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Do phase fluctuations influence the Fulde-Ferrell-Larkin-Ovchinnikov state in a 3D Fermi gas?

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Abstract

In ultracold Fermi gases, the effect of spin-imbalance on superfluidity has been the subject of intense study. One of the reasons for this is that spin-imbalance frustrates the Bardeen-Cooper-Schrieffer (BCS) superfluid pairing mechanism, in which fermions in different spin states combine into Cooper pairs with zero momentum. In 1964, it was proposed that an exotic superfluid state called the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, in which the Cooper pairs have nonzero momentum, could exist in a spin-imbalanced Fermi gas. At the saddle-point (mean field) level, it has been shown that the FFLO state only occupies a very small sliver in the ground state phase diagram of a 3D Fermi gas. However, a question that remains to be investigated is: what is the influence of phase fluctuations around the saddle point on the FFLO state? In this work we show that phase fluctuations only lead to relatively small quantitative corrections to the presence of the FFLO state in the saddle-point phase diagram of a 3D spin-imbalanced Fermi gas. Starting from the partition function of the system, we calculate the effective action within the path-integral adiabatic approximation. The action is then expanded up to second order in the fluctuation field around the saddle point, leading to the fluctuation free energy. Using this free energy, we calculate corrections due to phase fluctuations to the BCS-FFLO transition in the saddle-point phase diagram. At temperatures at which the FFLO state exists, we find only small corrections to the size of the FFLO area. Our results suggest that fluctuations of the phase of the FFLO order parameter, which can be interpreted as an oscillation of its momentum vector, do not cause an instability of the FFLO state with respect to the BCS state.

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I. INTRODUCTION

In the last two decades, ultracold quantum gases have been the subject of many theoretical and experimental investigations [1]. Among the many systems that have been studied, ultracold Fermi gases have received wide attention [2]. Due to the experimental controllability achieved with ultracold gases, quantum many-body phenomena such as fermionic superfluidity can be studied in great detail in these systems. By controlling and tuning the interaction strength between fermions in different states using Feshbach resonances [3], it has become possible to study the crossover from a Bardeen-Cooper-Schrieffer (BCS) superfluid state of weakly interacting Cooper pairs, to a Bose-Einstein condensate (BEC) of strongly coupled molecules [4, 5].

Aside from the interaction strength, another important parameter that can be tuned is the population imbalance between fermions in different states. This parameter is of importance because spin-imbalance frustrates the BCS superfluid pairing mechanism. In the BCS state, pairing between fermions occurs at the Fermi surface. However, a population imbalance will create a gap between the Fermi surfaces of the two spin states, making the BCS state energetically less favorable. Theoretically it was predicted that at a certain critical spin-imbalance, known as the Clogston-Chandrasekhar limit [6], a first order phase transition from the BCS state to the normal state would occur. By preparing a Fermi gas in one hyperfine state, and using a radio-frequency sweep to create a mixture of two hyperfine states (labeled spin-up and spin-down), the transition from a superfluid to a normal gas, induced by spin-imbalance, was demonstrated experimentally [7].

At this point, the question remains whether a non-uniform superfluid can exist in a spin-imbalanced 3D Fermi gas. The most prominent example of non-uniform superfluidity is the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, which was proposed independently by Fulde and Ferrell (FF) [8] and by Larkin and Ovchinnikov (LO) [9] in 1964. The FFLO state differs from the BCS state in that it has Cooper pairs with non-zero momentum, which in position space results in an oscillating superfluid order parameter. It was suggested that this exotic superfluid state could exist at non-zero polarization. In part of the literature, a further distinction is made between the FF state and the LO state: the former has one momentum component, whereas the latter is the superposition of two momentum components of equal magnitude but opposite sign. In this paper, we will focus on the FF state but we will henceforth call this the FFLO state, bearing in mind that we mean the superfluid state with one momentum component.

Following the success of creating a spin-imbalanced Fermi gas, the theoretical investigation of the FFLO superfluid state was intensified. The first studies focused on the three-dimensional (3D) Fermi gas, at the saddle-point (mean-field) level [10], and found that the FFLO state is only present in a very small sliver of the ground-state phase diagram [11]. The 1D case has also received wide theoretical attention, and has proven to be a promising setup for detecting the FFLO state. In 1D, the presence of the FFLO state in the ground-state phase diagram is much larger compared to the 3D case [12]. Following these theoretical predictions, the first indirect experimental evidence for the FFLO state was found in a 1D Fermi gas by the Hulet group at Rice University [13]. Inspired by this success, several new experimental detection techniques have been proposed [14], both for the 1D and for the 3D case. However, in the latter case, the FFLO state still eludes experimental observation.

To acquire a better understanding of the FFLO state in a 3D Fermi gas, it is necessary to go beyond the mean-field level, which, while resulting in quantitatively correct results in the limit of weak interaction (BCS limit) at temperature zero, offers a qualitative description at best for temperatures above zero or for stronger interactions. Up till now, little attention has been devoted to this subject and to the effect of fluctuations on the FFLO state in general. One important exception is the work by Radzihovsky, [15] in which a low-energy model for the Fulde-Ferrell state and for the Larkin-Ovchinnikov state is developed, with an in-depth focus on the nature of the emerging Goldstone modes for the latter state.

In this paper, we contribute to this subject by explicitly studying the effect of phase fluctuations on the presence of the FFLO state in the phase diagram of a 3D Fermi gas. Our main motivation is the following: the FFLO state is characterized by a momentum component $\mathbf{Q}$, which means that the rotational symmetry of the system is spontaneously broken by this state. The momentum component $\mathbf{Q}$ results in an oscillating phase of the order parameter in position space. Because of this, fluctuations of the phase of the order parameter are equivalent to fluctuations in the direction of the momentum $\mathbf{Q}$. Since a 3D Fermi gas exhibits spherical symmetry, these fluctuations cost zero energy. Hence, one would expect these fluctuations to proliferate and destabilize the FFLO state. Our main point of interest is to see whether the region of FFLO in the phase diagram of a 3D Fermi gas vanishes due to phase fluctuations, which would help to understand why this state has not been observed experimentally in 3D. To the best of our knowledge, this specific problem has not yet been studied in literature.

The rest of this paper is organized as follows. In section II we derive a hydrodynamic effective action, starting from the partition function of a 3D Fermi gas with spin-imbalance, within the path-integral adiabatic approximation. In section III we perform an expansion of the action up to second order in the fluctuation field, which leads to the fluctuation part of the action. From the fluctuation action, the fluctuation free energy is readily derived. Subsequently, in section IV, using this free energy, we calculate the phase diagram of the system and determine whether corrections emerge by taking into account phase fluctuations. Finally in section V we draw conclusions.

II. CALCULATING THE HYDRODYNAMIC EFFECTIVE ACTION

In this section, the effective action describing the FFLO state in a spin-imbalanced 3D Fermi gas, with the inclusion of phase fluctuations around the saddle point, is calculated within a hydrodynamic approach. The starting point of this derivation is the partition function of the system, written as a path integral over fermionic Grassmann fields $\bar{\psi}_{\mathbf{x},\tau,\sigma}$ and $\psi_{\mathbf{x},\tau,\sigma}$:

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}_{\mathbf{x},\tau,\sigma}\mathcal{D}\psi_{\mathbf{x},\tau,\sigma}\; e^{-S\left(\bar{\psi}_{\mathbf{x},\tau,\sigma},\psi_{\mathbf{x},\tau,\sigma}\right)}, \tag{1}$$

where the action consists of a single-particle term and an interaction term:

$$\begin{aligned} S\left(\bar{\psi}_{\mathbf{x},\tau,\sigma},\psi_{\mathbf{x},\tau,\sigma}\right) = &\int_0^\beta d\tau \int d\mathbf{x} \sum_\sigma \bar{\psi}_{\mathbf{x},\tau,\sigma}\left(\frac{\partial}{\partial\tau} - \nabla_{\mathbf{x}}^2 - \mu_\sigma\right)\psi_{\mathbf{x},\tau,\sigma} \\ &+ \int_0^\beta d\tau \int d\mathbf{x} \int d\mathbf{y}\; \bar{\psi}_{\mathbf{x},\tau,\uparrow}\bar{\psi}_{\mathbf{y},\tau,\downarrow} V\left(\mathbf{x}-\mathbf{y}\right)\psi_{\mathbf{y},\tau,\downarrow}\psi_{\mathbf{x},\tau,\uparrow}. \end{aligned} \tag{2}$$

The action (2) is written in imaginary time $\tau = it$, $\mathbf{x}$ and $\mathbf{y}$ represent 3D position vectors, $\beta = 1/k_B T$ is the inverse temperature and μ_σ is the chemical potential of the fermions in the spin state $\sigma = \{\uparrow,\downarrow\}$. Furthermore, we use $\hbar = 2m = 1$ as units. In the interaction term, $V\left(\mathbf{x}-\mathbf{y}\right)$ represents a general interparticle potential. In this paper, only s-wave scattering will be considered at ultracold temperatures, therefore the interaction potential will be replaced by a pseudo-potential:

$$V\left(\mathbf{x}-\mathbf{y}\right) = g\delta\left(\mathbf{x}-\mathbf{y}\right), \tag{3}$$

where the interaction strength g is related to the s-wave scattering length a_s through [16, 17]:

$$\frac{1}{g} = \frac{1}{8\pi a_s} - \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{2k^2}. \tag{4}$$

An alternative, elegant derivation of expression (4) can be found in [18].

Using the standard Hubbard-Stratonovich transformation, the fourth-degree interaction term in (2) is rewritten as a sum of two second-degree terms. The cost of this transformation is that an additional path integral, over auxiliary bosonic fields $\Delta_{\mathbf{x},\tau}$ and $\bar{\Delta}_{\mathbf{x},\tau}$, is introduced. These fields are physically relevant, as they are interpreted as the fields of the fermion pairs. Following the Hubbard-Stratonovich transformation, the partition function becomes

$$\begin{aligned} \mathcal{Z} = &\int \mathcal{D}\bar{\psi}_{\mathbf{x},\tau,\sigma} \mathcal{D}\psi_{\mathbf{x},\tau,\sigma} \int \mathcal{D}\bar{\Delta}_{\mathbf{x},\tau} \mathcal{D}\Delta_{\mathbf{x},\tau} \\ &\times \exp\left[-\int_0^\beta d\tau \int d\mathbf{x} \sum_\sigma \bar{\psi}_{\mathbf{x},\tau,\sigma} \left(\frac{\partial}{\partial\tau} - \nabla_{\mathbf{x}}^2 - \mu_\sigma \right) \psi_{\mathbf{x},\tau,\sigma} \right. \\ &\left. + \int_0^\beta d\tau \int d\mathbf{x} \left(\frac{\bar{\Delta}_{\mathbf{x},\tau}\Delta_{\mathbf{x},\tau}}{g} - \Delta_{\mathbf{x},\tau}\bar{\psi}_{\mathbf{x},\tau,\uparrow}\bar{\psi}_{\mathbf{x},\tau,\downarrow} - \bar{\Delta}_{\mathbf{x},\tau}\psi_{\mathbf{x},\tau,\downarrow}\psi_{\mathbf{x},\tau,\uparrow} \right) \right]. \end{aligned} \tag{5}$$

One way to introduce fluctuations into the partition function is to write the bosonic fields as the sum of a saddle-point contribution and a fluctuation contribution: $\Delta_{\mathbf{x},\tau} = \Delta_{\mathbf{x},\tau}^{(sp)} + \phi_{\mathbf{x},\tau}$ and $\bar{\Delta}_{\mathbf{x},\tau} = \Delta_{\mathbf{x},\tau}^{(sp)} + \bar{\phi}_{\mathbf{x},\tau}$. In this way, amplitude- and phase fluctuations are fully intertwined, and both are automatically taken into account [19]. An alternative way, which will be used in this paper since it allows (at a later stage in this calculation) to focus solely on phase fluctuations, is to write the bosonic fields in terms of an amplitude and a phase

$$\begin{cases} \Delta_{\mathbf{x},\tau} = |\Delta_{\mathbf{x},\tau}| \, e^{i\theta_{\mathbf{x},\tau}} \\ \bar{\Delta}_{\mathbf{x},\tau} = |\Delta_{\mathbf{x},\tau}| \, e^{-i\theta_{\mathbf{x},\tau}} \end{cases}, \tag{6}$$

where both $|\Delta_{\mathbf{x},\tau}|$ and $\theta_{\mathbf{x},\tau}$ are real fields. Upon substituting (6) into the partition function (5), it is convenient to apply the following gauge transformation to the fermionic fields: $\psi_{\mathbf{x},\tau,\sigma} \to \psi_{\mathbf{x},\tau,\sigma} e^{i\theta_{\mathbf{x},\tau}/2}$. After working out the derivatives in (5), the partition function becomes

$$\begin{aligned} \mathcal{Z} = &\int \mathcal{D}\bar{\psi}_{\mathbf{x},\tau,\sigma} \mathcal{D}\psi_{\mathbf{x},\tau,\sigma} \int \mathcal{D} |\Delta_{\mathbf{x},\tau}| \, \mathcal{D}\theta_{\mathbf{x},\tau} \\ &\times \exp\left(-\int_0^\beta d\tau \int d\mathbf{x} \, \bar{\eta}_{\mathbf{x},\tau} \left(-\mathbb{G}_{\mathbf{x},\tau}^{-1} \right) \eta_{\mathbf{x},\tau} + \int_0^\beta d\tau \int d\mathbf{x} \frac{|\Delta_{\mathbf{x},\tau}|^2}{g} \right), \end{aligned} \tag{7}$$

where the following Nambu spinors were used:

$$\bar{\eta}_{\mathbf{x},\tau} = \begin{pmatrix} \bar{\psi}_{\mathbf{x},\tau,\uparrow} & \psi_{\mathbf{x},\tau,\downarrow} \end{pmatrix} \text{ and } \eta_{\mathbf{x},\tau} = \begin{pmatrix} \psi_{\mathbf{x},\tau,\uparrow} \\ \bar{\psi}_{\mathbf{x},\tau,\downarrow} \end{pmatrix}, \tag{8}$$

in order to write the inverse Green's function in block-diagonal form, where the diagonal elements are given by a set of 2×2 matrices given by

$$\begin{aligned} -\mathbb{G}^{-1}_{\mathbf{x},\tau} = & \left(\frac{\partial}{\partial \tau} - \zeta - i \nabla_{\mathbf{x}} \left(\theta_{\mathbf{x},\tau} \right) . \nabla_{\mathbf{x}} - \frac{i}{2} \nabla^2_{\mathbf{x}} \left(\theta_{\mathbf{x},\tau} \right) \right) \sigma_0 \\ & - \left(\nabla^2_{\mathbf{x}} + \mu - \frac{i}{2} \frac{\partial \theta_{\mathbf{x},\tau}}{\partial \tau} - \frac{1}{4} \left[\nabla_{\mathbf{x}} \left(\theta_{\mathbf{x},\tau} \right) \right]^2 \right) \sigma_3 + \left| \Delta_{\mathbf{x},\tau} \right| \sigma_1 . \end{aligned} \tag{9}$$

Here σ_0, σ_1 and σ_3 are Pauli matrices, and furthermore the definitions of the total chemical potential $\mu = (\mu_\uparrow + \mu_\downarrow)/2$ and the imbalance chemical potential $\zeta = (\mu_\uparrow - \mu_\downarrow)/2$ have been introduced.

The path integral over the Bose field $|\Delta_{\mathbf{x},\tau}|$ in (7) cannot be calculated exactly and hence an approximation has to be made. The most basic approximation is to replace the field $|\Delta_{\mathbf{x},\tau}|$ by a constant Δ: this is the saddle-point approximation. Since in this article we want to describe phase fluctuations, we improve on this approximation by also considering fluctuations of the field $\theta_{\mathbf{x},\tau}$ around its saddle-point value. The specific choice of the saddle point is such that the FFLO state is included in the current formalism. The FFLO state is defined by fermionic pairs that have a non-zero momentum $\mathbf{Q}$. In position space, this is equivalent with an oscillating phase of the order parameter $\Delta \exp(i\mathbf{Q}\mathbf{x})$. Hence the choice of the saddle point, including phase fluctuations, is the following:

$$\begin{cases} |\Delta_{\mathbf{x},\tau}| = \Delta \\ \theta_{\mathbf{x},\tau} = \mathbf{Q}.\mathbf{x} + \delta\theta_{\mathbf{x},\tau} \end{cases}, \tag{10}$$

where $\delta\theta_{\mathbf{x},\tau}$ is the fluctuation field. At this point, we explicitly choose to neglect amplitude fluctuations and to focus solely on phase fluctuations. After substitution of (10) in the partition function (7), we find:

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}_{\mathbf{x},\tau,\sigma} \mathcal{D}\psi_{\mathbf{x},\tau,\sigma} \int \mathcal{D}\delta\theta_{\mathbf{x},\tau} \exp\left(-\int_0^\beta d\tau \int d\mathbf{x} \; \bar{\eta}_{\mathbf{x},\tau} \left(-\mathbb{G}^{-1}_{\mathbf{x},\tau} \right) \eta_{\mathbf{x},\tau} + \beta V \frac{\Delta^2}{g} \right). \tag{11}$$

The inverse Green's function in (11) is given by:

$$\begin{aligned} -\mathbb{G}^{-1}_{\mathbf{x},\tau} = & \left(\frac{\partial}{\partial \tau} - \zeta - i\mathbf{Q}.\nabla_{\mathbf{x}} - i \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) . \nabla_{\mathbf{x}} - \frac{i}{2} \nabla^2_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \right) \sigma_0 \\ & - \left(\nabla^2_{\mathbf{x}} + \mu - \frac{i}{2} \frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} - \frac{Q^2}{4} - \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) . \frac{\mathbf{Q}}{2} - \frac{1}{4} \left[\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \right]^2 \right) \sigma_3 + \Delta \sigma_1 . \end{aligned} \tag{12}$$

The partition function (11) still contains two path integrals: one over the phase fluctuation field and one over the fermionic fields. To calculate the fermionic path integral, a transformation to reciprocal space is necessary, because of the derivatives that are present in the inverse Green's function. However, since the field $\delta\theta_{\mathbf{x},\tau}$ is a general function of space and time, this will lead to an infinite number of non-diagonal terms in the action, making the calculation intractable. As a remedy, the path-integral adiabatic approximation will be used. This approximation assumes that the bosonic fluctuation field $\delta\theta_{\mathbf{x},\tau}$ varies slowly in time and space compared to the fermionic fields $\bar{\psi}_{\mathbf{x},\tau,\sigma}$ and $\psi_{\mathbf{x},\tau,\sigma}$. As a result, for a given configuration of the fluctuation field, the configuration of fermionic fields can be coarse-grained by averaging over the 'fast' degrees of freedom:

$$\bar{\psi}_{\mathbf{x},\tau,\sigma}\psi_{\mathbf{x},\tau,\sigma} \to \frac{1}{\beta V}\int d\tau' \int d\mathbf{x}' \bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}\psi^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}. \tag{13}$$

Here $(\mathbf{x},\tau)$ are the space-time points for the 'slow' subsystem, and $(\mathbf{x}',\tau')$ are the space-time points for the 'fast' subsystem. The appearance of additional degrees of freedom is due to the assumption that for boson configurations $\delta\theta_{\mathbf{x},\tau}$ the averaging over fermion configurations can be performed while keeping the bosonic field constant. This method was used successfully for a hydrodynamic description of the Berezinskii-Kosterlitz-Thouless transition in a 2D Fermi gas [20]. Using expression (13), the Fourier transformation of the fermionic fields can be performed independently of the fluctuation field (see appendix A). After Fourier transformation, the path integral over fermionic fields is quadratic and can be calculated exactly. The partition function then becomes:

$$\mathcal{Z} = \int \mathcal{D}\delta\theta_{\mathbf{x},\tau} \exp\left(\frac{1}{\beta V}\int_0^\beta d\tau \int d\mathbf{x} \sum_{\mathbf{k},\omega_n} \log\left(-\det\left\{-\mathbb{G}^{-1}_{\mathbf{k},\omega_n}\left[\delta\theta\left(\mathbf{x},\tau\right)\right]\right\}\right) + \beta V \frac{\Delta^2}{g}\right), \tag{14}$$

with $\mathbf{k}$ the momentum of the fermionic fields and $\omega_n = (2n+1)\pi/\beta$ the fermionic Matsubara frequencies. In (14) the inverse Green's function is given by

$$-\mathbb{G}^{-1}_{\mathbf{k},\omega_n}\left[\delta\theta\left(\mathbf{x},\tau\right)\right] = \begin{pmatrix} -i\omega_n - \tilde{\zeta}^{(\theta)}_{\mathbf{k},\mathbf{Q}} + \tilde{\xi}^{(\theta)}_{\mathbf{k},\mathbf{Q}} & \Delta \\ \Delta & -i\omega_n - \tilde{\zeta}^{(\theta)}_{\mathbf{k},\mathbf{Q}} - \tilde{\xi}^{(\theta)}_{\mathbf{k},\mathbf{Q}} \end{pmatrix}. \tag{15}$$

To write (15) in a compact form, the following notations were introduced:

$$\begin{cases} \tilde{\zeta}^{(\theta)}_{\mathbf{k},\mathbf{Q}} = \zeta_{\mathbf{k},\mathbf{Q}} + \zeta^{(\theta)}_{\mathbf{k}} \\ \zeta_{\mathbf{k},\mathbf{Q}} = \zeta \ - \mathbf{k}.\mathbf{Q} \\ \zeta^{(\theta)}_{\mathbf{k}} = - \ \nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\mathbf{k} + \frac{i}{2}\nabla^2_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right) \end{cases}, \tag{16}$$

and

$$\begin{cases} \tilde{\xi}_{\mathbf{k},\mathbf{Q}}^{(\theta)} = k^2 - \tilde{\mu}_{\mathbf{Q}}^{(\theta)} \\ \tilde{\mu}_{\mathbf{Q}}^{(\theta)} = \mu_{\mathbf{Q}} + \mu_{\mathbf{Q}}^{(\theta)} \\ \mu_{\mathbf{Q}} = \mu - \dfrac{Q^2}{4} \\ \mu_{\mathbf{Q}}^{(\theta)} = -\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau}\right) . \dfrac{\mathbf{Q}}{2} - \dfrac{i}{2} \dfrac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} - \dfrac{1}{4} \left[\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau}\right)\right]^2 \end{cases} . \tag{17}$$

These notations are divided in a part that depends explicitly on the phase field $\delta\theta_{\mathbf{x},\tau}$ (indicated by a superscript (θ)), and a part that does not. When $\delta\theta_{\mathbf{x},\tau}$ is set to zero in (14), the saddle-point result is obtained [21]. In (16) and (17) the two competing effects of the FFLO state are visible. The first effect is that the imbalance chemical potential ζ can be lowered by the term $\mathbf{k}.\mathbf{Q}$, which depends on the FFLO momentum $\mathbf{Q}$. This shows that the FFLO state is able to cope with spin-imbalance. The second effect is that the momentum $\mathbf{Q}$ increases the total chemical potential, i.e. the term $Q^2/4$ in $\mu_{\mathbf{Q}}$. This shows that forming Cooper pairs with non-zero momentum costs more energy than forming Cooper pairs with zero momentum. The FFLO state is a trade-off between these two competing effects.

Finally, the fermionic Matsubara summation in (14) can be performed. The resulting action is ultraviolet divergent, which is an artifact of the contact potential (3). This divergence can be removed by writing the interaction strength g as a function of the s-wave scattering length a_s, as in expression (4). The integral over momentum $\mathbf{k}$ in this expression cancels out the divergence of the action in (14). The latter then becomes:

$$S = -\int_0^\beta d\tau \int d\mathbf{x} \int \frac{d\mathbf{k}}{(2\pi)^3} \left\{ \frac{1}{\beta} \log\left[2\cosh\left(\beta \tilde{E}_{\mathbf{k},\mathbf{Q}}^{(\theta)}\right) + 2\cosh\left(\beta \tilde{\zeta}_{\mathbf{k},\mathbf{Q}}^{(\theta)}\right)\right] - \tilde{\xi}_{\mathbf{k},\mathbf{Q}}^{(\theta)} - \frac{\Delta^2}{2k^2} \right\} \tag{18}$$
$$- \beta V \frac{\Delta^2}{8\pi a_s},$$

with $\tilde{E}_{\mathbf{k},\mathbf{Q}}^{(\theta)} = \sqrt{\left(\tilde{\xi}_{\mathbf{k},\mathbf{Q}}^{(\theta)}\right)^2 + \Delta^2}$. This is the effective action for the FFLO state in a 3D Fermi gas with spin-imbalance.

III. THE FLUCTUATION ACTION

In the partition function (14), only the path integral over the fluctuation field $\delta\theta_{\mathbf{x},\tau}$ remains. However, at this point, the action is still a complicated functional of this field, for which the path integral cannot be calculated analytically. Therefore, the action will be expanded up to quadratic order in $\delta\theta_{\mathbf{x},\tau}$ and its derivatives. At this point we have made two approximations regarding the fluctuation field: (1) the fluctuations are assumed to be small compared to the saddle-point value, which justifies an expansion up to quadratic order in the fluctuation field and (2) the fluctuations are assumed to be slowly varying in time and space, compared to the fermionic fields, which forms the basis for the path-integral adiabatic approximation. The latter approximation implies that the momentum q, associated with the fluctuation field, is also small. For this reason, we will take into account terms only up to order q^4 in the action when Fourier transforming the fluctuation field, which will be done at a later stage in this work.

By expanding the action (18) up to quadratic order in $\delta\theta_{\mathbf{x},\tau}$ and its derivatives, the action can be written as the sum of two parts: $S = S_{sp} + S_{fl}$. The zeroth order term yields the saddle-point action $S(\delta\theta_{\mathbf{x},\tau} = 0) = S_{sp}$ and the second order term leads to the fluctuation action S_{fl}. The saddle-point action corresponds to the result obtained in [21]. The fluctuation action is given by

$$\begin{aligned}
S_{fl} = &-\int_0^\beta d\tau \int d\mathbf{x} \int \frac{d\mathbf{k}}{(2\pi)^3} \Bigg\{ \left(1 - \frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}} X(E_{\mathbf{k},\mathbf{Q}})\right) \left(-\frac{1}{4}\left[\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau})\right]^2\right) \\
&+ \frac{1}{2} Y(E_{\mathbf{k},\mathbf{Q}}) \left(\left[\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{k}\right]^2 - i\nabla_{\mathbf{x}}^2(\delta\theta_{\mathbf{x},\tau}) \nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{k} - \frac{1}{4}\left[\nabla_{\mathbf{x}}^2(\delta\theta_{\mathbf{x},\tau})\right]^2 \right) \\
&+ \frac{1}{2} \left[\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2 Y(E_{\mathbf{k},\mathbf{Q}}) + X(E_{\mathbf{k},\mathbf{Q}}) \frac{\Delta^2}{E_{\mathbf{k},\mathbf{Q}}^3} \right] \\
&\times \left[\left(\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\frac{\mathbf{Q}}{2}\right)^2 + \frac{i}{2}\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{Q}\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau} - \frac{1}{4}\left(\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau}\right)^2 \right] \\
&+ \tilde{Y}(E_{\mathbf{k},\mathbf{Q}}) \left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right) \Bigg[\left(\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\frac{\mathbf{Q}}{2}\right) \left[\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{k}\right] - \frac{i}{4}\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{Q}\ \nabla_{\mathbf{x}}^2(\delta\theta_{\mathbf{x},\tau}) \\
&+ \frac{i}{2}\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau}\nabla_{\mathbf{x}}(\delta\theta_{\mathbf{x},\tau}).\mathbf{k} \Bigg] \Bigg\},
\end{aligned} \tag{19}$$

where the following notations were used:

$$\begin{cases}
X(A) = \dfrac{\sinh(\beta A)}{\cosh(\beta E_{\mathbf{k},\mathbf{Q}}) + \cosh(\beta\zeta_{\mathbf{k},\mathbf{Q}})} \\
Y(E_{\mathbf{k},\mathbf{Q}}) = \beta\dfrac{1 + \cosh(\beta E_{\mathbf{k},\mathbf{Q}})\cosh(\beta\zeta_{\mathbf{k},\mathbf{Q}})}{\left[\cosh(\beta E_{\mathbf{k},\mathbf{Q}}) + \cosh(\beta\zeta_{\mathbf{k},\mathbf{Q}})\right]^2} \\
\tilde{Y}(E_{\mathbf{k},\mathbf{Q}}) = \dfrac{\beta\sinh(\beta E_{\mathbf{k},\mathbf{Q}})\sinh(\beta\zeta_{\mathbf{k},\mathbf{Q}})}{\left[\cosh(\beta E_{\mathbf{k},\mathbf{Q}}) + \cosh(\beta\zeta_{\mathbf{k},\mathbf{Q}})\right]^2}
\end{cases}. \tag{20}$$

More details about the derivation of (19) are given in appendix B. As a last step before calculating the path integral over the fluctuation field $\delta\theta_{\mathbf{x},\tau}$, we have to perform a Fourier transformation of this field, again to eliminate the derivatives in the action (19). The result is:

$$S_{fl} = \frac{1}{2}\sum_{\mathbf{q},m}\left\{\int \frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{1}{2}\left(1-\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}X\left(E_{\mathbf{k},\mathbf{Q}}\right)\right)q^2 - Y\left(E_{\mathbf{k},\mathbf{Q}}\right)\left((\mathbf{q}.\mathbf{k})^2 - \frac{1}{4}q^4\right)\right.\right.$$
$$-\left(\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2 Y\left(E_{\mathbf{k},\mathbf{Q}}\right) + X\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\Delta^2}{E^3_{\mathbf{k},\mathbf{Q}}}\right)\left(\frac{1}{4}(\mathbf{q}.\mathbf{Q})^2 - \frac{1}{2}i\varpi_m(\mathbf{q}.\mathbf{Q}) - \frac{1}{4}\varpi_m^2\right)$$
$$\left.\left.-\tilde{Y}\left(E_{\mathbf{k},\mathbf{Q}}\right)\left[(\mathbf{q}.\mathbf{Q})(\mathbf{q}.\mathbf{k}) - i\varpi_m(\mathbf{q}.\mathbf{k})\right]\right]\right\}\delta\theta_{\mathbf{q},m}\delta\theta^*_{\mathbf{q},m}. \tag{21}$$

In expression (21), $\mathbf{q}$ is the momentum of the fluctuation field and $\varpi_m = 2\pi m/\beta$ are bosonic Matsubara frequencies. The action in (21) contains three scalar products: $\mathbf{k}.\mathbf{q}$, $\mathbf{q}.\mathbf{Q}$ and $\mathbf{k}.\mathbf{Q}$. When calculating the integrals over momentum $\mathbf{k}$ and $\mathbf{q}$, one has to be careful when integrating over the polar and azimuthal angles in spherical coordinates. Here, for the benefit of clarity, we explicitly mention our conventions in the definition of these various angles. Within the integration over $\mathbf{k}$, $\mathbf{Q}$ points along the z-axis, such that $\mathbf{k}.\mathbf{Q} = |\mathbf{k}|\,|\mathbf{Q}|\cos(\alpha_{\mathbf{k},\mathbf{Q}})$ with $\alpha_{\mathbf{k},\mathbf{Q}}$ the polar angle of the spherical coordinate system. Analogously, the scalar product of $\mathbf{q}$ and $\mathbf{Q}$ is written as: $\mathbf{q}.\mathbf{Q} = |\mathbf{q}|\,|\mathbf{Q}|\cos(\alpha_{\mathbf{q},\mathbf{Q}})$. The vector $\mathbf{q}$, however, lies in an arbitrary direction with respect to the vector $\mathbf{k}$. As a consequence, the scalar product of these two vectors is given by $\mathbf{k}.\mathbf{q} = |\mathbf{k}|\,|\mathbf{q}|\cos(\alpha_{\mathbf{k},\mathbf{q}})$, where the angle $\alpha_{\mathbf{k},\mathbf{q}}$ can be written in terms of both the polar and the azimuthal angle between $\mathbf{k}$ and $\mathbf{Q}$ on the one hand and between $\mathbf{q}$ and $\mathbf{Q}$ on the other hand, by using:

$$\cos(\alpha_{\mathbf{k},\mathbf{q}}) = xy + \sqrt{1-x^2}\sqrt{1-y^2}\cos(\varphi_{\mathbf{k},\mathbf{Q}} - \varphi_{\mathbf{q},\mathbf{Q}}), \tag{22}$$

where $x = \cos(\alpha_{\mathbf{k},\mathbf{Q}})$ and $y = \cos(\alpha_{\mathbf{q},\mathbf{Q}})$. Substituting (22) in the action (21) and integrating out $\varphi_{\mathbf{k},\mathbf{Q}}$, the fluctuation partition function, defined by $\mathcal{Z}_{fl} = \exp(-S_{fl})$ becomes

$$\mathcal{Z}_{fl} = \int \mathcal{D}\delta\theta_{\mathbf{q},m}\exp\left(-\frac{1}{2}\sum_{\mathbf{q},m}\delta\theta^*_{\mathbf{q},m}\mathbb{A}_{\mathbf{q},m}\delta\theta_{\mathbf{q},m}\right). \tag{23}$$

Here, $\mathbb{A}_{\mathbf{q},m}$ equals

$$\mathbb{A}_{\mathbf{q},m} = A_{\mathbf{Q},x}\; q^2 + B_{\mathbf{Q},x}\; q^2y^2 + C_{\mathbf{Q},x}\; q^4 + D_{\mathbf{Q},x}\; i\varpi_m\; qy - E_{\mathbf{Q},x}\;\left(i\varpi_m\right)^2, \tag{24}$$

where the following coefficients have been introduced

$$\left\{\begin{array}{l} A_{\mathbf{Q},x}=\dfrac{1}{2}\displaystyle\int\frac{d\mathbf{k}}{(2\pi)^3}\left(1-\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}X\left(E_{\mathbf{k},\mathbf{Q}}\right)-k^2\left(1-x^2\right)Y\left(E_{\mathbf{k},\mathbf{Q}}\right)\right) \\ B_{\mathbf{Q},x}=\displaystyle\int\frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{1}{2}k^2\left(1-3x^2\right)Y\left(E_{\mathbf{k},\mathbf{Q}}\right)-kQx\tilde{Y}\left(E_{\mathbf{k},\mathbf{Q}}\right)\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)\right. \\ \qquad\left.-\dfrac{Q^2}{4}\left[\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2Y\left(E_{\mathbf{k},\mathbf{Q}}\right)+X\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\Delta^2}{E_{\mathbf{k},\mathbf{Q}}^3}\right]\right\} \\ C_{\mathbf{Q},x}=\dfrac{1}{4}\displaystyle\int\frac{d\mathbf{k}}{(2\pi)^3}Y\left(E_{\mathbf{k},\mathbf{Q}}\right) \\ D_{\mathbf{Q},x}=\displaystyle\int\frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{Q}{2}\left[\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2Y\left(E_{\mathbf{k},\mathbf{Q}}\right)+X\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\Delta^2}{E_{\mathbf{k},\mathbf{Q}}^3}\right]\right. \\ \qquad\left.+kx\tilde{Y}\left(E_{\mathbf{k},\mathbf{Q}}\right)\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)\right\} \\ E_{\mathbf{Q},x}=\dfrac{1}{4}\displaystyle\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2Y\left(E_{\mathbf{k},\mathbf{Q}}\right)+X\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\Delta^2}{E_{\mathbf{k},\mathbf{Q}}^3}\right] \end{array}\right. . \tag{25}$$

At this point only the path integral over the fluctuation field remains in the partition function given by expression (23). When calculating this path integral, one has to be careful not to double-count the fields. The reason is that $\delta\theta_{\mathbf{q},m}=\delta\theta^*_{-\mathbf{q},-m}$ because $\delta\theta_{\mathbf{x},\tau}$ is a real function. To circumvent this problem, only half the total momentum domain is taken into account:

$$\mathcal{Z}_{fl}=\prod_{\substack{\mathbf{q},m\\q_z>0}}\int d\delta\theta_{\mathbf{q},m}\int d\delta\theta^*_{\mathbf{q},m}\exp\left(-\sum_{\substack{\mathbf{q},m\\q_z>0}}\delta\theta^*_{\mathbf{q},m}\mathbb{A}_{\mathbf{q},m}\delta\theta_{\mathbf{q},m}\right), \tag{26}$$

where the symmetry property

$$\mathbb{A}_{\mathbf{q},m}=\mathbb{A}_{-\mathbf{q},-m}, \tag{27}$$

was used. Now, the standard expression for a quadratic bosonic path integral can be used, which leads to

$$\mathcal{Z}_{fl}=\exp\left(-\frac{1}{2}\sum_{\mathbf{q},m}\ln\left[-E_{\mathbf{Q},x}\ (i\varpi_m)^2+D_{\mathbf{Q},x}\ i\varpi_m\ qy+A_{\mathbf{Q},x}\ q^2+B_{\mathbf{Q},x}\ q^2y^2+C_{\mathbf{Q},x}\ q^4\right]\right). \tag{28}$$

As a final step, the bosonic Matsubara summation can be calculated, which eventually results in the fluctuation free energy:

$$\Omega_{fl}\left(\mu,\zeta,\beta;\Delta,Q\right)=\frac{1}{2\beta V}\sum_{\mathbf{q}}\left\{\ln\left[2\cosh\left(\beta\sqrt{\Psi}\right)-2\cosh\left(\beta\Xi\right)\right]-\beta\sqrt{\Psi}\right\}, \tag{29}$$

with

$$\begin{cases} \sqrt{\Psi} = \dfrac{1}{2E_{\mathbf{Q},x}} \sqrt{D_{\mathbf{Q},x}^2 q^2 y^2 + 4E_{\mathbf{Q},x} \left(A_{\mathbf{Q},x} \; q^2 + B_{\mathbf{Q},x} \; q^2 y^2 + C_{\mathbf{Q},x} \; q^4 \right)} \\ \Xi = \dfrac{D_{\mathbf{Q},x}}{2E_{\mathbf{Q},x}} q y \end{cases} . \tag{30}$$

IV. THE T>0 PHASE DIAGRAM

A. Method and caveat

In this subsection we briefly explain the method for constructing the phase diagram of the system, including phase fluctuations, starting from the fluctuation free energy (29). In our previous work [22], the saddle-point phase diagram for a 3D spin-imbalanced Fermi gas at zero temperature was calculated, as a function of the chemical potentials μ and ζ. For a given value of μ and ζ, the saddle-point free energy was minimized with respect to the variational parameters Δ (the superfluid band gap) and Q (the FFLO momentum). The values of Δ and Q at a given minimum then determined which state this minimum corresponds to: the BCS state ($\Delta \neq 0, Q = 0$), the FFLO state ($\Delta \neq 0, Q \neq 0$) or the normal state ($\Delta = 0$). Now, phase fluctuations around the saddle point can be included by using the fluctuation free energy (29). For each value of μ and ζ, the contribution of the fluctuation free energy to the different minima of the saddle-point free energy can be calculated. This will result in a shift of these minima relative to each other, leading to corrections to the phase diagram.

However, an important caveat has to be kept in mind. In this paper, phase fluctuations are calculated by using an expansion of the action around the saddle point up to quadratic order in the fluctuation field. This expansion only makes sense when the saddle point is a local or a global minimum. If this is not the case, the quadratic terms will result in negative contributions to the action, which leads to an exponential divergence of the partition function because $\mathcal{Z} \sim e^{-S}$. Because of this fact, only corrections to the BCS-FFLO transition and to the BCS-normal transition can be calculated within the current formalism, whereas this is not possible for the FFLO-normal transition. The reason for this is that the former two transitions are both of first order, which means there is a competition between two local minima. For given values of μ and ζ, the fluctuation corrections to both these minima can be calculated and thus their relative shift can be determined. In contrast, the FFLO-normal transition is of second order, so that there is a continuous transition between the FFLO minimum and the normal minimum. From this it follows that for given values of μ and ζ, either the FFLO minimum or the normal minimum is present, but never both at the same time. Hence, fluctuation corrections can only be calculated for one of the two states, for given values of μ and ζ. As a result, it is not possible to calculate corrections to the FFLO-normal transition.

Finally it should be mentioned that in order to calculate fluctuation corrections to the normal state, both amplitude- and phase fluctuations should be taken into account. This is because when Δ goes to zero, both types of fluctuations become indistinguishable. Because of this reason, we will focus solely on fluctuation corrections to the BCS-FFLO transition in this paper.

B. The (μ, ζ)-phase diagram at $T > 0$

Figure 1 shows the saddle-point phase diagram of a 3D Fermi gas with spin-imbalance, as a function of the chemical potentials μ and ζ, at different temperatures. Three different states are indicated: the BCS state (light blue area), the FFLO state (light green area), and the normal state (light red area). As is well known, at the saddle-point level, the presence of the FFLO state in the phase diagram of a spin-imbalanced 3D Fermi gas is minimal. With increasing temperature, the area of the FFLO state decreases in size, because the free energy of the normal state lowers relatively to the free energy of the FFLO state (as well as the BCS state) when temperature increases. We find that at $T = 0.04$, the FFLO state vanishes for all values of μ.

When adding fluctuation corrections to these phase diagrams, we find, surprisingly, that these corrections are very small so that their influence is not visible at the scale of fig. 1. To give an idea of the order of magnitude of the fluctuation corrections, we 'zoom in' on the FFLO area at $T = 0.03$ by re-scaling the ordinate of the phase diagram in fig. 1(C). More specifically, we use $\zeta - \zeta_C$ instead of ζ, where $\zeta_C = \zeta_C(\mu)$ is the value of ζ (for a given value of μ) at which the BCS-FFLO transition occurs within the saddle-point approximation. Using this rescaling method, the phase diagram at $T = 0.03$, now including fluctuation corrections, is given by fig. 2. We find that the FFLO area is slightly enlarged when fluctuations are taken into account (green + orange area) compared to the saddle-point result (green area), i.e. the BCS-FFLO transition line lies at lower values of $\zeta - \zeta_C$. The orange (fluctuation)

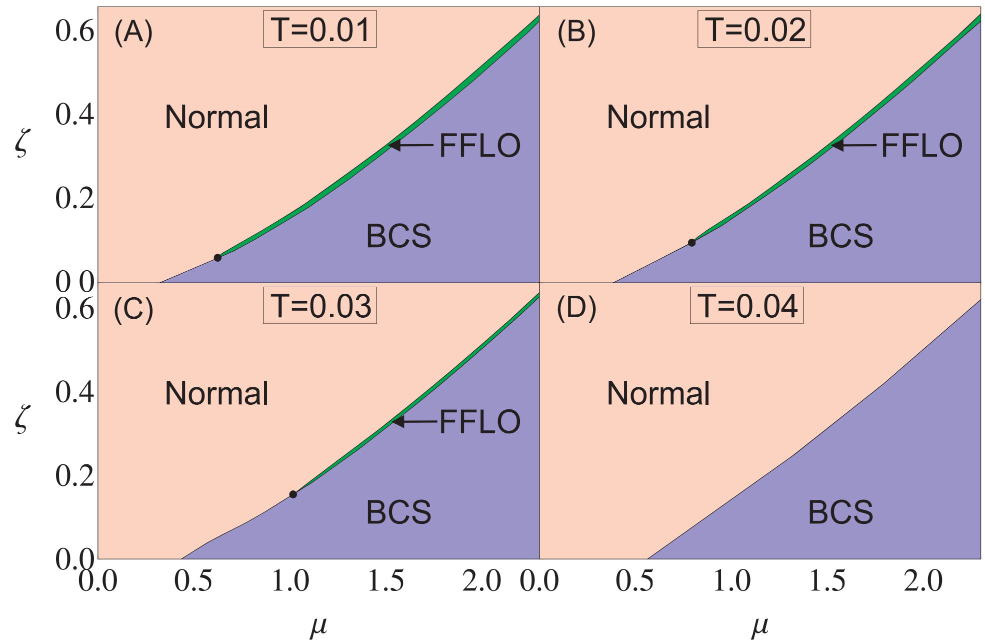

FIG. 1: The saddle-point phase diagram of a 3D Fermi gas with spin-imbalance, as a function of the total chemical potential μ and the imbalance chemical potential ζ, for several values of temperature T (in units $\hbar = 2m = |a_s| = 1$). The tri-critical point at which the BCS-normal transition meets the BCS-FFLO transition is indicated by a black dot in figures (A), (B) and (C). The presence of the FFLO state decreases when temperature increases, and at $T \approx 0.04$, the FFLO state vanishes for all values of μ. It should be noted that T/T_F is not constant in these phase diagrams, because we use $|a_s|$ as a unit of length instead of k_F. When converting temperature to units of the Fermi temperature, we find that the tri-critical point lies at $T/T_F \approx 0.03$.

area is relatively small, even compared to the FFLO area, which in itself is only a small sliver on the phase diagram, as can be seen in fig. 1. Thus we find that phase fluctuations, at least within a hydrodynamic approach, have only a relatively small quantitative effect on the FFLO state, when considering the BCS-FFLO transition. In fig. 2 we have omitted the values of μ at which the FFLO state is not present, since we only calculated corrections for the BCS-FFLO transition and not for the BCS-normal transition. The somewhat irregular shape of the fluctuation area is due to the resolution of our numerical grid. At higher values than $\mu = 2.0$, the contribution of fluctuations is negligible, even at the scale of this figure. This may seem counterintuitive, because the interaction strength increases when the density and hence the total chemical potential μ is increased, and fluctuations are expected

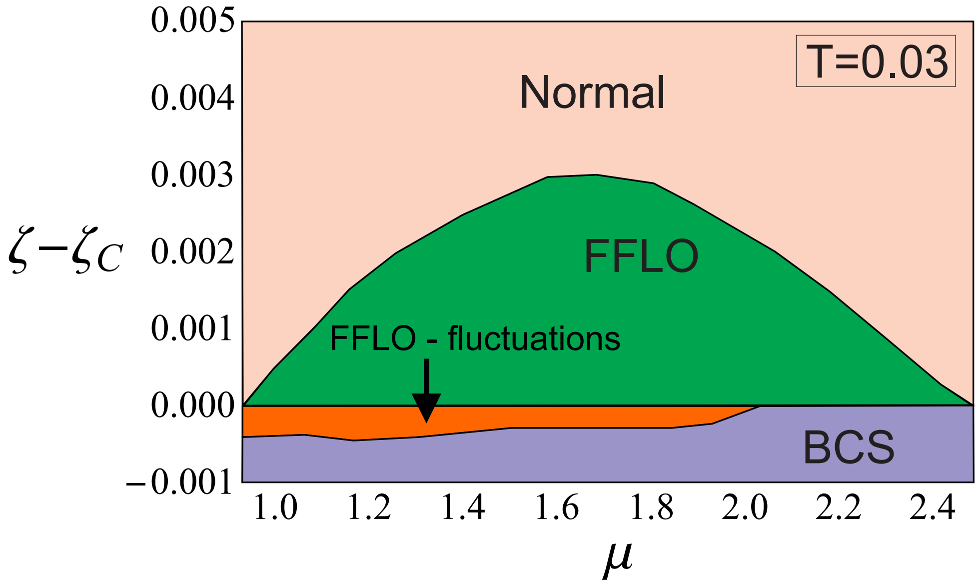

FIG. 2: Phase diagram of a 3D Fermi gas with spin-imbalance at $T = 0.03$ (in units $\hbar = 2m = |a_s| = 1$). This phase diagram is a rescaled version of fig. 1(C), with the addition of corrections due to phase fluctuations. The rescaling is performed by plotting ζ relative to the value of ζ at which the BCS-FFLO transition occurs (ζ_C) at the saddle-point level. The fluctuation corrections lead to a small enlargement of the FFLO region (green + orange area) compared to the saddle-point result (green area). Given the size of the FFLO area in fig. 1(c), this figure demonstrates that fluctuations only lead to small quantitative corrections to the FFLO area. We did not plot lower values than $\mu = 1.0$ because we only calculated corrections to the BCS-FFLO transition. At higher values than $\mu = 2.0$, the contribution of fluctuations is negligible, even at the scale of this figure. The somewhat irregular shape of the fluctuation area is due to the resolution of our numerical grid.

to contribute more at higher values of the interaction strength. The reason, however, is that because we use the scattering length as a unit of length (instead of the Fermi vector k_F) the temperature relative to the Fermi temperature T/T_F is not a constant in figure 2: T/T_F decreases with increasing μ, because the density and hence $T_F = k_F^2/k_B$ (in units $\hbar = 2m = 1$) increases when μ is increased. This explains why the fluctuation corrections, which decrease in magnitude at lower temperatures, become smaller for increasing values of μ in fig. 2(C). When converting the temperature in fig. 1 to units of the Fermi temperature, we find that the tri-critical point (indicated by a black dot in this figure) lies at approximately $T/T_F \approx 0.03$.

V. CONCLUSIONS

In this paper, we have studied the effect of phase fluctuations on the FFLO state in a 3D Fermi gas with spin-imbalance. Starting from the partition function of the system, the complex fields of the Cooper pairs were written in terms of an amplitude field and a phase field. By choosing a suitable saddle point, in which Cooper pairs were allowed to have a non-zero momentum **Q**, the FFLO state was included into the mathematical description. We then considered fluctuations of the phase field around the saddle point. Because the resulting path integrals over the fermionic fields and the fluctuation field could not be calculated exactly, we used the path-integral adiabatic approximation. Subsequently, the action was expanded up to second order in the fluctuation field, after which the path integral over this field could be calculated. This led to the fluctuation free energy. This free energy was used to calculate corrections to the saddle-point phase diagram of the system. For each value of the chemical potentials μ and ζ, the contribution of the fluctuation free energy to each minimum of the saddle-point free energy was calculated. This method allows to calculate corrections to the BCS-FFLO transition, but not to the FFLO-normal transition. We have found that phase fluctuations only lead to relatively small quantitative corrections to the BCS-FFLO transition. Our results suggest that the fluctuation of the phase of the FFLO order parameter, which can be interpreted as an oscillation of the FFLO momentum vector around its saddle-point value, does not cause an instability of the FFLO state with respect to the BCS state.

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Appendix A: Fourier transformation within the path-integral adiabatic approach

When applying the averaging given by (13) to the partition function (11), the latter becomes

$$\mathcal{Z}=\prod_{\mathbf{x},\tau,\sigma}\prod_{\mathbf{x}',\tau'}\int d\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}d\psi^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}\int\mathcal{D}\delta\theta_{\mathbf{x},\tau}$$
$$\times\exp\left(-\int_0^\beta d\tau\int d\mathbf{x}\frac{1}{\beta V}\int_0^\beta d\tau'\int d\mathbf{x}'\;\bar{\eta}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau'}\left\{-\mathbb{G}^{-1}_{\mathbf{x}',\tau'}\left[\delta\theta\left(\mathbf{x},\tau\right)\right]\right\}\eta^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau'}+\beta V\frac{\Delta^2}{g}\right),\tag{A1}$$

where the Nambu spinors are given by (8). Let us first transform the diagonal terms of the inverse Green's function, given by expression (12). Our definitions for the Fourier transform are as follows:

$$\left\{\begin{array}{l}\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}=(\beta V)^{-\frac{1}{2}}\sum_{\mathbf{k},\omega_n}e^{i\omega_n\tau'-i\mathbf{k}.\mathbf{x}'}\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\sigma}\\ \psi^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\sigma}=(\beta V)^{-\frac{1}{2}}\sum_{\mathbf{k},\omega_n}e^{-i\omega_n\tau'+i\mathbf{k}.\mathbf{x}'}\psi^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\sigma}\end{array}\right..\tag{A2}$$

The term proportional to $\bar{\psi}_\uparrow\psi_\uparrow$ can be transformed as

$$-\int_0^\beta d\tau\int d\mathbf{x}\frac{1}{\beta V}\int_0^\beta d\tau'\int d\mathbf{x}\;\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\uparrow}\left(\frac{\partial}{\partial\tau}-\nabla^2_{\mathbf{x}}-\mu+\frac{Q^2}{4}-\zeta-i\mathbf{Q}.\nabla_{\mathbf{x}}\right.$$
$$\left.+\frac{i}{2}\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau}-i\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)\nabla_{\mathbf{x}}-\frac{i}{2}\nabla^2_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)+\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\frac{\mathbf{Q}}{2}+\frac{1}{4}\left[\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)\right]^2\right)\psi^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\uparrow}$$
$$=-\frac{1}{\beta V}\int_0^\beta d\tau\int d\mathbf{x}\sum_{\mathbf{k},\omega_n}\left(-i\omega_n+k^2-\mu+\frac{Q^2}{4}-\zeta+\mathbf{Q}.\mathbf{k}\right.$$
$$\left.+\frac{i}{2}\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau}+\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\mathbf{k}-\frac{i}{2}\nabla^2_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)+\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\frac{\mathbf{Q}}{2}+\frac{1}{4}\left[\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)\right]^2\right)\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\uparrow}\psi^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\uparrow},\tag{A3}$$

and analogously for the term proportional to $\bar{\psi}_\downarrow\psi_\downarrow$. The off-diagonal term becomes

$$-\int_0^\beta d\tau\int d\mathbf{x}\frac{1}{\beta V}\int_0^\beta d\tau'\int d\mathbf{x}'\;\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\uparrow}\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{x}',\tau',\downarrow}\Delta=-\int_0^\beta d\tau\int d\mathbf{x}\frac{1}{\beta V}\sum_{\mathbf{k},\omega_n}\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\uparrow}\bar{\psi}^{(\mathbf{x},\tau)}_{-\mathbf{k},-\omega_n,\downarrow}\Delta.\tag{A4}$$

Now the partition function has become

$$\mathcal{Z}=\prod_{\mathbf{x},\tau}\prod_{\mathbf{k},\omega_n,\sigma}\int d\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\sigma}d\psi^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\sigma}\prod_{\mathbf{x},\tau}\int\mathcal{D}\delta\theta_{\mathbf{x},\tau}$$
$$\times\exp\left(-\frac{1}{\beta V}\int_0^\beta d\tau\int d\mathbf{x}\sum_{\mathbf{k},\omega_n}\;\bar{\eta}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n}\left\{-\mathbb{G}^{-1}_{\mathbf{k},\omega_n}\left[\delta\theta\left(\mathbf{x},\tau\right)\right]\right\}\eta^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n}+\beta V\frac{\Delta^2}{g}\right),\tag{A5}$$

where $-\mathbb{G}^{-1}_{\mathbf{k},\omega_n}\left[\delta\theta\left(\mathbf{x},\tau\right)\right]$ is given by

$$-\mathbb{G}^{-1}_{\mathbf{k},\omega_n}\left[\delta\theta\left(\mathbf{x},\tau\right)\right]=\left(-i\omega_n-\zeta+\mathbf{Q}.\mathbf{k}+\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\mathbf{k}-\frac{i}{2}\nabla^2_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)\right)\sigma_0$$
$$+\left(k^2-\mu+\frac{Q^2}{4}+\frac{i}{2}\frac{\partial\delta\theta_{\mathbf{x},\tau}}{\partial\tau}+\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right).\frac{\mathbf{Q}}{2}+\frac{1}{4}\left[\nabla_{\mathbf{x}}\left(\delta\theta_{\mathbf{x},\tau}\right)\right]^2\right)\sigma_3+\Delta\sigma_1, \tag{A6}$$

with the Nambu-spinors

$$\bar{\eta}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n}=\begin{pmatrix}\bar{\psi}^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\uparrow} & \psi^{(\mathbf{x},\tau)}_{-\mathbf{k},-\omega_n,\downarrow}\end{pmatrix}\text{ en }\eta^{(\mathbf{x},\tau)}_{\mathbf{k}',\omega_n'}=\begin{pmatrix}\psi^{(\mathbf{x},\tau)}_{\mathbf{k},\omega_n,\uparrow}\\ \bar{\psi}^{(\mathbf{x},\tau)}_{-\mathbf{k},-\omega_n,\downarrow}\end{pmatrix}. \tag{A7}$$

Note that in (A5) the term proportional to $\bar{\psi}_{\downarrow}\psi_{\downarrow}$ has had its sum over momentum re-indexed as follows: $\mathbf{k}\to-\mathbf{k}$.

Appendix B: Obtaining the fluctuation action after Fourier expansion of the effective hydrodynamic action

The expansion of the action

$$S=-\int_0^\beta d\tau\int d\mathbf{x}\int\frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{1}{\beta}\log\left[2\cosh\left(\beta\tilde{E}^{(\theta)}_{\mathbf{k},\mathbf{Q}}\right)+2\cosh\left(\beta\tilde{\zeta}^{(\theta)}_{\mathbf{k},\mathbf{Q}}\right)\right]-\tilde{\xi}^{(\theta)}_{\mathbf{k},\mathbf{Q}}\right\}-\beta V\frac{\Delta^2}{g}, \tag{B1}$$

up to order $\delta\theta^2_{\mathbf{x},\tau}$ is done by first expanding the action to second order in $\mu^{(\theta)}_{\mathbf{Q}}$ and $\zeta^{(\theta)}_{\mathbf{k}}$, which are given by (17) and (16) respectively. After expansion of the integrand of (B1), the action can be divided into a saddle-point contribution (zeroth order in $\delta\theta_{\mathbf{x},\tau}$):

$$S_{sp}=-\int_0^\beta d\tau\int d\mathbf{x}\int\frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{1}{\beta}\log\left[2\cosh\left(\beta E_{\mathbf{k},\mathbf{Q}}\right)+2\cosh\left(\beta\zeta_{\mathbf{k},\mathbf{Q}}\right)\right]-\xi_{\mathbf{k},\mathbf{Q}}\right)-\beta V\frac{|\Delta|^2}{g}, \tag{B2}$$

and a fluctuation contribution (terms of higher order in $\delta\theta_{\mathbf{x},\tau}$ and its derivatives):

$$S_{fl}=-\int_0^\beta d\tau\int d\mathbf{x}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\left(1-\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}X\left(E_{\mathbf{k},\mathbf{Q}}\right)\right)\mu^{(\theta)}_{\mathbf{Q}}\right.$$
$$+\left[X\left(\zeta_{\mathbf{k},\mathbf{Q}}\right)\right]\zeta^{(\theta)}_{\mathbf{k}}+\left(\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)^2Y\left(E_{\mathbf{k},\mathbf{Q}}\right)+X\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\Delta^2}{E^3_{\mathbf{k},\mathbf{Q}}}\right)\frac{\left(\mu^{(\theta)}_{\mathbf{Q}}\right)^2}{2}$$
$$\left.+Y\left(E_{\mathbf{k},\mathbf{Q}}\right)\frac{\left(\zeta^{(\theta)}_{\mathbf{k}}\right)^2}{2}+2\tilde{Y}\left(E_{\mathbf{k},\mathbf{Q}}\right)\left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}}\right)\frac{\mu^{(\theta)}_{\mathbf{Q}}\zeta^{(\theta)}_{\mathbf{k}}}{2}\right], \tag{B3}$$

where the definitions in (20) were used. The saddle-point action corresponds to the result of [21]. Now the fluctuation action can be simplified further. Firstly, only the terms up to order q^4 are taken into account. Secondly, some terms may vanish due to e.g. boundary conditions. Now we briefly point out the main simplifications which can be performed. In the integral over $\mu_{\mathbf{Q}}^{(\theta)}$, the first two terms (we use the order of terms as given in (20)) vanish because of

$$\int dx \, \frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial x} Q_x = Q_x \left(\delta\theta_{+\infty,\tau} - \delta\theta_{-\infty,\tau} \right) = 0, \tag{B4}$$

and the boundary condition for bosons

$$\int_0^\beta d\tau \frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} = \delta\theta_{\mathbf{x},\beta} - \delta\theta_{\mathbf{x},0} = 0, \tag{B5}$$

respectively. In the integral over $\left(\mu_{\mathbf{Q}}^{(\theta)}\right)^2$, three terms vanish because they are of other $\delta\theta_{\mathbf{x},\tau}^3$ or higher. In the integral over $\zeta_{\mathbf{k}}^{(\theta)}$, the first term (again respecting the order as given in (20)) vanishes for the same reason as (B4), while the second term is zero because of another boundary condition

$$\int d\mathbf{x} \, \nabla_{\mathbf{x}}^2 \left(\delta\theta_{\mathbf{x},\tau} \right) = \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \Big|_{-\infty}^{+\infty} = 0. \tag{B6}$$

Furthermore, in the integral over $\mu_{\mathbf{Q}}^{(\theta)} \zeta_{\mathbf{k}}^{(\theta)}$, two terms again vanish because they are of order $\delta\theta_{\mathbf{x},\tau}^3$ or higher. Finally, the term

$$\frac{1}{4} \sum_k \int_0^\beta d\tau \int d\mathbf{x} \, \tilde{Y}\left(E_{\mathbf{k},\mathbf{Q}}\right) \left(\frac{\xi_{\mathbf{k},\mathbf{Q}}}{E_{\mathbf{k},\mathbf{Q}}} \right) \frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} \nabla_{\mathbf{x}}^2 \left(\delta\theta_{\mathbf{x},\tau} \right) \tag{B7}$$

equals zero. Using Green's first identity, the term (B7) becomes

$$\begin{aligned} \int_0^\beta d\tau \int d\mathbf{x} \, \frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} \nabla_{\mathbf{x}}^2 \left(\delta\theta_{\mathbf{x},\tau} \right) &= - \int_0^\beta d\tau \int d\mathbf{x} \, \nabla \left(\frac{\partial \delta\theta_{\mathbf{x},\tau}}{\partial \tau} \right) . \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \\ &= - \int_0^\beta d\tau \int d\mathbf{x} \, \frac{\partial \left[\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \right]}{\partial \tau} . \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right), \end{aligned} \tag{B8}$$

because $\int \left(\partial\delta\theta / \partial\tau \ \nabla\delta\theta \right) . d\mathbf{S} = 0$. Subsequently, we can write

$$- \int_0^\beta d\tau \int d\mathbf{x} \, \frac{\partial \left[\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \right]}{\partial \tau} . \nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) = - \int_0^\beta d\tau \int d\mathbf{x} \, \frac{1}{2} \frac{\partial \left\{ \left[\nabla_{\mathbf{x}} \left(\delta\theta_{\mathbf{x},\tau} \right) \right]^2 \right\}}{\partial \tau} = 0, \tag{B9}$$

again because of boundary conditions. Putting it all together, the fluctuation action becomes equal to (19).

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