ be the inflation rate, then for the non-tradable goods, this equation is given by

$$\pi_{N,t} = \kappa_{b,N}\pi_{N,t-1} + \kappa_{F,N}E_t\pi_{N,t+1} + \lambda_N mc_{N,t} \tag{3.58}$$

where

$$\begin{aligned}
\kappa_{b,N} &= \frac{\varsigma_N}{\epsilon_N + \varsigma_N(1-\epsilon_N(1-\beta))}, \\
\kappa_{F,N} &= \frac{\beta\epsilon_N}{\epsilon_N + \varsigma_N(1-\epsilon_N(1-\beta))}, \\
\lambda_N &= \frac{(1-\varsigma_N)(1-\epsilon_N)(1-\beta\epsilon_N)}{\epsilon_N + \varsigma_N(1-\epsilon_N(1-\beta))}.
\end{aligned}$$

3.4.2 Final tradable goods producing and exporting firms

There is a continuum of monopolistically competitive firms given in the unit interval that combine the two types of domestically produced intermediate tradable goods [or as it is commonly referred to as in the literature they "package" or "brand name"] to produce

[14] For detailed derivations of the Hybrid New Keynesian Phillips Curve for small open economy, see Holmberg (2006).

differentiated tradable goods that are sold on both domestic and foreign markets. The production function used by the i^{th} firm to produce the i^{th} final tradable goods can be given by the following CES production function

$$Y^i_{H,t} = \left[(1-\gamma_3)^{\frac{1}{\theta_3}} \left(Y^i_{HF,t}\right)^{\frac{(\theta_3-1)}{\theta_3}} + (\gamma_3)^{\frac{1}{\theta_3}} \left(Y^i_{HI,t}\right)^{\frac{(\theta_3-1)}{\theta_3}}\right]^{\theta_3/(\theta_3-1)} \tag{3.59}$$

where $Y^i_{H,t}$ is the specialized final tradable goods produced by firm i, $Y^i_{HF,t}$ and $Y^i_{HI,t}$, respectively, are the intermediate tradable goods produced by firms operating in the formal and the informal sectors and used as inputs by final tradable goods producer, firm i. θ_3 measures the elasticity of substitution between the two inputs and γ_3 is the share parameter. The aggregate production function for final tradable goods may be given by

$$\int_0^1 Y^i_{H,t} di = Y_{H,t} = \left[(1-\gamma_3)^{\frac{1}{\theta_3}} \left(Y_{HF,t}\right)^{\frac{(\theta_3-1)}{\theta_3}} + (\gamma_3)^{\frac{1}{\theta_3}} \left(Y_{HI,t}\right)^{\frac{(\theta_3-1)}{\theta_3}}\right]^{\theta_3/(\theta_3-1)}$$

The demand for the two inputs of production (the intermediate tradable goods) can be derived by solving the objective function of firms that minimize the expenditure on the two inputs while producing certain level of differentiated output. This expenditure minimization yields the conditional demand functions for the two varieties of intermediate tradable goods as in the following equations:

$$Y_{HF,t} = (1-\gamma_3)\left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\theta_3} Y_{H,t} \tag{3.60}$$

$$Y_{HI,t} = \gamma_3\left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\theta_3} Y_{H,t} \tag{3.61}$$

where $P_{HF,t}$, $P_{HI,t}$ and $P_{H,t}$ are, respectively, prices of $Y_{HF,t}$, $Y_{HI,t}$ and $Y_{H,t}$. We assume that the only cost incurred by firms that are producing final tradable goods are the expenditure on the intermediate tradable goods. Therefore, the marginal cost of firms that are producing final tradable goods is given by[15]

$$MC_{H,t} = \left[(1-\gamma_3)\left(P_{HF,t}\right)^{1-\theta_3} + \gamma_3\left(P_{HI,t}\right)^{1-\theta_3}\right]^{\frac{1}{(1-\theta_3)}} \tag{3.62}$$

[15] Detailed derivation of the final tradable goods production and pricing process is given in a separate technical appendix to this paper.

3.5 Importing Firms

As with the non-tradable and final tradable goods producing firms discussed above, we assume that there are a continuum of monopolistically competitive firms that import and distribute foreign goods. Each importing firm buys homogenous goods from foreign firms and produce (package them into) differentiated products and sell to domestic consumers. The existence of market power with the fact that prices are set to the market (prices are set in terms of domestic currency), the price index of imports in domestic currency is no longer equal to the nominal exchange rate times the foreign price index- the Law of One Price Gap discussed in earlier sections. These firms, like the domestic final goods producers, face price setting friction that is captured via Calvo price setting mechanism that we discussed above. Suppose, at a given point in time a random fraction ϵ_M of firms cannot adjust their prices while the remaining $1-\epsilon_M$ can do. Furthermore, assume that of those firms who can reset their prices fraction ς_M of firms are "backward looking" while the fraction $(1-\varsigma_M)$ of firms are "forward-looking". Fallowing the same procedure discussed for non-tradable goods producing firms, the rate of inflation in the average domestic currency price of imports is given by the following equation:

$$\pi_{M,t} = \kappa_{b,M}\pi_{M,t-1} + \kappa_{f,M}E_t\pi_{M,t+1} + \lambda_M\psi_t \tag{3.63}$$

where

$$\begin{aligned}
\kappa_{b,M} &= \frac{\varsigma_M}{\epsilon_M + \varsigma_M(1-\epsilon_M(1-\beta))},\\
\kappa_{f,M} &= \frac{\beta\epsilon_M}{\epsilon_M + \varsigma_M(1-\epsilon_M(1-\beta))},\\
\lambda_M &= \frac{(1-\varsigma_M)(1-\epsilon_M)(1-\beta\epsilon_M)}{\epsilon_M + \varsigma_M(1-\epsilon_M(1-\beta))}.
\end{aligned}$$

This implies that the inflation dynamics of the tradable goods in the economy can be derived from the weighted average of the inflation in the home produced tradable and imported goods inflation and the weights are given by the proportion of these goods in the consumption of households as given by (3.10). Log-linearizing (3.10) around a steady-state and subtracting the lags from both sides gives the following equation of the New Keynesian Phillips Curve of tradable goods:

$$\pi_{T,t} = (1-\gamma_2)\,\pi_{H,t} + \gamma_2\pi_{M,t}$$

$$\pi_{T,t} = \pi_{H,t} - \gamma_2 \left(\pi_{H,t} - \pi_{M,t}\right) = \pi_{H,t} - \gamma_2 \left(v_t - v_{t-1}\right) \tag{3.64}$$

Similarly, the overall inflation rate of the economy can be given by log-linearizing (3.6) around a steady-state and subtracting the lags from both sides which is the average of the inflation in tradable and non-tradable goods

$$\pi_t = \left(1 - \gamma_1\right) \pi_{T,t} + \gamma_1 \pi_{N,t} \tag{3.65}$$

3.6 Goods market clearing conditions

Goods market clearing in the domestic economy requires that domestic output is equal to the sum of domestic consumption and foreign consumption of domestically produced goods or exports. This implies

$$Y_t = Y_{H,t} + Y_{N,t} = C_{H,t} + C^*_{H,t} + C_{N,t} \tag{3.66}$$

We know that

$$C_{H,t} = (1 - \gamma_2) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta_2} C_{T,t} \text{ and, in turn, } C_{T,t} = (1 - \gamma_1) \left(\frac{P_{T,t}}{P_t}\right)^{-\theta_1} C_t$$

therefore we obtain

$$C_{H,t} = \left(1 - \gamma_1\right) \left(1 - \gamma_2\right) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta_2} \left(\frac{P_{T,t}}{P_t}\right)^{-\theta_1} C_t \tag{3.67}$$

Given the domestic consumption of domestically produced tradable goods as

$$C_{H,t} = (1 - \gamma_2) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta_2} C_{T,t}$$

following Liu (2006) we argue that the foreign consumption of domestically produced tradable goods (exports) must be

$$C^*_{H,t} = \gamma_2 \left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\theta_2} C^*_t = \gamma_2 \left(\frac{P_{H,t}}{Q_t P_t}\right)^{-\theta_2} C^*_t \tag{3.68}$$

In the non-tradable sector the market clearing condition is given by the equality of production and consumption

$$Y_{N,t} = C_{N,t} \tag{3.69}$$

3.7 Monetary policy rules

We use the simple Taylor type rule where the monetary authority is assumed to act to stabilize inflation, output and exchange rate

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_r} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y} \left(\frac{\varepsilon_t}{\varepsilon_{t-1}}\right)^{\phi\Delta e}\right]^{(1-\rho_r)} \epsilon_{r,t} \tag{3.70}$$

According to this rule, the monetary authority adjusts the nominal interest rate in response to current inflation rate, deviation of output from some target level, and the appreciation/depreciation/ of the nominal exchange rate. ϕ_π, ϕ_y, and $\phi_{\Delta e}$ are weights put by monetary authority, respectively, on inflation, target GDP, and change in the nominal exchange rate. The lagged interest rate serves for interest rate smoothing while ρ_r denotes the extent of persistence of interest rate. The monetary policy shock is captured by $\epsilon_{r,t}$ which is independently, identically distributed normal error term with zero mean and standard deviation $\sigma_{\epsilon r}$.

3.8 The external sector

The small open economies by definition are small relative to the rest of the world and hence they cannot affect the foreign variables like income, inflation, interest rate, etc. Therefore, the foreign economy can be modelled as exogenous. Following the literature in this area, we assume that the foreign variables GDP, inflation, and policy interest rate follow first order autoregressive processes in their logarithms.

3.9 Log-linear approximation to the model[16]

We log-linearized the equations that characterize the equilibrium conditions of the model around a steady-state to reduce the computational complexity of the original model that makes it difficult to solve and simulate. Note that all lower-cases indicate log-deviation from a steady state, i.e., $x_t \equiv \ln X_t - \ln \overline{X}$ where $\overline{X}$ is the steady state value of X. The exceptions are the real wages and the unemployment rates for which the log-deviations are expressed as the respective variables with a hat. Accordingly, in the log-linearized

[16] Detailed derivation of all the log-linearized model equations and calculations of some key steady-state conditions are given in a separate technical appendix.

model we have c_t, $y_{HF,t}$, $y_{HI,t}$, $y_{H,t}$, $y_{N,t}$, y_t, l_t, $l_{F,t}$, $l_{I,t}$, $\widehat{u}_t$, $x_{F,t}$, $g_{F,t}$, $\widehat{w}_{F,t}$, $\widehat{w}_{I,t}$, $z_{H,t}$, $z_{N,t}$, $mc_{HF,t}$, $mc_{HI,t}$, $mc_{H,t}$, $mc_{N,t}$, $\pi_{H,t}$, $\pi_{N,t}$, $\pi_{M,t}$, $\pi_{T,t}$, π_t, Δe, $\Delta\psi$, v_t, q_t, μ_t, r_t, y_t^*, π_t^*, r_t^* in the equations that follow.

Consumption

$$c_t = \frac{b}{1+b}c_{t-1} + \frac{1}{1+b}E_t[bc_t + y_{t+1}^* - by_t^* + \frac{(1-b)}{\sigma}q_{t+1}] - \frac{1-b}{\sigma(1+b)}(r_t - E_t\pi_{t+1})$$

Production

$$y_{HF,t} = l_{F,t} + z_{H,t} \tag{3.71}$$

$$y_{HI,t} = l_{HI,t} + z_{H,t} \tag{3.72}$$

$$y_{N,t} = l_{NI,t} + z_{N,t} \tag{3.73}$$

The resource (labour input) constraint is given by

$$l_t = \tau_L l_{F,t} + (1-\tau_L) l_{I,t} \tag{3.74}$$

where

$$l_{I,t} = \tau_{HI} l_{HI,t} + (1-\tau_{HI}) l_{NI,t}$$

where τ_L and $(1-\tau_L)$ are, respectively, $\frac{\overline{L}_F}{\overline{L}}$ and $\frac{\overline{L}_I}{\overline{L}}$ while τ_{HI} and $(1-\tau_{HI})$ are, respectively, $\frac{\overline{L}_{H,I}}{\overline{L}_I}$ and $\frac{\overline{L}_{NI}}{\overline{L}_I}$.

Total factor productivity in the two sectors can be log-linearized to yield

$$z_{H,t} = \rho_H z_{H,t-1} + \epsilon_{H,t} \tag{3.75}$$

and

$$z_{N,t} = \rho_N z_{N,t-1} + \epsilon_{N,t} \tag{3.76}$$

The final tradable goods production is given by

$$y_{Ht} = (1-\gamma_3)\, y_{HF,t} + \gamma_3 y_{HI,t} \tag{3.77}$$

Marginal cost for the intermediate tradable goods produced in the formal sector $mc_{HF,t}$ is log-linearized to yield

$$\frac{1}{\overline{MC}_{HF}} \left[\begin{array}{c} \overline{w}_F \widehat{w}_{F,t} + \alpha\Theta\overline{X}_F^{\alpha} x_{F,t} - \overline{w}_F z_{H,t} \\ + (1-\delta)\,\beta\Theta\overline{X}_F^{\alpha}\left(z_{H,t} + \frac{\sigma b}{1-b}c_{t-1} - \frac{\sigma(1+b)}{1-b}c_t + \frac{\sigma}{1-b}c_{t+1} - \pi_{t+1} - \alpha x_{F,t+1} - z_{H,t+1}\right) \end{array} \right] \tag{3.78}$$

where $\overline{MC}_{HF} = \overline{w}_F + \Theta\overline{X}_F^{\alpha} - (1-\delta)\,\beta\Theta\overline{X}_F^{\alpha}$.

The marginal cost for the tradable goods produced by the firms operating in the informal sector is given by

$$mc_{HI,t} = \widehat{w}_{I,t} - z_{H,t} \tag{3.79}$$

The log-linearized version of the marginal cost of the final tradable goods is given by

$$mc_{H,t} = (1-\gamma_3)\,mc_{HF,t} + \gamma_3 mc_{HI,t} \tag{3.80}$$

Similarly, the marginal cost for the non-tradable goods is given by

$$mc_{NI,t} = \widehat{w}_{I,t} - z_{N,t} \tag{3.81}$$

Goods market clearing condition

$$y_{H,t} = c_{H,t} + c^*_{H,t}$$

$$c_{H,t} = -\gamma_2(\theta_2 - \theta_1\gamma_1)v_t + \theta_1\gamma_1\,(p_{N,t} - p_{H,t}) + c_t$$

$$c^*_{H,t} = -\theta_2\gamma_2(1-\gamma_1)v_t + \theta_2\gamma_1\,(p_{N,t} - p_{H,t}) + c^*_t + \theta_2 q_t$$

$$y_{H,t} = -\gamma_2(\theta_2-\theta_1\gamma_1)v_t+\theta_1\gamma_1\,(p_{N,t} - p_{H,t})+c_t-\theta_2\gamma_2(1-\gamma_1)v_t+\theta_2\gamma_1\,(p_{N,t} - p_{H,t})+c^*_t+\theta_2 q_t \tag{3.82}$$

And

$$y_{N,t} = c_{N,t} = -\theta_1\gamma_2(1-\gamma_1)v_t + \theta_1(\gamma_1 - 1)(p_{N,t} - p_{H,t}) + c_t \tag{3.83}$$

Finally,

$$y_t = (1-\gamma_1)\,y_{H,t} + \gamma_1 y_{N,t} \tag{3.84}$$

Labour market

Log-linearizing the labour market tightness and the hiring cost, we obtain, respectively

$$x_t = \frac{1}{\delta}(l_{F,t} - (1-\delta)\,l_{F,t-1}) - \widehat{U}_t \tag{3.85}$$

where

$$\widehat{U}_t = -\frac{(1-\delta)\left(\overline{L}_F l_{F,t-1} + \overline{L}_I l_{I,t-1}\right)}{1-(1-\delta)\left(\overline{L}_F + \overline{L}_I\right)} \tag{3.86}$$

and

$$g_{F,t} = z_{H,t} + \alpha x_t \tag{3.87}$$

Note that U_t in the above equations is the beginning of period unemployment rate, as discussed in household's preference section of the paper. The log-linearized version of the end of period unemployment rate is given by

$$\widehat{u}_t = -\frac{\left(\overline{L}_F l_{F,t} + \overline{L}_I l_{I,t}\right)}{\overline{u}} \tag{3.88}$$

Log-linearizing the demand for labour in the formal and the informal sector we obtain, respectively,

$$\widehat{w}_{F,t} = \frac{1}{\overline{w}_F}\begin{bmatrix} \frac{1}{\overline{P}}\overline{MC}_{HF} mc_{HF,t} - \frac{\overline{MC}_{HF}}{\overline{P}} p_t - (1-\delta)\beta\Theta\overline{X}_F^{\alpha} p_t + \frac{\overline{MC}_{HF}}{\overline{P}} z_{H,t} - \Theta\overline{X}_F^{\alpha} z_{H,t} \\ +\alpha\Theta\overline{X}_F^{\alpha} x_{F,t} + (1-\delta)\beta\left(\frac{\sigma+\sigma b}{1-b}\right)\Theta\overline{X}_F^{\alpha} c_t - (1-\delta)\beta\left(\frac{\sigma b}{1-b}\right)\Theta\overline{X}_F^{\alpha} c_{t-1} \\ -(1-\delta)\beta\left(\frac{\sigma}{1-b}\right)\Theta\overline{X}_F^{\alpha} c_{t+1} + (1-\delta)\beta\Theta\overline{X}_F^{\alpha} p_{t+1} + (1-\delta)\beta\Theta\overline{X}_F^{\alpha} z_{H,t+1} \\ +\alpha(1-\delta)\beta\Theta\overline{X}_F^{\alpha} x_{F,t+1} \end{bmatrix} \tag{3.89}$$

and

$$l_{I,t} = \left[\frac{1}{\overline{L}_I}\frac{\overline{Y}_{HI}}{\overline{P}\overline{w}_I}\overline{MC}_{HI} mc_{HI,t} - \widehat{w}_{I,t} - p_t + \frac{\overline{Y}_{HI}}{\overline{L}_I}\frac{\overline{MC}_{HI}}{\overline{P}\overline{w}_I} y_{HI,t} + \frac{1}{\overline{L}_I}\frac{\overline{Y}_N}{\overline{w}_I} p_{N,t} + \frac{1}{\overline{L}_I}\frac{\overline{Y}_N}{\overline{w}_I} p_{N,t}\right] \tag{3.90}$$

On the other hand, the supply of labour to the formal sector can be log-linearized to yield

$$\widehat{w}_{F,t} = \frac{1}{\overline{w}_F}\begin{bmatrix} \xi\Theta\overline{X}_F^{\alpha} z_{H,t} + \alpha\xi\Theta\overline{X}_F^{\alpha} x_{F,t} \\ +\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}\eta\left(\overline{L}\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}}\left(\overline{C}(1-b)\right)^{\sigma}\left(\frac{\overline{L}_F}{\tau}\right)^{\frac{1}{\theta_L}} l_t \\ +\frac{1}{\theta_L}\eta\left(\overline{L}\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}}\left(\frac{1}{\tau_L}\right)^{\frac{1}{\theta_L}}\left(\overline{C}(1-b)\right)^{\sigma}\overline{L}_F^{\frac{1}{\theta_L}} l_{F,t} \\ +\left(\frac{1}{(1-b)}\begin{bmatrix} \sigma\eta\left(\overline{L}\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}}\left(\frac{\overline{L}_F}{\tau_L}\right)^{\frac{1}{\theta_L}}\left(\overline{C}(1-b)\right)^{\sigma} \\ +\beta(1-\delta)(\sigma+\sigma b)\xi\left(1-\overline{X}_F\right)\Theta\overline{X}_F^{\alpha} \end{bmatrix} c_t\right) \\ +\left(\frac{1}{(1-b)}\begin{bmatrix} -\sigma b\eta\left(\overline{L}\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}}\left(\frac{\overline{L}_F}{\tau_L}\right)^{\frac{1}{\theta_L}}\left(\overline{C}(1-b)\right)^{\sigma} \\ -\sigma b\beta(1-\delta)\xi(1-\overline{x}_F)\Theta\overline{X}_F^{\alpha} \end{bmatrix} c_{t-1}\right) \\ -\sigma\beta(1-\delta)\frac{1}{(1-b)}\xi(1-\overline{x}_F)\Theta\overline{X}_F^{\alpha} c_{t+1} \\ +\beta(1-\delta)\xi\left(1-\overline{X}_F\right)\Theta\overline{X}_F^{\alpha} z_{H,t+1} \\ \beta(1-\delta)\xi[\alpha-(\alpha+1)\overline{x}_F]\Theta\overline{X}_F^{\alpha} x_{F,t+1} \end{bmatrix} \tag{3.91}$$

where $\overline{w}_F = \xi \overline{Z}_H \Theta \overline{X}_F^{\alpha} + \eta \left(\overline{L}\right)^{\frac{\varphi+\varphi\theta_L-1}{1+\theta_L}} \left(\frac{\overline{L}_F}{\tau_L}\right)^{\frac{1}{\theta_L}} \left(\overline{C}(1-b)\right)^{\sigma} + (1-\delta_F)\beta\xi\left(1-\overline{X}_F\right)\overline{Z}_H\Theta\overline{X}_F^{\alpha}$.
And the log-linearized version of the labour supply to the informal sector is given by

$$\widehat{w}_{I,t} = \frac{\varphi+\varphi\theta_L-1}{1+\theta_L} l_t + \frac{1}{\theta_L} l_{I,t} + \frac{\sigma}{1-b} c_t - \frac{\sigma b}{1-b} c_{t-1} \tag{3.92}$$

Domestically produced tradable goods inflation

$$\pi_{H,t} = \kappa_{b,H}\pi_{T,t-1} + \kappa_{F,H} E_t \pi_{H,t+1} + \lambda_H mc_{H,t} \tag{3.93}$$

Non-tradable goods inflation

$$\pi_{N,t} = \kappa_{b,N}\pi_{N,t-1} + \kappa_{F,N} E_t \pi_{N,t+1} + \lambda_N mc_{N,t} \tag{3.94}$$

Imported inflation

$$\pi_{F,t} = \kappa_{b,F}\pi_{F,t-1} + \kappa_{f,F} E_t \pi_{F,t+1} + \lambda_F \psi_{F,t} \tag{3.95}$$

Tradable goods inflation

$$\pi_{T,t} = (1-\gamma_2)\,\pi_{H,t} + \gamma_2 \pi_{F,t} \tag{3.96}$$

Overall CPI inflation

$$\pi_t = (1-\gamma_1)\,\pi_{T,t} + \gamma_1 \pi_{N,t} \tag{3.97}$$

The evolution of law of one price gap

$$\psi_t - \psi_{t-1} = e_t - e_{t-1} + \pi_t^* - \pi_{F,t} \tag{3.98}$$

The evolution of the terms of trade

$$v_t = v_{t-1} + \pi_{F,t} - \pi_{H,t} + \epsilon_{tot,t} \tag{3.99}$$

The relationship between real exchange rate and terms of trade

$$q_t = \psi_t - (1-\gamma_2(1-\gamma_1))\,v_t - \gamma_1\left(p_{N,t} - p_{H,t}\right) \tag{3.100}$$

The Uncovered Interest Parity Condition

$$E_t e_{t+1} - e_t = r_t - r_t^* + \epsilon_{uip,t} \tag{3.101}$$

The evolution of price differential between the tradable and non-tradable sectors of the domestic economy

$$\mu_t = p_{N,t} - p_{H,t}$$
$$\mu_t = \mu_{t-1} + \pi_{N,t} - \pi_{H,t} \tag{3.102}$$

Monetary policy rule

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t + \phi_e \Delta e_t) + \epsilon_{r,t} \tag{3.103}$$

The rest of the world

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_{y^*,t},\ 0< \rho_{y^*} <1 \tag{3.104}$$
$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_{\pi^*,t},\ 0< \rho_{\pi^*} <1 \tag{3.105}$$
$$r_t^* = \rho_{R^*} r_{t-1}^* + \epsilon_{r^*,t},\ 0< \rho_{r^*} <1 \tag{3.106}$$

where π_t^*, and r_t^* represent the foreign economy variables inflation and interest rate, respectively. y_t^* is the log-deviation of foreign GDP from its steady-state and $\epsilon_{i,t}$ is an independently and identically distributed normal error term with zero mean and standard deviation of σ_i, where i stands for y_t^*, π_t^* and r_t^*.

Now once the values of the parameters are fixed the model can be solved and simulated to assess the responses of the economy to both policy and non-policy shocks. Therefore, in the next section we address the calibration and simulation of the model.

4 Calibration and simulation

4.1 Calibration of parameters

In addition to being theoretically consistent, we would like our model to be in accordance with stylized facts of the economies that are characterized by dual labour market. Accordingly, to solve and simulate the model and then to assess the dynamics of some fundamental macroeconomic variables in response to various shocks, parameters of the model are calibrated. Most of the parameter values are from the literature on emerging and low-income economies. However, there is no literature available on some of the model parameters of this study, such as the parameters of the labour market friction and

price stickiness. For some of the model parameters for which there are no references, unavoidably, the values are assigned based on subjective judgment using the values of the parameters reported in the literature on developed countries as a point of reference. Some of the parameters are calculated to match the stylized facts of the economies in the region. For example, the labour market tightness and separation rate are determined by assuming that the long-run average rate of unemployment for the region is equal to the steady-state unemployment rate. Similarly, the proportions of the labour force employed in the formal and informal sector together with the proportions employed in the production of tradable and non-tradable goods in the informal sector are calculated based on the long-run averages from the ILO Key Indicators of Labour Markets (KILM) database. The DYNARE[17] toolbox (Adjemian et al. (2012)) is used to solve the model numerically and generate the impulse response functions to different domestic and external shocks. The complete list of the parameters of the model and their values are given in Table 1 below.

Table 1: Model parameter values*

$\alpha = 1.5$	$\gamma_2 = 0.3$	$\phi_e = 0.80$
$\beta = 0.9951$	$\gamma_3 = 0.5$	$\rho_{ZH} = 0.90$
$\Theta = 2.5$	$\varsigma_F = 0.20$	$\rho_{ZN} = 0.90$
$\sigma = 2.61$	$\varsigma_H = 0.75$	$\rho_{y^*} = 0.75$
$\varphi = 3$	$\varsigma_N = 0.80$	$\rho_{\pi^*} = 0.60$
$\eta = 0.24$	$\epsilon_F = 0.40$	$\rho_{r^*} = 0.66$
$\delta = 0.2$	$\epsilon_H = 0.45$	$\omega = 0.95$
$\theta_1 = 12$	$\epsilon_N = 0.10$	$\tau_L = 0.10$
$\theta_2 = 12$	$\xi = 0.50$	$\tau_{HI} = 0.70$
$\theta_3 = 12$	$\rho_r = 0.82$	$\theta_L = 0.75$
$b = 0.25$	$\phi_y = 0.50$	
$\gamma_1 = 0.731$	$\phi_\pi = 0.30$	

*The sources of the values for the parameters are given in the Appendix

[17]DYNARE (version 4.3.0, released in 2012) is used in this paper. DYNARE is a free MATLAB toolkit to solve, simulate and estimate DSGE and a wide variety of other models. It is downloadable at http://www.dynare.org/.

4.2 The Impulse Responses

In this section we present the impulse responses of key macroeconomic variables in response to six types of shocks. Three of these shocks are external and the other three are domestic. The external shocks considered are the shocks to three foreign variables included in the model: income, inflation, and interest rate. The domestic shocks are the productivity shocks to the tradable and non-tradable goods, and the domestic monetary policy shock. In all cases, we consider positive shocks of the same magnitude (a 1 percent shock). Since the model is log-linearized, the responses are interpreted as percentage deviations of the variable concerned to a 1 percentage point increase in the variable hit by the shock. Furthermore, all shocks are temporary, which implies that the variables are expected to come back to their steady-state (in this case zero) after the shock.

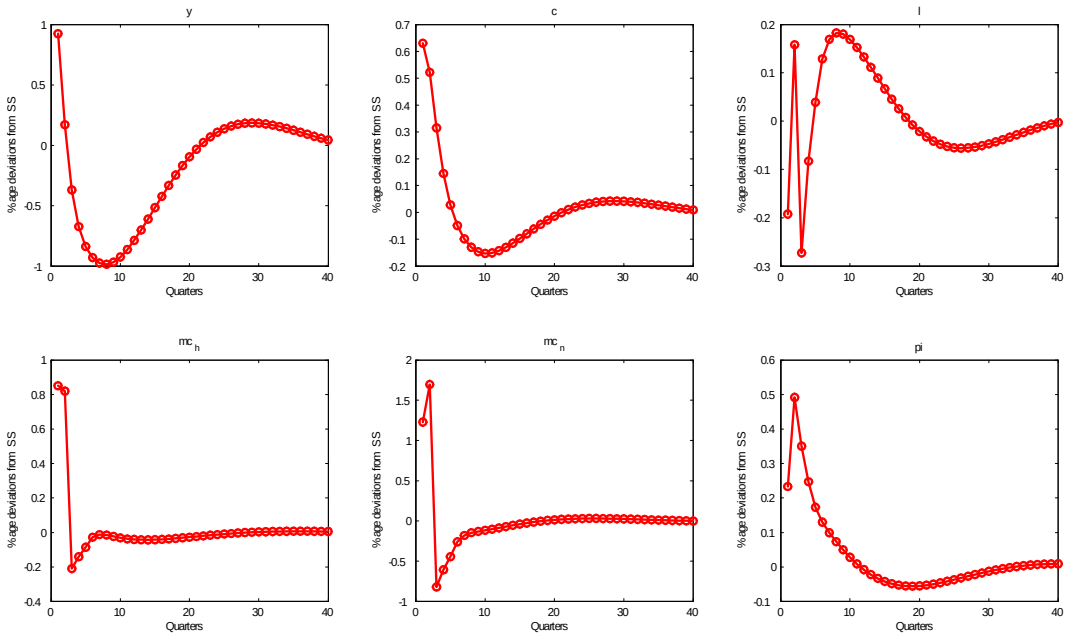

Figure 1: Impulse responses to a foreign income shock

Figure 1[18] shows the impulse responses to foreign income shock. Economic theory

[18]Note: y =income, c = consumption, l = labour, mc_h = marginal cost of tradable sector, mc_n = marginal cost of non-tradable sector, pi = inflation rate.

predicts that an increase in income of the rest of the world leads to an increasing income, consumption, and employment in a small open economy due to various reasons. For instance, income of the home country increases since increasing income of the rest of the world means increasing demand for the exports of the home country. This increased demand will also lead to increasing employment. Consumption increases through the international risk sharing introduced into the model. However, both consumption and income fall eventually and the response changes to negative (after 3 quarters for income and after 6 quartes for consumption). The process may be explained as follows. The increasing foreign income leads to an increasing demand for the exports of the home country. This, in turn, leads to an increasing demand for labour by firms that produce tradable goods and hence puts upward pressure on the marginal cost of the tradable goods. At the same time, when the demand for domestically produced tradable goods (exports) increases, households substitute some of their consumption of these goods by non-tradable goods since domestic consumers consume both tradable and non-tradable goods and substitute one for the other depending on their relative prices. This leads to the same process we indicated for tradable goods: increasing demand for labour by firms that produce non-tradable goods, therefore, increasing marginal cost. The increasing marginal cost forces firms to reduce their output (the fall in income) since they cannot adjust their prices automatically. Developments in both sectors in response to a foreign income shock put upward pressure on the overall inflation rate relative to its steady-state value. This leads monetary authority to respond by raising the policy interest rate which helps the economy to adjust back to the steady-state.

As can be seen from Figure 2, the initial effect of a foreign inflation shock is that output and consumption respond positively while employment, marginal costs, and inflation respond negatively. The increase in inflation in the rest of the world, other things remaining the same, is expected to have two effects on the domestic economy. First, the inflationary pressure in the rest of the world leads to improvement of the competitiveness of the domestic economy since the initial impact of this event is a depreciation of the real exchange rate of the domestic economy. This leads to an increasing demand for tradable goods of the domestic economy by foreigners which in turn leads to an increasing demand for labour and, therefore, increasing marginal cost. Second, the same event makes

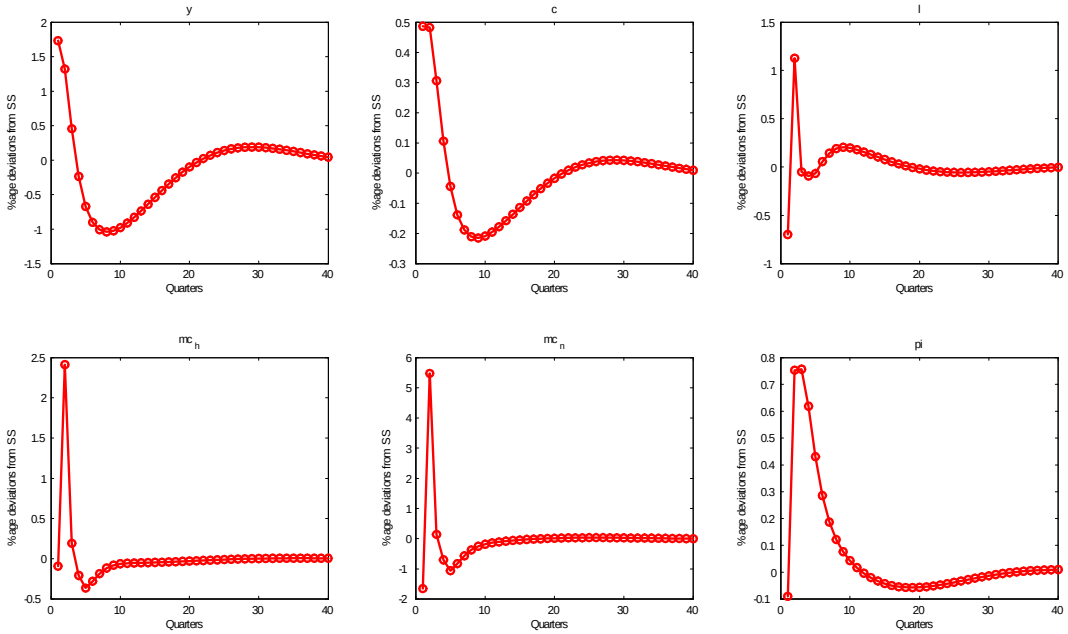

Figure 2: Impulse responses to a foreign inflation shock

imported consumption goods more expensive for domestic consumers. As highlighted above, households attempt to substitute some goods for others depending on their relative prices. Accordingly, when the exchange rate depreciates and the demand from foreign consumers for domestic tradable goods (exports) increases and imports become expensive in domestic currency, domestic households substitute part of their consumption of tradable goods by non-tradable goods. This leads to increasing demand for non-tradable goods. Both effects reinforce each other and lead the output of the domestic economy and to some extent consumption to increase. The overall effect of the event on marginal costs and inflation seems to be higher in our model compared to the effect of the same event in the standard small open economy New Keynesian models. This is due partly to the duality of the labour market introduced in our model. In the standard model, the labour market is single and homogenous which implies that there is perfect mobility of labour across the sectors and, therefore, equalization of wage rates. Therefore, when there is an increasing demand for labour in one sector, the wage rate (and, therefore, marginal cost) increases in both sectors. But the adjustment is fast. In the segmented

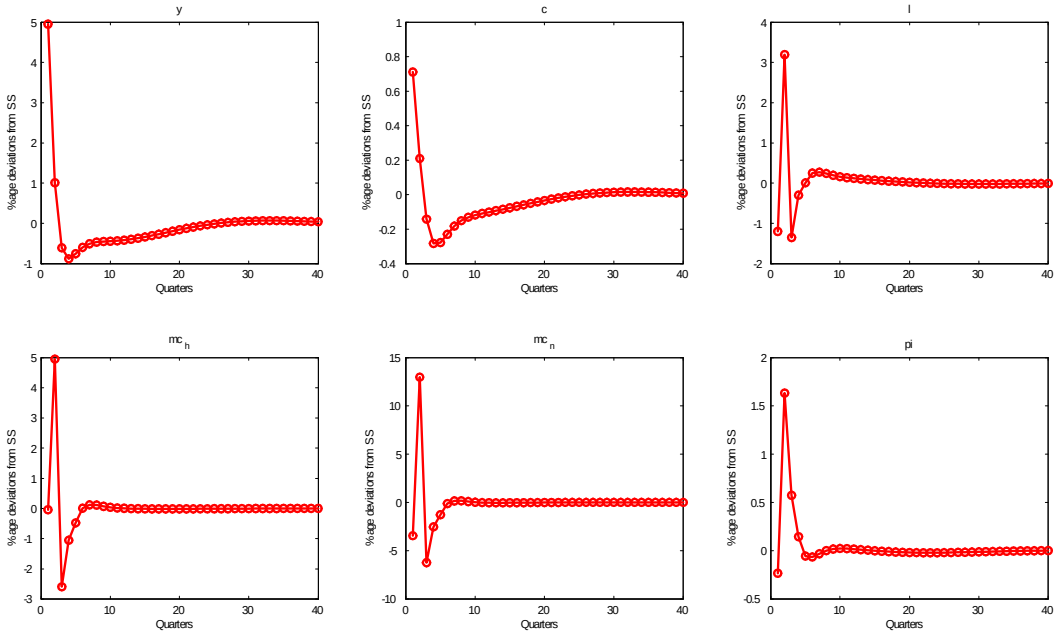

Figure 3: Impulse responses to a foreign monetary policy shock

labour market, due to absence of perfect mobility of labour (limited mobility of labour between the two sectors) the impact on marginal costs is stronger than in the standard model.

Similarly, the responses of most of the variables to the foreign monetary policy shock (increasing the foreign policy interest rate, say, due to contractionary monetary policy), shown in Figure 3, are consistent with what is expected based on economic theory. The foreign monetary policy shock has two effects: an effect on consumption and income through the international risk sharing, and another effect through the depreciation of the nominal exchange rate of the domestic economy. That is, as the interest rate of the rest of the world increases consumption increases through the international risk sharing which leads to increasing demand for goods. Furthermore, the increasing foreign interest rate leads to depreciation of domestic currency that leads to increasing demand for exports of the home country and decreasing domestic demand for foreign goods. However, the increasing demand for exports leads to an increasing demand for labour by firms operating in the formal sector which in turn leads to increasing labour market tightness. The

increasing labour market tightness leads to increasing cost of hiring new workers and, therefore, marginal cost. This forces firms to reduce their production. Eventually the flow of workers from the informal sector to the unemployment pool (to search for job in the formal sector) and the intervention from monetary authority by decreasing the interest rate leads the economy to adjust to the steady-state.

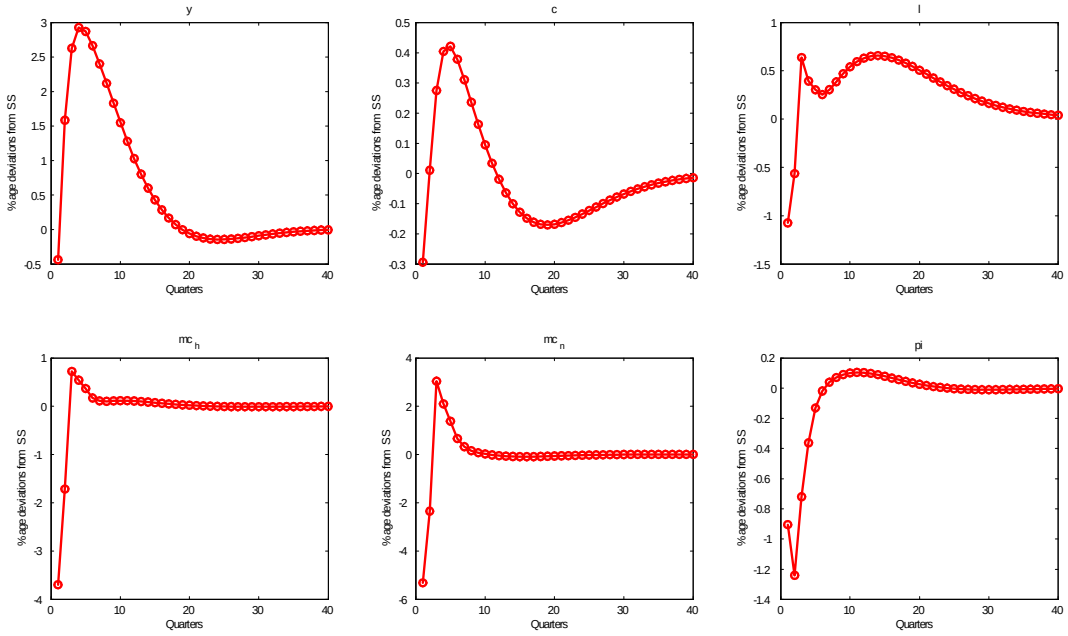

Figure 4: Impulse responses to a productivity shock (tradable goods)

Figure 4 and 5 show the responses to the shocks in the productivity of the tradable and non-tradable goods sectors, respectively. Closer examination of these figures shows that the productivity shocks to both tradable and the non-tradable goods sectors generate responses of the variables that seem counterintuitive. That is, the initial impact of the productivity shock in both sectors is contractionary though one expects output and consumption to increase and employment, marginal costs, and inflation to fall. However, these responses to the shock in productivity are in line with some findings of the literature. For example, Basu et al. (2006) report that the initial impact of an improvement in productivity could be contractionary when firms cannot adjust their prices in a sticky price model. They document that output and employment increase in the log-run

in response to the improvement to technology, once firms make adjustment and prices change.

After the initial fall in output and employment, the economy starts expanding once all firms and consumers make adjustments to the technology shock. Since the model is an open economy model, this expansionary effect of the productivity shocks is strengthened by its effect on the real exchange rate of the economy. That is, the increased productivity reduces the real marginal cost of firms. This leads to decreasing prices since prices are constant mark-ups over the marginal cost which, in turn, leads to a significant depreciation of the real exchange rate. The depreciation of the real exchange rate results into an increase in exports and a fall in imports.

However, this process is slowed and then reversed by the labour market friction. That is, during the initial expansion period the economy is just absorbing (or fully utilizing) the labour that was made redundant by the technology shock. Once this absorption is complete, the expansion in production in the formal sector leads to an increasing demand for labour. The increasing demand for labour, in turn, leads to increasing hiring cost since labour market tightness increases. This will lead to increasing marginal costs of firms that are operating in the formal sector and forces them to reduce production. This same effect sends a signal to the workers in the informal sector to separate from their informal sector jobs and search for jobs in the formal sector. This process eases the labour market tightness in the formal sector which leads to decreasing marginal costs in the sector. Therefore, the eventual fall in output and consumption as well as the eventual rise in employment can be explained by the existence of the labour market friction.

As can be seen from comparing Figures 4 and 5, the contractionary effect is pronounced in the non-tradable sector. This can be explained by the relative shares of the prices of tradable and non-tradable goods in the aggregate price index and, therefore, the magnitude of their relative effects on the real exchange rate. As the proportion of non-tradable goods in the consumer price index is very large ($\gamma_1 = 0.731$) compared to the domestically produced tradable goods ($(1-\gamma_2)(1-\gamma_1) = 0.1883$), the increasing productivity in the non-tradable goods sector affects the aggregate price level significantly and thereby the real exchange rate (see also Dotsey and Duarte (2008) for discussion on similar issue).

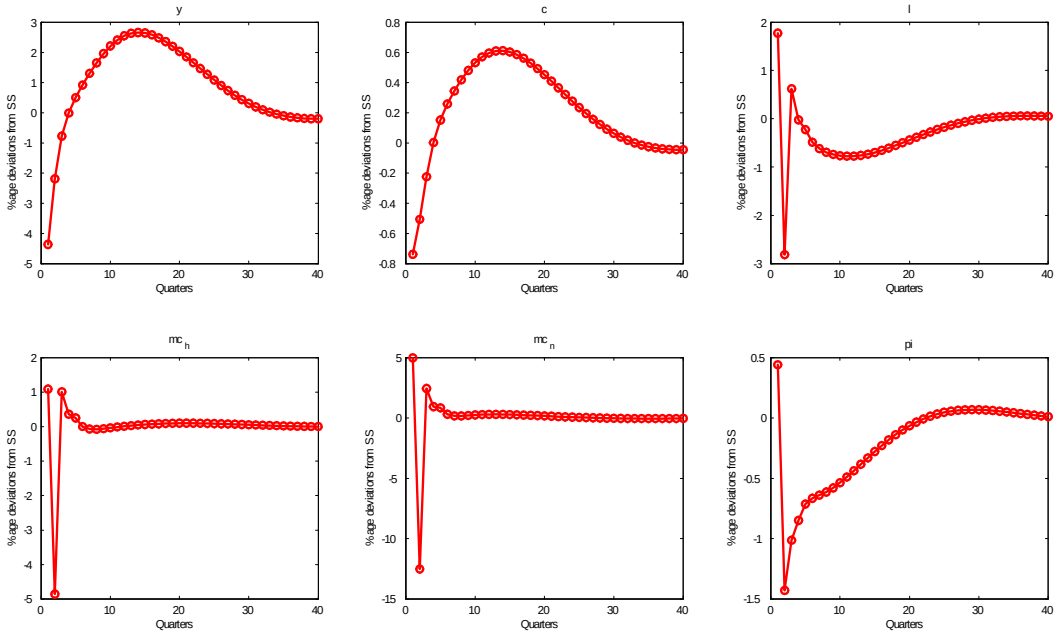

Figure 5: Impulse Responses to a productivity shock (non-tradable goods)

The model yields qualitative results that are consistent with what the theory predicts for most of the variables in response to a domestic monetary policy shock. Economic theory indicates that there are two channels through which the monetary policy shock affects macroeconomic variables of an open economy. First, an increasing interest rate leads to appreciation of the nominal exchange rate which leads the real exchange rate to appreciate since the domestic price index adjusts slowly. The appreciation of the real exchange rate leads a decreasing demand for domestic tradable goods by foreigners (that is, exports of the home country decrease). On the other hand, the same event (that is, the appreciation of the exchange rate of the home country) makes imports cheaper relative to home produced goods which leads to an increasing demand for imports by domestic consumers. These decreasing imports and increasing imports reinforce each other and lead to a decreasing domestic income.

The second channel through which the monetary policy shock affects the economy is through the effect of the interest rate on households' expenditure. That is, an increasing interest rate encourages domestic consumers to postpone their current consumption.

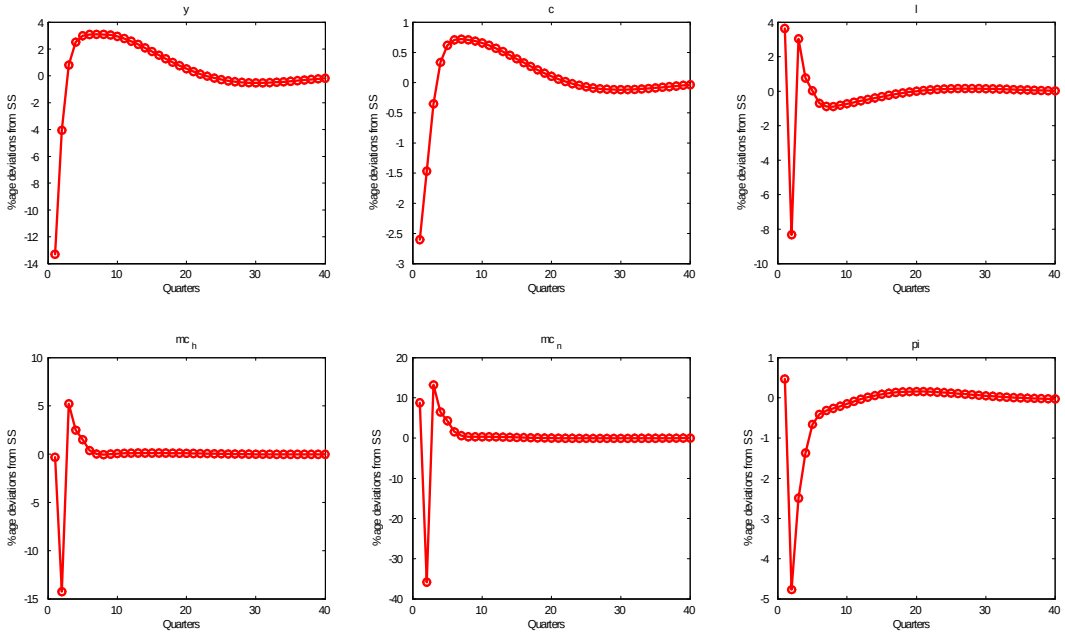

Figure 6: Impulse Responses to a Monetary policy shock

The effects through both channels augment each other in reducing output and, therefore, consumption. As in the previous cases, the observed magnified variations seem to emanate from the existence of the labour market friction that makes adjustment slow in the model.

5 Sensitivity Analysis

We assessed the sensitivity of the model to changes in the calibrated parameter values. Accordingly, we varied the values of some of the basic parameters and examined the behaviour of the model. This attempt is constrained by the fact that there are very few empirical studies that are conducted on the economies of the region in a DSGE framework to see the range of parameters used by other works or obtained from estimating the models on real data. Furthermore, all of the studies conducted for countries in the region so far have not included labour market frictions. As a result, there is no reference point to find a range within which to vary the parameters of the labour market. In spite of all this, for

the basic parameters of preferences, technology and monetary policy, we used the range of values assigned by earlier studies conducted for countries in the region and we found that the impulse responses of most of the variables remain the same (qualitatively). However, the model seems to be very sensitive to variations in the parameters of the labour market: the elasticity of substitution of labour supply between formal and informal sectors, θ_L, the job-separation rate, δ, and the proportions of labour supply between the two sectors, τ_L. The results are also sensitive to variations in the parameters of price setting.

6 Conclusion

In this paper we attempted to address the role of labour market duality in explaining macroeconomic fluctuations in response to various domestic and external shocks. Specifically, we tried to answer the question "How do key macroeconomic variables of a small open economy with labour markets segmented into formal and informal, and where the informal sector employs a large proportion of the labour force, behave in response to domestic and external shocks?". We argue that this question is one of the most important issues that need to be answered by macroeconomists who try to understand the behaviour of the emerging and low-income economies since these economies are characterized not only by formal-informal labour market duality but also by significantly large informal sectors. Therefore, to understand how these economies respond to both domestic and external (both policy and non-policy) shocks one needs to understand how these sectors respond to shocks and how they interact among themselves.

This study contributes to the literature on labour market friction in the New Keynesian framework in general and on modeling labour market friction in low-income economies in particular. As we indicated in the text, only a couple of works have attempted to introduce dual labour markets into the New Keynesian model. Furthermore, these works deal with a closed economy and the dualities they model are not those that are of concern for low income countries. We tried to introduce labour market duality which is consistent with the evidence accumulated in the literature outside the DSGE framework on the labour market frictions of developing countries. Therefore, the model in this study captures the stylized facts about labour markets of low-income countries.

However, the model can still be extended in many directions, some of which we highlight below, so that it can be more complete and used for policy evaluation.

Most of the impulse responses of the variables considered in our simulations are consistent with what is expected based on economic theory and empirical works. However, our results do not seem to support the conclusion of some works that suggest that the existence of a flexible informal sector serves as a stabilizing force of the economy in the event of shocks (see Castillo and Montoro (2012) and the references in that paper). The impulse responses of the key macroeconomic variables to both policy and non-policy shocks show more variability in our model than in the model with a homogenous, flexible labour market that can be found in a large number of works. The argument that a relatively flexible informal sector plays a stabilizing role might work in models where firms employ both types of labour (such as the model in Castillo and Montoro (2012)) which allows firms to immediately change their labour input by increasing or decreasing the informal component of employment. This is so since the informal labour market is assumed to be perfectly competitive and the formal labour market is characterized by search-matching frictions. But as we argued in the text the duality of labour market in low-income countries is not about firms having a mix of employees with formal and informal employment arrangement. In our model the duality is the nature of the whole economy - some firms operate in the formal sector while others operate in the informal sector. Hence, the fast adjustment that could exist in the case where firms can employ both types of workers does not exist in our model.

However, given the following two factors the results reported in this paper seem to be more appealing than those reported in works that employ the standard model. First, low-income economies are characterized more by overall duality than just coexistence of two types of employment contracts in the firms. Second, it is a stylized fact that low-income economies are more vulnerable and have less capacity to absorb shocks. These two factors imply that the model in this paper better represents low-income countries, and the higher variability generated in response to various domestic and external shocks seems to reflect more accurately the realities of these economies.

As we have just highlighted, the model in this study, though an extended version of the basic small open economy New Keynesian model, is still simple in the sense that

there are no financial markets and, therefore, no financial market frictions. Furthermore, we abstracted from capital accumulation. Though we are not sure of their overall effect on the results reported in this paper, the aforementioned factors could have significant implications for the performance of the dual economy model for two reasons. First, low income countries are characterized not only by dual labour markets but also by dual financial markets where the informal sector is more constrained to access credit for both working capital and investment for expansion. Second, due to the imperfect access to the financial markets discussed above and partly due to its nature, the informal sector is characterized by a low capital-labour ratio compared to the formal sector. This low capital-labour ratio is one of the factors that explain the low productivity of the sector compared to the formal sector. Hence, abstracting from capital accumulation as if it were the same and innocuous in the short run might lead to misleading results.

Therefore, incorporating these two features will make the model more realistic in capturing the interaction between the duality in the labour market and imperfections in other markets of low-income countries. Such complete models will have rich dynamics and the results obtained from them might be more reliable and enlightening than the results from simple models like ours.

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7 Appendix

7.1 Parameter values and sources (Domestic)

Parameter	Value	Sources
α	1.5	Castillo and Montoro (2012) which is α_F in their case
β	0.9951	Dagher et al. (2012)
Θ	2.5	Castillo and Montoro (2012) which is B_F in their case
σ	2.61	Kose and Reizman (2001)
φ	3	Dagher et al. (2012)
η	0.24	Dagher et al. (2012)
δ	0.2	Castillo and Montoro (2012)
$\theta_1 = \theta_2$	12	Dagher et al. (2012) and $\theta_3 = \theta_1 = \theta_2$ assumed
b	0.25	Peiris and Saxegaard (2007)
γ_1	0.731	Peiris and Saxegaard (2007)
γ_2	0.3	Assumed
γ_3	0.5	Assumed
ς_H	0.20	Assumed
ς_M	0.75	Assumed
ς_N	0.80	Assumed
ϵ_M	0.40	Assumed
ϵ_H	0.45	Assumed
ϵ_N	0.10	Assumed
ξ	0.50	Assumed (following Blanchard and Gali (2010))
ρ_r	0.82	Peiris and Saxegaard (2007)
ϕ_y	0.50	Peiris and Saxegaard (2007)
ϕ_π	0.30	Peiris and Saxegaard (2007)
ϕ_e	0.80	Peiris and Saxegaard (2007)
ρ_{ZH}	0.90	Assumed based on Matheson (2010)-estimated two sector SOE DSGE
ρ_{ZN}	0.90	Assumed based on Matheson (2010)-estimated two sector SOE DSGE
ω	0.95	Castillo and Montoro (2012)
τ_L	0.10	Calculated based on figures in ILO KILM database
τ_{HI}	0.70	Calculated based on figures in ILO KILM database
θ_L	0.75	Assumed (based on Dagher et al. (2012))

7.2 Parameter values and sources (External sector)

Parameter	value	source
ρ_{r^*}	0.66	Peiris and Saxegaard (2007)
ρ_{y^*}	0.75	"
ρ_{π^*}	0.60	"

7.3 Shocks

ϵ_{ZH}	Productivity shoch (tradable sector)
ϵ_{ZN}	Productivity shock (non-tradable sector)
ϵ_r	Monetary policy shock
ϵ_{tot}	Terms of trade shock
ϵ_{r^*}	Foreign monetary policy shock
ϵ_{y^*}	Foreign income shock
ϵ_{π^*}	Foreign inflation shock