INTERNATIONAL FINANCIAL INTEGRATION, CREDIT FRICTIONS AND EXCHANGE RATE REGIMES

Mara Pirovano
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Abstract

This paper studies the stabilisation properties of different exchange rate policies in a small open economy with cross-border balance sheet interdependence. The model features price and wage rigidities, credit frictions à la Bernanke, Gertler and Gilchrist (1999) both between households and banks and between banks and entrepreneurs, as well as international financial linkages à la Ueda (2012). I find that, overall, the argument in favor of flexible exchange rates holds irrespectively of the degree of financial integration. In fact, for all shocks considered, a fixed exchange rate policy delivers larger output losses and higher volatility of real and financial variables. Furthermore, my results reveal that the cost of pegging the exchange rate is inversely related to the degree of financial integration. Finally, I find that the presence of financial linkages increases the trade-off between inflation and output volatility faced by the central bank of a small open economy.

Keywords: Monetary policy rules, credit frictions, open economy, international transmission

JEL Classification: E44, E52, F41, F42

1 Introduction

Understanding the role of banks in cross-border finance has become an urgent research priority since the onset of the global crisis, as issues related to cross-border banking have played a central role in its origin and propagation. While the tightening of financial linkages is a worldwide phenomenon, it gained particular importance for the ex-members of the Soviet Union which joined the European Union in the enlargement waves of 2004 and 2007. Since the beginning of the transition process, financial integration between Eastern and Western Europe increased at a fast pace, especially in light of prospects of EU membership. In recent years, the degree of interdependence between old and new EU members increased markedly,

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particularly with the intensification of cross-border ownership in the banking sector. According to a study performed by the Committee of the Global Financial System (CGFS, 2010), in 2007 foreign bank participation, measured as consolidated lending of foreign banks as a share in total non-bank lending, amounted to 85% in Central and Eastern Europe compared to 45% in Western Europe.

The drawbacks of financial interdependence became evident during the financial crisis. As the new EU member states weren’t directly exposed to toxic assets, in a financially autarkic world they would have been hit by the crisis through exchange rate and external demand effects. However, given the high degree of financial interdependence with Western Europe (in turn heavily connected to American banks), the new EU member states got dragged into the spiral and suffered major losses in terms of GDP growth. In particular, the greatest magnitude of the economic downturn was observed in countries which adopted a pegged exchange rate regime: while in the Baltic countries GDP growth declined by more than 14% in 2009, in the Czech Republic and Hungary the decline was more modest (in the neighborhood of -5%), and Poland managed to achieve positive GDP growth. Motivated by this background, this paper studies the interplay between exchange rate regimes and financial integration in a two-country, general equilibrium setting characterized by real and financial frictions.

The recent global downturn led academics and practitioners to reinterpret a strand of literature dealing with imperfections in financial markets, pioneered by the seminal contributions of Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). These papers focused on the role of asymmetric information in credit contracts between borrowers and lenders in amplifying economic fluctuations in response to shocks (the so-called financial accelerator mechanism). In their framework, asymmetric information arises from the impossibility of lenders to adequately monitor borrowers: as borrowers have the incentive to misreport the outcome of a project, lenders are forced to verify the borrowers’ output, incurring a monitoring cost. Breaking the Modigliani-Miller theorem, they show that the cost of external finance is positively related to borrowers’ leverage and default probability. Hence, asset prices and borrowers’ net worth play a key role in determining the cost of funds, giving rise to a self-reinforcing mechanism in response to shocks.

The recent literature explored the effect of financial frictions in New Keynesian general equilibrium models in various directions, two of which are particularly relevant for the present study. On one side, a great number of studies concentrated on the monetary policy implications of financial frictions, both in closed economy context. In the latter case, particular attention has been devoted to the choice of exchange rate regime in economies characterized by liability dollarization, a feature common to many emerging economies which implies an explicit role of exchange rate fluctuations on borrowers’ balance sheets, thereby amplifying the effect of a shock with devaluation pressures on the domestic currency. This literature concludes that, in the presence of credit frictions, a flexible exchange rate regime delivers better stabilization in event of foreign shocks. Moreover, the presence of liability

1 Curdia and Woodford (2008), De Fiore and Tristani (2009), Kamber and Thonissen (2012).
dollarization increases the cost of a fixed exchange rate even further.

On the other hand, studies have begun to appear embedding banking linkages in macroeconomic models of the New Open Economy Macroeconomics paradigm and largely focused on explaining international business cycle co-movements. These studies affirm the importance of international financial linkages for the synchronization of business cycles. Furthermore, they highlight the role of financial frictions in reinforcing the international transmission of real and financial shocks.

This study aims at bridging the gap between these two strands of literature, and analyzes the stabilization properties of different exchange rate regimes in a small open economy obtained as a limit of a two country DSGE model, characterized by internationally operating banks subject to balance sheet constraints. To this end, I augment a two country version of the standard new keynesian DSGE model with price and wage rigidities (Smets and Wouters (2003), Christiano et al. (2005)) with chained credit contracts and cross-border loans following Ueda (2012). My analysis has multiple purposes. First, I analyze the transmission mechanism of foreign real and financial shocks under different exchange rate regimes and different degrees of financial integration. Second, I explore the implications of financial linkages for the ranking between fixed and flexible exchange rate regimes: in particular, I examine whether the stabilization properties of each exchange rate regime are affected by the degree of financial integration. Finally, I look at the implications of international financial linkages for the output-inflation trade-off faced by a small open economy’s monetary policy authority.

My analysis differs from previous studies mainly because it examines exchange rate policy in a world where even country-specific shocks generate positive international business cycle co-movements, obtained by modeling explicitly cross-country balance sheet interdependence. While in the literature analyzing exchange rate regimes in financially dollarized economies the degree of financial dollarization is taken as a proxy of financial interdependence, issues related to international spillovers through direct bank exposure to foreign borrowers are discarded. In a small open economy context, this rules out the analysis of foreign financial shocks, while in two country models, it limits the transmission of these shocks to real channels. In fact, in this context, the foreign financial shock feeds to the domestic economy only to the extent that changes in foreign investment and output alter international relative prices and demand, and trigger a reaction of the domestic central bank. In a model with explicit financial linkages, a second direct effect is present. A foreign financial shock is directly transmitted to the domestic economy, as domestic banks and lenders are involved in lending contracts with foreign agents. Furthermore, in this context the exchange rate plays a role in the balance sheets of banks and entrepreneurs in both countries, giving rise to additional inter-country dynamics.

Overall, I find that the existence of financial linkages does not alter the ranking between flexible and fixed exchange rate regimes. In particular, in response to negative real and financial shocks, a flexible exchange rate delivers smaller decline in output and lower volatility of real and financial variables, irrespective of the degree of financial integration. Furthermore, the degree of financial integration matters within a flexible exchange rate regime, but not within a fixed exchange rate regime. Under a flexible exchange rate regime, higher financial

\footnote{Mendoza and Quadrini (2009), Davis (2011), Kollmann et al. (2011), Yao (2012), Ueda (2012), Dedola, Karadi and Lombardo (2012).}
interdependence generates a stronger transmission of the foreign shock and leads to larger output and investment effects. The difference in the responses between financial integration regimes in less pronounced in the case of fixed exchange rate regime.

2 Literature Review

In recent years, many economists have explored the implications of imperfections in credit markets for monetary policy in small open economies and for the international transmission of shocks.

The analysis of the role of financial market imperfections in originating and transmitting business cycle fluctuations has become the subject of a large body of economic research. The majority of studies models credit market imperfections assuming the presence of asymmetric information between borrowers and lenders that generate an incentive for borrowers to misreport the outcomes of their project, making it costly for lenders to verify the quality of firms' investment (Carlstrom and Fuerst, 1997, Bernanke, Gertler and Gilchrist (henceforth, BGG) 1999). In this framework, internal and external finance are no longer substitutes, and borrowers are charged with an external finance premium tied to their balance sheet conditions. Therefore variables like net worth, leverage and borrowers' default probability play a key role in determining the cost of external funds, as opposed to the case of perfect credit markets.

In the context of small, open, emerging market economies, issues as vulnerability to external shocks, limited access to credit and foreign currency borrowing are particularly relevant, and have been incorporated in New Keynesian open economy models framework in order to study their monetary policy implications, with a particular focus on the choice of exchange rate regime. Although the insulating properties of flexible exchange rate regimes have been advocated since the times of Friedman (1953) and Mundell (1960), researchers started to question the validity of this claim in the presence of credit frictions and liability dollarization. While in a non-dollarized economy exchange rate movements affect primarily aggregate demand through a change in relative prices, when debt is denominated in foreign currency an additional balance sheet effect arises, which increases the domestic currency value of debt in case of a depreciation, increasing leverage and reducing investment. Therefore, the negative balance sheet effect offsets the expansion of aggregate demand brought about by the currency depreciation and, if it prevails, it offers an incentive for the central bank to limit exchange rate fluctuations adopting a pegged exchange rate. Cespedes, Chiang

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4In the literature, financial market imperfections have been introduced either in the form of collateral constraints (Kiyotaki and Moore (1997)) or assuming asymmetric information between borrowers and lenders leading to the existence of a premium on external finance (Carlstrom and Fuerst (1997), Bernanke et al. (1999)). In what follows, I concentrate on the latter approach since it is the focus of the presented model. Brzoza-Brezina et al. (2011) compare the properties of the two approaches in a consistent way, and find that the business cycle properties of the external finance premium framework are more in line with empirical evidence.

5Estimated DSGE models provide quantitative evidence in favor of the financial accelerator, and find that its presence improves the ability of models to capture the dynamics observed in the data (Elekdag et al.(2005), Christensen and Dib (2008) and Saxegaard et al. (2010)). Furthermore, Christiano et al. (2010) affirm the importance of financial shocks in accounting for a substantial portion of economic fluctuations.
and Velasco (2004) explore the stabilization properties of fixed and flexible exchange rate regimes in a dynamic general equilibrium model of a small open economy characterized by a financial accelerator and liability dollarization, concluding that, although balance sheet effects magnify the effect of external disturbances, a flexible exchange rate is still successful in insulating the economy from external shocks. This happens because adverse external shocks call for a real devaluation: while in case of flexible exchange rate this is achieved through a nominal devaluation that leaves real wages and employment unaffected, under a fixed exchange rate regime the real devaluation is obtained through deflation, which increases real wages and unemployment, thereby leading to a larger drop in investment and welfare. The superior stabilization properties of flexible exchange rates are confirmed by Devereux et al. (2006), which subject their small open economy to foreign interest rate and terms of trade shocks. However, their conclusion crucially hinges on the degree of exchange rate pass-through. With high pass-through, stabilizing the exchange rate implies a high trade-off between output and inflation volatility, since it requires a stronger interest rate response; when pass-through is low, exchange rate movements do not have a strong destabilizing effect on the price level and it is better for the central bank to focus on stabilizing inflation, while allowing for the currency to float. In a similar framework, Gertler et al. (2007) explore the issue of whether the exchange rate regime influences a country’s response to a financial crisis, defined as an exogenous increase in the country’s risk premium. They find that while the financial accelerator amplifies the effect of the shock, it does not alter the ranking between fixed and flexible exchange rate regimes: in particular, they find that the effect of the financial accelerator is more muted with a floating currency. Concerning liability dollarization, they conclude that, although it lowers the attractiveness of a flexible exchange rate, this still leads to a smaller output drop. While these papers treat the foreign economy as exogenous, Batini et al. (2007) study the monetary policy implications of increased degrees of financial frictions and dollarization in a small open economy obtained as the limit case of a two-country DSGE model and characterize the optimal monetary policy in this setting. They conclude that the financial accelerator has a larger impact on the performance of monetary policy rules than the presence of liability dollarization: in particular, targeting the exchange rate is not optimal, as exchange rate movements attenuate the effect of financial frictions. Furthermore, the cost of pegging the exchange rate increases with the degree of financial frictions. 

Models with frictions in credit markets have also been used to explore the role of financial markets in the international transmission of shocks and international business cycle correlations. While increased business cycle co-movement has been observed even when countries are hit by asymmetric shocks, this result does not emerge from traditional open economy models (see for example Gali’ and Monacelli (2002)). In fact, in these models the international transmission of shocks happens through international trade and demand switching effects. In a series of papers, Faia (2001, 2002, 2007, 2010) extends the financial accelerator model to a two country framework, and finds that credit frictions enrich the international transmission mechanism with an "indirect financial spillover effect" which can be strong enough to offset the expenditure-switching effect and yield a wide range of business cycle correlations. In particular, Faia (2002) finds that the magnitude of the financial spillover effect increases with the degree of financial similarity between countries, leading to positive
In a similar framework, Gilchrist (2003) explores the role of financial leverage in the international transmission of shocks. Specifically, he focuses on the transmission of shocks from developed countries (characterized by lower levels of leverage) and developing countries (highly leveraged). His results suggest that, not only slowdowns in economic activity are severely amplified by financial frictions, but high-leverage economies are particularly vulnerable to external shocks, and that asymmetries between lending conditions across economies provide a strong source of transmission for shocks from developed to developing economies. Concerning the interplay between financial frictions and exchange rate regimes in a similar model, Faia (2010) argues for the superiority of a floating exchange rate in isolating a country from foreign shocks. Furthermore, the desirability of a floating currency is enhanced by the presence of financial frictions: not only the output-inflation trade-off is steeper when the currency is pegged, but its intensity increases with the degree of credit frictions. While this modeling approach offers interesting theoretical insights on the role of financial frictions in altering the international transmission of shocks, it still fails on empirical grounds, to replicate observed business cycle correlations. Alpanda and Aysun (2012) estimate a model in the spirit of Gilchrist (2003) and Faia (2010) with Bayesian methods in order to test its ability to reproduce Euro Area responses to US shocks. They find that the model is able to generate meaningful business cycle correlations only when allowing for correlated shocks across countries.

Extending the framework of an increasing number of studies incorporating a banking sector is standard DSGE models to an open economy setting, a recent strand of literature proved successful in producing models capable of accounting for the observed business cycle correlations and international transmission of country-specific financial shocks. Motivated by the large observed cross-country spillovers of financial shocks during the financial crisis, many studies have appeared embedding international financial linkages in two country models of the New Keynesian paradigm (Mendoza and Quadrini (2009), Davis (2011), Kollmann et al. (2011), Yao (2012), Ueda (2012), Dedola, Karadi and Lombardo (2012)). The key feature of these models is the simultaneous presence of frictions in credit markets, and financial institutions engaged in cross-border lending. In this setting, international credit contracts generate cross-country financial interdependence, as balance sheet conditions of borrowers in one country will affect financial institutions in any country financially linked to it. On one side, international lending constitutes an additional channel through which foreign shocks}

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6In a later paper, Faia (2007) explores the effect of different monetary policy rules (currency union, unilateral peg and inflation targeting) in a similar model with financial differences. She finds that international business cycle synchronization is enhanced in a currency area compared to an independent policy regime. Furthermore, under the unilateral peg, the business cycle co-movements are very close to the ones arising under the currency area.

7The recognition of the key importance of the banking sector in originating and propagating shocks led many researchers to model financial intermediation in closed economy DSGE models. A non-exhaustive list of such studies includes Hirakata et al. (2009), Davis (2010), Meh and Moran (2010), Dib (2010), Gerali et al. (2010), Christiano et al. (2010), Gertler and Karadi (2011), Rannenberg (2011). Estimated versions of these models reveal their ability to fit the data (especially financial variables) quite well and that banking sector shocks are important in explaining macroeconomic fluctuations (Hirakata et al (2010), Villa and Yang (2011)).
are transmitted. While in case of financial autarky, say, a foreign monetary policy shock is transmitted internationally to the extent that it alters bilateral exchange rates, relative prices and international demand patterns, in a financially integrated world it has an additional direct effect through its impact on the banking sector and on lending rates in any country borrowing from on the one where the shock originates. On the other hand, modeling financial linkages explicitly allows to study the inter-country transmission of financial shocks, which directly impact other countries through cross-border financial exposures. Adopting different modeling approaches, these studies broadly affirm the importance of cross-country banking exposures in the propagation of country-specific real and financial shocks. Compared to the case of financial autarky, the interdependence of balance sheet conditions resulting from cross-border banking activities yields larger co-movement of business cycles and a stronger spillover of disturbances.

3 Model

The model portrays two countries with symmetric structures but different sizes. It is assumed that the home country (denoted by the superscript $H$) is of size $n$, while the foreign country ($F$) is of size $(1-n)$, where $n \in [0,1]$. In the calibration, I set $n \rightarrow 0$, allowing me to model the Home country as a small open economy while still allowing for trade and financial linkages. On the real side, the two economies are characterized by imperfectly competitive product and labor markets, coupled with Calvo pricing and wage setting whose implied inefficiencies warrant an explicit role for monetary policy. Each economy produces a variety of internationally traded intermediate goods, where the total number of producers is normalized to unity, so that $n$ firms are located in the Home country, while the remaining fraction resides in the Foreign country. While international financial markets are complete from the point of view of households, implying that the real exchange rate is related to the ratio of marginal utilities of consumption, credit markets are characterized by frictions at different stages of the intermediation process, in the spirit of Ueda (2012). In particular, in each country a continuum of financial intermediaries lends to entrepreneurs in both countries and finances its loan portfolio by collecting deposits from domestic and foreign households. The presence of asymmetric information between households and financial intermediaries and between financial intermediaries and entrepreneurs implies a costly state verification problem leading to an optimal contract whereby the cost of external finance is tied to balance sheet conditions. Hence, in contrast with the standard model of the financial accelerator model, the leverage of financial institutions, together with that of entrepreneurs, plays a role in determining the tightness of credit conditions in the economy. Furthermore, integration in international credit markets implies interdependency of credit conditions between countries, strengthening the degree of business cycle correlation, as shown by Ueda (2012).
3.1 Households

In each country the preferences of the representative household are represented by the utility function:

\[ E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( C_t - hC_{t-1}\right)^{1-\sigma} \frac{1}{1-\sigma} - \chi H_t^{1+\varphi} \right\} \]

Where \( C_t \) and \( H_t \) are composites of consumption goods and labor services respectively, and \( h \) is the degree of consumption habit formation. The consumption index \( C \) is a Cobb-Douglas aggregate of home produced \( (C_{H,t}) \) and imported \( (C_{F,t}) \) goods, where \( \gamma \) is the share of domestic good in the consumption basket of Home households:

\[ C_t = \frac{C_{H,t}^{\gamma} C_{F,t}^{1-\gamma}}{\gamma(1-\gamma)^{1-\gamma}} \]

As in De Paoli (2009) and Corsetti and Müller (2011), I assume that the relative weight of domestic and foreign goods in the consumption bundle is a function of the relative country size and the degree of trade openness. In particular, I assume \( (1 - \gamma) = (1 - n)\lambda \), where \( \lambda \in (0, 1) \) is the openness parameter: when \( \lambda = 1 \) there is no home bias, and the share of imported goods in consumption equals \((1 - n)\). A similar specification holds for the Foreign economy, where the aggregate consumption bundle is

\[ C_t^* = \frac{C_{H,t}^{\gamma^*} C_{F,t}^{1-\gamma^*}}{\gamma^*(1-\gamma^*)^{1-\gamma^*}} \]

and \( \gamma^* = n\lambda. \) Hence, in the limit case when the Home economy becomes small \((n \to 0)\), \( \gamma \to \lambda \) and \( \gamma^* \to 0 \), the Foreign country becomes closed, but, as long as \( \lambda > 0 \), the Home country consumes Foreign goods.

The consumption sub-indices \( C_{H,t}, C_{F,t}, C^*_{H,t} \) and \( C^*_{F,t} \) are in turn aggregates of intermediate goods produced in the Home and foreign country, i.e.:

\[ C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_t(j) \frac{1}{\varepsilon} dj \right]^{\frac{1}{\varepsilon-1}} \; ; \;
C^*_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n c_t(j) \frac{1}{\varepsilon} dj \right]^{\frac{1}{\varepsilon-1}} \]

\[ C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_t(j) \frac{1}{\varepsilon} dj \right]^{\frac{1}{\varepsilon-1}} \; ; \;
C^*_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 c_t^*(j) \frac{1}{\varepsilon} dj \right]^{\frac{1}{\varepsilon-1}} \]

Where \( \varepsilon > 0 \) is the elasticity of substitution between different varieties of intermediate goods. The price indices corresponding to the consumption bundles in the two countries are, respectively:

\[ P_t = P_{H,t}^\gamma P_{F,t}^{1-\gamma} \] (1)

\(^8\)Here I follow a notation whereby subscripts refer to the country where the good is produced (H or F). The presence (absence) of an asterisk indicates that the good is consumed or used as an input in the foreign (domestic) country.
Households choose the optimal allocation of expenditure between domestic and imported consumption goods solving an expenditure minimization problem, which results in the following optimality conditions:

\[ P_t^* = P_{H,t}^{\gamma^*} P_{F,t}^{1-\gamma^*} \]  

(2)

Following Schmitt-Grohé and Uribe (2006), I assume the existence of a continuum of labor markets of measure 1 indexed by \( i \in [0, 1] \), in each of which wages are set by a monopolistically competitive union facing downward sloping labor demand, given by \( \frac{W_t(i)}{W_t} = \frac{H_t^d}{H_t} \), where \( W_t(i) \) denotes the wage set by the wage union for the \( i-th \) labor market, \( H_t^d \) denotes labor demand by firms, \( \varepsilon_w \) represents the elasticity of substitution between different labor types, and the aggregate wage prevailing in the economy is given by

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\varepsilon_w} di \right]^{\frac{1}{1-\varepsilon_w}} \]  

(5)

Given the contracted \( W_t(i) \), the union is assumed to supply enough labor to satisfy demand, i.e. \( H_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} H_t^d \). This condition, coupled with the requirement that the total labor supply satisfies the market clearing condition \( H_t = \int_0^1 H_t(i) di \), yields:

\[ H_t = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} H_t^d \]  

(6)

The union then takes \( W_t \) and \( H_t^d \) as given and sets the optimal wage \( \bar{W}_t(i) \) so as to equate the union’s expected average marginal return with the marginal cost of supplying labor. However, in doing so the union faces nominal rigidities in the Calvo fashion. Specifically, in each period the wage can be optimized only in a fraction \((1 - \theta^w)\) of labor markets. In the remaining fraction \( \theta^w \) the real wage is indexed to past inflation and it is therefore given by:

\[ W_t(i) = W_{t-1}(i) \pi_{t-1} \]

The reoptimizing union sets the optimal wage \( \bar{W}_t(i) \) so as to maximize:

\[
E_t \sum_{k=0}^{\infty} (\beta \theta^w)^k \mu_{t+k} \left( \frac{\bar{W}_t \prod_{i=1}^{k} \left( \frac{\pi_{t+i-1}}{\pi_{t+i}} \right)}{W_{t+k}} \right)^{-\varepsilon_w} H_t^d \left[ \bar{W}_t \prod_{i=1}^{k} \left( \frac{\pi_{t+i-1}}{\pi_{t+i}} \right) - \frac{U_H'}{\mu_{t+k}} \right]
\]

\[ \]  

9In what follows I drop the index \((i)\) as all firms allowed to reoptimize in a given period set the same wage.
Where the first term in parenthesis represents the marginal gain for the union of supplying an extra unit of labor, and the second term represents the marginal disutility of doing so. The first order conditions can be formulated in the following recursive fashion:

\[
K^w_t = \left( \frac{\varepsilon_w - 1}{\varepsilon_w} \right) \bar{W}_t H_t \left( \frac{W_t}{\bar{W}_t} \right)^{\varepsilon_w} + \beta \theta^w \left( \frac{\pi_{H,t+1}}{\pi_{H,t}} \right)^{\varepsilon_w} K^w_{t+1} \tag{7}
\]

\[
F^w_t = \chi_H \left( H^\phi_t \right)^{\varphi} \left( \frac{W_t}{W^*_{t+1}} \right)^{\varepsilon_w} + \beta \theta^w \left( \frac{\pi_{H,t+1}}{\pi_{H,t}} \right)^{\varepsilon_w} F^w_{t+1} \tag{8}
\]

\[
K^w_t = F^w_t \tag{9}
\]

It follows from (5) and staggered wage setting that the law of motion of the aggregate wage is\textsuperscript{10}:

\[
W_t = \left( 1 - \theta^w \right) \bar{W}_t^{1-\varepsilon_w} + \theta^w \left( W_{t-1(i)} \pi_{t-1} \right)^{1-\varepsilon_w} \tag{10}
\]

Besides labor income, households receive dividends from ownership of domestic firms and returns from their investments in domestic and international asset markets. Households have access to a complete set of internationally traded Arrow-Debreu securities. They acquire a portfolio \( B(s_{t+1}) \) of real state-contingent securities, each of which pays one unit in \( t + 1 \) at the occurrence of the state of nature \( s_{t+1} \), which carries the pricing kernel \( m(s_{t+1}|s_t) \). Furthermore, they can invest in real deposits in Home or Foreign financial intermediaries (denoted respectively \( D_t \) and \( D^*_t \))\textsuperscript{11}, which yield a one period return of \( D_t R_t \) and \( D^*_t R^*_t \).

The budget constraint of the representative household in the Home country can then be formulated in real terms as:

\[
C_t + D_t + \varepsilon_t D^*_t + \sum_{s_{t+1}} m(s_{t+1}|s_t) B(s_{t+1}) \leq H_t \int_0^1 W_t(i) \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} di + R_{t-1} D_{t-1} + R^*_{t-1} \varepsilon_{t-1} D^*_{t-1} + D_t + \Pi^H_t - T_t
\]

Where \( \varepsilon_t = \frac{S_t P_t^*}{P_t} \) is the real exchange rate. The first order conditions deriving from the household’s optimization problem define the optimal intertemporal consumption path and labor supply which, denoting as \( \mu_t \) the Lagrange multiplier associated with the budget constraint, are given by:

\textsuperscript{10}Staggered wage setting implies an inefficient wage dispersion, arising from the fact that wages are not set simultaneously. The law of motion of such wage dispersion, defined as \( \Delta_{w,t} = \frac{1}{\varepsilon_w} \int_0^1 \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} di \), is given by:

\[
\Delta_{w,t} = (1 - \theta^w) \left( \frac{W_t}{\bar{W}_t} \right)^{\varepsilon_w} + \theta^w \left( \frac{W_t}{W_{t-1(i)}} \pi_{H,t-1} \right)^{\varepsilon_w} \Delta_{w,t-1}
\]

So that the effective labor supply is \( H_t = H^\mu_t \Delta_{w,t} \).

\textsuperscript{11}Although deposits are redundant, their presence is needed as they are demanded from domestic and foreign financial institutions, and necessary to satisfy the market clearing conditions in the general equilibrium. For a similar specification, cfr. Faia (2010).
\[ \mu_t = (C_t - hC_{t-1})^{-\sigma} \]  
(11)

\[ \beta E_t \left\{ \frac{\mu_{t+1}}{\mu_t} \frac{\pi_{t+1}}{\pi_t} \right\} = m(s_{t+1}|s_t); \quad \frac{1}{R_t} = \sum_{s_{t+1}} m(s_{t+1}|s_t) \]  
(12)

\[ w_t = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi_H H_t^E}{\mu_t} \]  
(13)

Where equation (12) results from maximization with respect to bonds and deposits. In particular, it states that the price of the state contingent portfolio relates to the changes in marginal utility of consumption and that, in equilibrium, the expected return of the state contingent portfolio has to equal that of deposits (arbitrage condition). Note that in the absence of staggered wage setting, (13) reduces to a standard labor supply equation.

An analogous utility maximization problem applies in the Foreign economy. In particular, the Foreign counterpart of equation (12) reads:

\[ \beta E_t \left\{ \frac{\mu^*_t}{\mu_t} \frac{\pi_{t+1}}{\pi_t} \frac{\varepsilon_t}{\varepsilon_{t+1}} \right\} = m(s_{t+1}|s_t); \quad \frac{1}{R^*_t} \frac{\varepsilon_t}{\varepsilon_{t+1}} = \sum_{s_{t+1}} m(s_{t+1}|s_t) \]  
(14)

From (12) and (14) I obtain:

\[ \frac{\mu_{t+1}}{\mu_t} = \frac{\mu^*_t}{\mu_t} \frac{\varepsilon_t}{\varepsilon_{t+1}} \]  
(15)

\[ \frac{1}{R_t} = \frac{1}{R^*_t} \frac{\varepsilon_t}{\varepsilon_{t+1}} \]  
(16)

From which an expectational version of the uncovered interest parity condition results:

\[ \sum_{s_{t+1}} m(s_{t+1}|s_t) \left[ R_t - R^*_t \frac{\varepsilon_{t+1}}{\varepsilon_t} \right] = 0 \]

### 3.2 Production

There exist a continuum of monopolistic intermediate good producers in each country, indexed \( j_H \) and \( j_F \) respectively. Each producer operates under monopolistic competition and is owned by households, with the demand for its products given by:

\[ Y_t(j_H) = \left( \frac{P_{H,t}(j_H)}{P_{H,t}} \right)^{-\varepsilon_H} Y_t \]

Producers use capital and three types of labor inputs (\( H_t, H^E_t \) and \( H^F_t \), supplied respectively by households, entrepreneurs and bankers) to produce differentiated goods. The production function for domestic intermediate good producers is given by:

\[ Y_t(j_H) = A_t K_t^\alpha (j_H) H(j_H)^{(1-\alpha)(1-\Omega_E-\Omega_F)} H^E_t(j_H)^{(1-\alpha)\Omega_E} H^F_t(j_H)^{(1-\alpha)\Omega_F} \]  
(17)
Where $\alpha$ is the share of capital in production, $\Omega_E$ and $\Omega_F$ are the shares of entrepreneurial and bankers’ labor in production. Cost minimization implies the following standard factor demand functions, where $r^K_t$ denotes the rental rate of capital:

\[
W_t = MC_t (1 - \alpha) (1 - \Omega_E - \Omega_F) \frac{Y_t}{H_t} \quad (18)
\]
\[
W^E_t = MC_t (1 - \alpha) \Omega_E \frac{Y_t}{H^E_t} \quad (19)
\]
\[
W^F_t = MC_t (1 - \alpha) \Omega_F \frac{Y_t}{H^F_t} \quad (20)
\]
\[
r^K_t = MC_t \alpha \frac{Y_t}{K_t} \quad (21)
\]

### 3.3 Price setting

Price setting is staggered. In each period, only a fraction $(1 - \theta_H)$ of firms are allowed to reset their price optimally. The fraction $\theta_H$ that is not allowed to optimize in each period sets the price equal to that prevailing in the previous period, indexing it to past inflation at a rate $\gamma_p$ and to the steady state inflation rate at rate $(1 - \gamma_p)$. Hence, denoting as $\tilde{P}_{H,t}$ the optimal reset price, the law of motion of the domestic good price evolves as:

\[
P_{H,t} = \left[ \theta_H \left( P_{H_{t-1}, \pi_{t-1}^{\gamma_p}} (\pi_{ss}^{\gamma_p})^{1-\gamma_p} \right)^{1-\epsilon_H} + (1 - \theta_H) \tilde{P}_{H,t} \right]^{\frac{1}{1-\epsilon_H}} \quad (22)
\]

The firm then chooses the optimal price $\tilde{P}_{H,t}(j_H)$ so as to maximize the discounted sum of future real profits. Hence, each producer maximizes:

\[
E_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_H)^k \frac{\mu_{t+k}}{\mu_t} \left[ \left( \frac{\tilde{P}_{H,t}(j_H)}{P_{H_{t+k}}} \prod_{i=1}^{k} \pi_{t+i-1}^{\gamma_p} (\pi_{ss}^{\gamma_p})^{1-\gamma_p} \right)^{1-\epsilon_H} - mc_{t+k} \left( \frac{\tilde{P}_{H,t}(j_H)}{P_{H_{t+k}}} \prod_{i=1}^{k} \pi_{t+i-1}^{\gamma_p} (\pi_{ss}^{\gamma_p})^{1-\gamma_p} \right)^{-\epsilon_H} \right] Y_{t+k} \right\}
\]

The first order conditions can be written in a recursive manner as follows:

\[
\tilde{P}_{H,t} \left( \frac{F_{H,t}}{P_{H,t}} \right)^{\frac{\epsilon_H}{\epsilon_H - 1}} D_{H,t} \quad (23)
\]
\[
F_{H,t} = \frac{\Lambda}{P_t} P_{H,t}mc_{H,t}Y_t^H + \beta \theta_H E_t \left\{ (\bar{\pi}_{t+1})^{\epsilon_H} F_{H,t+1} \right\} \quad (24)
\]
\[
D_{H,t} = \frac{\Lambda}{P_t} P_{H,t}Y_t^H + \beta \theta_H E_t \left\{ (\bar{\pi}_{t+1})^{\epsilon_H-1} D_{H,t+1} \right\} \quad (25)
\]

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3.4 Capital goods producers

Capital producers operate in a regime of perfect competition. In each period, they combine investment goods \((I_t, \text{ with price } P^I_t)\) and old undepreciated capital \(((1 - \delta)K_t, \text{ purchased from entrepreneurs at price } Q^H_t)\) to produce new capital goods, which will be sold at the real price \(Q^H_t\). Investment is subject to adjustment costs, represented by the function \(\Phi_t = \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2\) (Smets and Wouters (2003)). Capital producers choose the optimal amount of investment\(^{12}\) so as to maximize the following profit function:

\[
E_t \left\{ \sum_{k=0}^{\infty} \beta^k \frac{\mu_{t+k}}{\mu_t} \frac{1}{P^H_{t+k}} \left[ q^H_t P^H_{t+k} ((1 - \delta) K_{t+k} + (1 - \Phi_t) I_{t+k} - K_{t+k}) - P^H_{t+k} I_{t+k} \right] \right\}
\]

The first order condition with respect to \(I_t\) yields:

\[
1 = q^H_t \left[ 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} q^H_{t+1} \left[ \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \right\}
\]

Where \(q^H_t\) is the real price of the capital stock defined as \(\frac{Q^H_t}{P^H_t}\). The law of motion of the economywide capital stock is:

\[
K_{t+1} = \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t + (1 - \delta) K_t
\]

3.5 Entrepreneurs, banks and international lenders

Credit markets are characterized by chained credit contracts in the spirit of Hirakata et al. (2009) and Ueda (2012). Entrepreneurs in a given country borrow from domestic and foreign financial intermediaries (banks) to finance capital purchases. In turn, financial intermediaries in each country borrow from domestic and foreign investors in order to finance their loan portfolio. The presence of financial frictions in both contracts (i.e. between entrepreneurs and financial intermediaries and between financial intermediaries and international investors) makes the external finance premium faced by entrepreneurs in each country dependent on balance sheet conditions in the other country.

In each country a continuum of entrepreneurs purchase unfinished capital goods from capital producers and transform them into finished capital goods through a stochastic technology. Capital is then rented to firms at the rental rate \(r^K_t\). Entrepreneurs finance capital purchases partly using their own net worth \((NW^E_{H,t})\) and partly borrowing from domestic

\(^{12}\)The investment bundle has a similar composition as the consumption bundle, and can therefore be defined as:

\[
I_t = \frac{I^E_{H,t} T_t^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}
\]
entrepreneurs in the home country use a fraction \( (1 - \tau_H^E) \) of net worth to borrow from home intermediaries and purchase an amount \( (1 - \tau_H^E) Q_{H,t} K_{HH,t} \) of capital, and a fraction \( \tau_H^E \) to borrow from foreign intermediaries and purchase \( \tau_H^E Q_{H,t} K_{HF,t} \). Entrepreneurs in the foreign economy behave analogously. The credit contract is characterized by asymmetric information in the spirit of Bernanke, Gertler and Gilchrist (1999).

Denote \( R_{HH,t}^E \) and \( R_{HF,t}^E \) the expected return from capital investment of home entrepreneurs borrowing from domestic and foreign financial intermediaries respectively. The return to capital is made of the return from selling capital to production firms and the return from selling undepreciated capital to capital producers\(^{14}\):

\[
R_{HH,t}^E = \frac{r_t^K + (1 - \delta) Q_{H,t}}{Q_{H,t-1}}
\]

\[
R_{HF,t}^E = \frac{r_t^K + (1 - \delta) Q_{H,t}}{Q_{H,t-1}}
\]

### 3.5.1 Contract between financial intermediaries and entrepreneurs

Let us consider the problem of financial intermediaries in the home country, which stipulate credit contracts with entrepreneurs in the home and foreign country.

Entrepreneurs in the Home country owns net worth \( NW_{H,t}^E \) and uses a fraction \( (1 - \tau_H^E) \) of it to finance a capital expenditure of \( (1 - \tau_H^E) Q_{H,t} K_{HH,t} \). Hence, the Home entrepreneur borrows an amount given by \( L_{HH,t}^E = (1 - \tau_H^E) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) \) from domestic financial intermediaries. Entrepreneurs borrowing in the Home country are subject to a stochastic shock \( \omega_{H,t}^E \), so that the return to capital is \( R_{HH,t+1}^E \omega_{H,t+1}^E \).\(^{15}\) Foreign entrepreneurs borrowing from domestic financial intermediaries behave analogously, using a fraction \( \tau_F^E \) of their net worth \( NW_{F,t}^E \) to borrow \( L_{HF,t}^E = \tau_F^E (Q_{F,t} K_{HF,t} - NW_{F,t}^E) \) from domestic financial intermediaries, and they are characterized by the stochastic technology \( \omega_{HF,t+1}^E \).

The optimal contract (see Calstrom ad Fuest (1997)) specifies a state-contingent loan rate \( Z_{ij,t+1}^E \) and a threshold \( \omega_{ij,t+1}^E \), such that for realizations \( \omega_{ij,t+1}^E > \omega_{ij,t+1}^E \), entrepreneurs repay the loan at the contractual rate and keep the remaining proceeds of their investment, while for realizations \( \omega_{ij,t+1}^E < \omega_{ij,t+1}^E \), entrepreneurs default on their debt, financial intermediaries pay a monitoring cost to verify entrepreneurial output and seize the entrepreneur’s remaining assets, leaving the defaulting entrepreneur with a zero payoff. It is then possible to define the threshold productivity level as the minimum realization of productivity that

\(^{13}\)In what follows, the subscripts \( i,j = H,F \) refer, respectively, to the nationality of the agent and the origin of the loan. Furthermore, the superscript \( E \) pertains to entrepreneurs, while \( F \) denotes financial intermediaries. Therefore, \( R_{HF,t}^E \) denotes the return to capital for entrepreneurs in country \( H \) borrowing from country \( F \).

\(^{14}\)Note that capital is homogeneous within each country. The notation \( K_{HH} \) and \( K_{HF} \) is introduced for convenience in the calculations, but within each country there is one capital stock (which results from aggregating the quantities purchased by the two types of entrepreneurs) and one asset price \( Q \).

\(^{15}\)As in Bernanke, Gertler and Gilchrist (1999) I assume that the stochastic shock is \( iid \) across entrepreneurs and time, and follows a log-normal distribution with density \( f(\cdot) \) and cdf \( F(\cdot) \) and \( E(\omega) = 1 \).
allows entrepreneurs to repay their debts. For domestic and foreign entrepreneurs borrowing from Home financial intermediaries the thresholds \( \tilde{\omega}_{HH,t+1}^E \) and \( \tilde{\omega}_{FH,t+1}^E \) are defined as:

\[
\tilde{\omega}_{HH,t+1}^E R_{HH,t+1}^E Q_{HH,t} K_{HH,t} = Z_{HH,t+1}^E \left( Q_{HH,t} K_{HH,t} - NW_{HH,t}^E \right)
\]

\[
\tilde{\omega}_{FH,t+1}^E R_{FH,t+1}^E Q_{FH,t} K_{FH,t} = Z_{FH,t+1}^E \left( Q_{FH,t} K_{FH,t} - NW_{FH,t}^E \right)
\]

Where the left hand side represents the return to the entrepreneur corresponding to productivity level \( \tilde{\omega}_{ij,t+1}^E \), and the right hand side represents the required repayments on the contracted loan.

The expected returns of Home and Foreign entrepreneurs from the contract with domestic financial intermediaries are given, respectively, by:

\[
(1 - \tau_H^E) R_{HH,t+1}^E Q_{HH,t} K_{HH,t} \left[ \int_{-\infty}^{\tilde{\omega}_{HH,t+1}^E} \omega_{HH,t+1}^E f(\omega_{HH,t+1}^E) d\omega_{HH,t+1} - \tilde{\omega}_{HH,t+1}^E \int_{-\infty}^{\tilde{\omega}_{HH,t+1}^E} f(\omega_{HH,t+1}^E) d\omega_{HH,t+1} \right]
\]

\[
\tau_F^E R_{FH,t+1}^E Q_{FH,t} K_{FH,t} \left[ \int_{-\infty}^{\tilde{\omega}_{FH,t+1}^E} \omega_{FH,t+1}^E f(\omega_{FH,t+1}^E) d\omega_{FH,t+1} - \tilde{\omega}_{FH,t+1}^E \int_{-\infty}^{\tilde{\omega}_{FH,t+1}^E} f(\omega_{FH,t+1}^E) d\omega_{FH,t+1} \right]
\]

Which, defining

\[
\Gamma_{HH}^E (\tilde{\omega}_{HH,t+1}^E) = \omega_{HH,t+1}^E \left( 1 - \int_{0}^{\tilde{\omega}_{HH,t+1}^E} f(\omega_{HH,t+1}^E) d\omega_{HH,t+1} \right) + \int_{0}^{\tilde{\omega}_{HH,t+1}^E} \omega_{HH,t+1}^E f(\omega_{HH,t+1}^E) d\omega_{HH,t+1}
\]

\[
\Gamma_{FH}^E (\tilde{\omega}_{FH,t+1}^E) = \omega_{FH,t+1}^E \left( 1 - \int_{0}^{\tilde{\omega}_{FH,t+1}^E} f(\omega_{FH,t+1}^E) d\omega_{FH,t+1} \right) + \int_{0}^{\tilde{\omega}_{FH,t+1}^E} \omega_{FH,t+1}^E f(\omega_{FH,t+1}^E) d\omega_{FH,t+1}
\]

can be rewritten as:

\[
\left[ 1 - \Gamma_{HH}^E (\tilde{\omega}_{HH,t+1}^E) \right] \left( 1 - \tau_H^E \right) R_{HH,t+1}^E Q_{HH,t} K_{HH,t}
\]

\[
\left[ 1 - \Gamma_{FH}^E (\tilde{\omega}_{FH,t+1}^E) \right] \tau_F^E R_{FH,t+1}^E Q_{FH,t} K_{FH,t}
\]

Where \( \left[ 1 - \Gamma_{ij}^E (\tilde{\omega}_{ij,t+1}^E) \right] \) represents the share of payoff captured by the entrepreneur, i.e. the payoff from her capital investment minus loan repayments multiplied by the probability that the entrepreneur does not default\(^\text{16}\).

Entrepreneurs engage in the debt contract only if the expected return of doing so is at least equal to the payoff they would obtain if they invested only their own net worth. Hence, the following participation constraints for domestic and foreign entrepreneurs have to hold:

\(^\text{16}\)Recall that given a pdf \( f(\omega) \), \( \int_{0}^{\tilde{\omega}} f(\omega) d\omega = \Pr(\omega \leq \tilde{\omega}) \), which in this case corresponds to the probability of default, and \( \int_{0}^{\bar{\omega}} \omega f(\omega) d\omega = E(\omega|\omega \leq \tilde{\omega}) \). Furthermore, recall that the payoff of defaulting entrepreneurs is zero.
\[ [1 - \Gamma^E_{HH}(\omega^E_{HH,t+1})] R^E_{HH,t+1} Q_{HH,t} K_{HH,t} \geq R^E_{HH,t+1} NW^E_{HH,t} \]  
\[ [1 - \Gamma^E_{FH}(\omega^E_{FH,t+1})] R^E_{FH,t+1} Q_{FH,t} K_{FH,t} \geq R^E_{FH,t+1} NW^E_{FH,t} \]  

The expected payoff of the domestic financial intermediary from lending to home entrepreneurs is given by the loan repayment in the case the entrepreneur does not default and by the remaining payoff of the entrepreneurs minus monitoring costs in case the entrepreneur defaults:

\[
\begin{align*}
&\left\{ \begin{array}{l}
Z^E_{HH,t+1} (1 - \tau^E_H) (Q_{HH,t} K_{HH,t} - NW^E_{HH,t}) \left[ \int_{\omega^E_{HH,t+1}}^{\infty} f(\omega_{HH,t+1}) d\omega_{HH,t+1} \right]
\end{array} \right. \\
&\quad + (1 - \tau^E_H) R^E_{HH,t+1} Q_{HH,t} K_{HH,t} \left[ (1 - \mu^E_H) \int_{\omega^E_{HH,t+1}}^{\infty} \omega_{HH,t+1} f(\omega_{HH,t+1}) d\omega_{HH,t+1} \right] \\
&= (1 - \tau^E_H) R^E_{HH,t+1} Q_{HH,t} K_{HH,t} \left[ \int_{0}^{\omega^E_{HH,t+1}} \omega_{HH,t+1} f(\omega_{HH,t+1}) d\omega_{HH,t+1} + (1 - \mu^E_H) \int_{0}^{\omega^E_{HH,t+1}} \omega_{HH,t+1} f(\omega_{HH,t+1}) d\omega_{HH,t+1} \right]
\end{align*}
\]

Denoting:

\[ \Gamma^E_{HH}(\omega^E_{HH,t+1}) = \omega^E_{HH,t+1} \left( 1 - \int_{0}^{\omega^E_{HH,t+1}} f(\omega^E_{HH,t+1}) d\omega^E_{HH,t+1} \right) + \int_{0}^{\omega^E_{HH,t+1}} \omega^E_{HH,t+1} f(\omega^E_{HH,t+1}) d\omega^E_{HH,t+1} \]

\[ G^E_{HH}(\omega^E_{HH,t+1}) = \int_{0}^{\omega^E_{HH,t+1}} \omega^E_{HH,t+1} f(\omega^E_{HH,t+1}) d\omega^E_{HH,t+1} \]

Defining

\[ \Phi^E(\omega^E_{HH,t+1}) = \Gamma^E_{HH}(\omega^E_{HH,t+1}) - \mu^E_H G^E_{HH}(\omega^E_{HH,t+1}) \]

as the share of payoff of Home entrepreneurs borrowing from Home financial intermediaries that is captured by Home financial intermediaries, made of the share of expected payoff not retained by entrepreneurs (\( \Gamma^E_{HH}(\omega^E_{HH,t+1}) \)) minus the expected monitoring cost the financial intermediary has to incur if the entrepreneur defaults (\( \mu^E_H G^E_{HH}(\omega^E_{HH,t+1}) \)), I can write more compactly\(^{17}\):

\[
(1 - \tau^E_H) R^E_{HH,t+1} Q_{HH,t} K_{HH,t} \Phi^E(\omega^E_{HH,t+1})
\]

The financial intermediary in the Home country will engage in the contracts with domestic and foreign entrepreneurs only when the payoff of doing so (e.g. the expected earnings of the loan portfolio) will at least be equal to the return the intermediary expects to receive, denoted \( R^E_{t+1} \). Hence, the following participation constraint has to hold:

\[
\begin{align*}
(1 - \tau^E_H) R^E_{HH,t+1} Q_{H,t} K_{HH,t} \Phi(\omega^E_{HH,t+1}) + \tau^E_F R^E_{FH,t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} Q_{F,t} K_{FH,t} \Phi(\omega^E_{FH,t+1})
&= R^E_{H,t+1} \left( (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW^E_{H,t}) + \tau^E_F (Q_{F,t} K_{FH,t} - NW^E_{F,t}) \right)
\end{align*}
\]

\(^{17}\) Analogous expressions hold in the contract stipulated between Home financial intermediaries and Foreign entrepreneurs.
3.5.2 Contract between financial intermediaries and international investors

The contract between financial intermediaries and lenders is similar to the one just described for entrepreneurs: only, in this case, the financial intermediary is the debtor party in the credit contract. Let us still consider the point of view of financial intermediaries in the Home country. Financial intermediaries are endowed with net worth \(NW_{H,t}\) and stipulate credit contracts with domestic and foreign lenders (households) in order to finance the part of their loan portfolio exceeding net worth. Given the amount of loans granted to home and foreign entrepreneurs, the domestic financial intermediary borrows

\[
L_{H,t}^{F} = [(1 - \tau_{H}^E) (Q_{H,t}K_{HH,t} - NW_{H,t}^{E}) + \tau_{E}^{F} \varepsilon_t (Q_{F,t}K_{FH,t} - NW_{F,t}^{E})] - NW_{H,t}^{F}.
\]

In particular, the financial intermediary uses a fraction \(1 - \tau_{H}^E\) of its net worth to borrow from domestic lenders and a fraction \(\tau_{H}^E\) to borrow from foreign lenders, and uses profits from its loan portfolio to honor its debts. Each financial intermediary is subject to an idiosyncratic \(iid\) productivity shock \(\omega_{i,j,t+1}^{F}\), so that the effective return to its assets is given by \(\omega_{i,j,t+1}^{F} R_{i,j,t}^{F}\). The financial intermediary stipulates two credit contracts, one with domestic and one with foreign lenders\(^{18}\), which determine the contractual lending rate for domestic (foreign) borrowing \(Z_{i,j,H,t+1}^{F} (Z_{i,j,F,t+1}^{F})\) and the default thresholds \(\bar{\omega}_{i,j,H,t+1}^{F} (\bar{\omega}_{i,j,F,t+1}^{F})\):

\[
\bar{\omega}_{i,j,H,t+1}^{F} R_{i,j,H,t+1}^{F} [(1 - \tau_{H}^E) (Q_{H,t}K_{HH,t} - NW_{H,t}^{E}) + \tau_{E}^{F} \varepsilon_t (Q_{F,t}K_{FH,t} - NW_{F,t}^{E})] = Z_{i,j,H,t+1}^{F} L_{i,j,H,t}^{F} (30)
\]

\[
\bar{\omega}_{i,j,F,t+1}^{F} R_{i,j,F,t+1}^{F} [(1 - \tau_{H}^E) (Q_{H,t}K_{HH,t} - NW_{H,t}^{E}) + \tau_{E}^{F} \varepsilon_t (Q_{F,t}K_{FH,t} - NW_{F,t}^{E})] = Z_{i,j,F,t+1}^{F} L_{i,j,F,t}^{F} (31)
\]

Once again, if \(\omega_{i,j,t+1}^{F} \geq \bar{\omega}_{i,j,t+1}^{F}\) the financial intermediary does not default and keeps the profits after honoring its debt; if \(\omega_{i,j,t+1}^{F} < \bar{\omega}_{i,j,t+1}^{F}\) the financial intermediary goes bankrupt, and lenders incur a monitoring cost in order to seize the bank’s remaining assets. The expected payoff of the financial intermediary from the debt contract can be expressed as:

\[
\left\{ (1 - \tau_{H}^E) R_{i,j,H,t+1}^{F} [(1 - \tau_{H}^E) (Q_{H,t}K_{HH,t} - NW_{H,t}^{E}) + \tau_{E}^{F} \varepsilon_t (Q_{F,t}K_{FH,t} - NW_{F,t}^{E})] \right\} +
\left\{ \int_{\omega_{i,j,H,t+1}^{F}}^{\infty} \omega_{i,j,H,t+1}^{F} f(\omega_{i,j,H,t+1}^{F}) d\omega_{i,j,H,t+1}^{F} - \bar{\omega}_{i,j,H,t+1}^{F} R_{i,j,H,t+1}^{F} \int_{\omega_{i,j,H,t+1}^{F}}^{\infty} f(\omega_{i,j,H,t+1}^{F}) d\omega_{i,j,H,t+1}^{F} \right\} +
\left\{ \tau_{H}^E R_{i,j,F,t+1}^{F} [(1 - \tau_{H}^E) (Q_{H,t}K_{HH,t} - NW_{H,t}^{E}) + \tau_{E}^{F} \varepsilon_t (Q_{F,t}K_{FH,t} - NW_{F,t}^{E})] \right\} +
\left\{ \int_{\omega_{i,j,F,t+1}^{F}}^{\infty} \omega_{i,j,F,t+1}^{F} f(\omega_{i,j,F,t+1}^{F}) d\omega_{i,j,F,t+1}^{F} - \bar{\omega}_{i,j,F,t+1}^{F} R_{i,j,F,t+1}^{F} \int_{\omega_{i,j,F,t+1}^{F}}^{\infty} f(\omega_{i,j,F,t+1}^{F}) d\omega_{i,j,F,t+1}^{F} \right\}
\]

As in the previous case, lenders participate in the contract only if it is worthy to do so. In particular, lenders in each country require that lending funds to financial intermediaries yields an expected return at least equal to what they would obtain by investing funds in the risk-free asset. The participation constraints of domestic and foreign lenders in the contract with home financial intermediaries are respectively:

\(^{18}\)As in the previous section, I use the subscript \(ij, i, j = H, F\) to denote a financial intermediary in country \(i\) borrowing from lenders in country \(j\).
(1 - \tau^F_H) R^F_{H,t+1} \left\{ \left[ (1 - \tau^E_H) (Q_{F,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) \right] \right\} 
\geq R_t (1 - \tau^E_H) \left[ (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) \right] - NW_{H,t}^E 
\geq R^*_{t, \varepsilon_{t+1}} (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) - NW_{H,t}^E \tag{32}

3.5.3 Optimal contract

The optimal contract is characterized by the levels of capital investment (K_{HH,t} and K_{FH,t}), and the threshold values \bar{w}_{HH,t+1}^F, \bar{w}_{HH,t+1}^F, \bar{w}_{HH,t+1}^E, \bar{w}_{FH,t+1}^E that maximize the financial intermediary’s payoff subject to the participation constraints of Home and Foreign lenders (33, 32) and of Home and Foreign entrepreneurs (26, 27).\(^{19}\)

The first order conditions of the optimal contract resulting from the constrained maximization problem are:

\[
0 = \left\{ R^E_{HH,t+1} \left[ (1 - \Gamma^E_{HH} (\omega^E_{HH,t+1}) \Phi^E (\omega^E_{HH,t+1}) + \Gamma^E_{HH} (\omega^E_{HH,t+1}) \Phi^E (\omega^E_{HH,t+1}) \right] \right\} 
+ (1 - \tau^E_H) \frac{\Gamma^F_{HH} (\omega^F_{HH,t+1})}{\Phi^F (\omega^F_{HH,t+1})} \left[ \Gamma^E_{HH} (\omega^E_{HH,t+1}) \Phi^E (\omega^E_{HH,t+1}) R^E_{HH,t+1} \Phi^E (\omega^E_{HH,t+1}) \right] 
+ \frac{\Gamma^F_{HH} (\omega^F_{HH,t+1})}{\Phi^F (\omega^F_{HH,t+1})} \left[ \Gamma^E_{HH} (\omega^E_{HH,t+1}) \Phi^E (\omega^E_{HH,t+1}) R^E_{HH,t+1} \Phi^E (\omega^E_{HH,t+1}) \right] 
\geq R^*_{t, \varepsilon_{t+1}} (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) - NW_{H,t}^E \tag{33}
\]

\[
0 = \left\{ R^E_{FH,t+1} \left[ (1 - \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) + \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) \right] \right\} 
+ (1 - \tau^E_H) \frac{\Gamma^F_{FH} (\omega^F_{FH,t+1})}{\Phi^F (\omega^F_{FH,t+1})} \left[ \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) R^E_{FH,t+1} \Phi^E (\omega^E_{FH,t+1}) \right] 
+ \frac{\Gamma^F_{FH} (\omega^F_{FH,t+1})}{\Phi^F (\omega^F_{FH,t+1})} \left[ \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) R^E_{FH,t+1} \Phi^E (\omega^E_{FH,t+1}) \right] 
\geq R^*_{t, \varepsilon_{t+1}} (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) - NW_{H,t}^E \tag{34}
\]

\[
0 = \left\{ R^E_{FH,t+1} \left[ (1 - \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) + \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) \right] \right\} 
+ (1 - \tau^E_H) \frac{\Gamma^F_{FH} (\omega^F_{FH,t+1})}{\Phi^F (\omega^F_{FH,t+1})} \left[ \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) R^E_{FH,t+1} \Phi^E (\omega^E_{FH,t+1}) \right] 
+ \frac{\Gamma^F_{FH} (\omega^F_{FH,t+1})}{\Phi^F (\omega^F_{FH,t+1})} \left[ \Gamma^E_{FH} (\omega^E_{FH,t+1}) \Phi^E (\omega^E_{FH,t+1}) R^E_{FH,t+1} \Phi^E (\omega^E_{FH,t+1}) \right] 
\geq R^*_{t, \varepsilon_{t+1}} (1 - \tau^E_H) (Q_{H,t} K_{HH,t} - NW_{H,t}^E) + \tau^E_{F,t} (Q_{F,t} K_{FH,t} - NW_{F,t}^E) - NW_{H,t}^E \tag{35}
\]

\(^{19}\)For computational details, refer to the Appendix.
\[(1 - \tau^F_H)^\Phi^F (\omega_{HH,t+1}^F) \cdot \left[ (1 - \tau^E_H ) R^{E}_{HH,t+1} Q_{H,t} K_{HH,t} \Phi^E (\omega_{HH,t+1}^E) + \right] = \]

\[R^t (1 - \tau^E_H ) \left[ (1 - \tau^E_H ) (Q_{H,t} K_{HH,t} - NW_{E, H,t}^E) + \tau^E_{e_{t}} (Q_{F,t} K_{FH,t} - NW_{F, t}^E) - NW_{H,t}^F \right] \]

\[\tau^E_H \Phi^F (\omega_{H, F, t+1}^E) \cdot \left[ (1 - \tau^E_H ) R^{E}_{HH,t+1} Q_{H,t} K_{HH,t} \Phi^E (\omega_{H, H, t+1}^E) + \right] = \]

\[R^t_{e_{t} \in t} \tau^F_H \left[ (1 - \tau^E_H ) (Q_{H,t} K_{HH,t} - NW_{E, H,t}^E) + \tau^E_{e_{t}} (Q_{F,t} K_{FH,t} - NW_{F, t}^E) - NW_{H,t}^E \right] \]

\[\left[ 1 - \Gamma_{HH}^E (\omega_{HH,t+1}^E) \right] R^{E}_{HH,t+1} Q_{H,t} K_{HH,t} = R^{E}_{HH,t+1} NW_{H,t}^E \]

\[\left[ 1 - \Gamma_{FF}^E (\omega_{FH,t+1}^E) \right] R^{E}_{FH,t+1} Q_{F,t} K_{FH,t} = R^{E}_{FH,t+1} NW_{F,t}^E \]

### 3.5.4 Net worth

As in BGG (1999) entrepreneurs and financial institutions have a finite life horizon. In particular, in each period the probability to exit the market is equal to \((1 - \gamma^E)\) for entrepreneurs and \((1 - \gamma^F)\) for financial institutions. The dying agents are immediately replaced by an equal number of newly born entrepreneurs and financial institutions, so that the population remains constant. This assumption ensures that borrowers do not accumulate enough net worth to become fully self-sufficient. Furthermore, both entrepreneurs and financial intermediaries receive payments, denoted respectively \(W_{E, H,t}^E\) and \(W_{H, F,t}^F\), for labor services supplied to firms in their country of origin\(^{20}\). Surviving agents accumulate net worth, which is made of the return from investment net of debt repayments. The dynamic evolution of net worth for entrepreneurs and financial institutions in the home country can be expressed as:

\[NW_{H,t}^E = \gamma^E V_{H,t}^E + W_{H,t}^E \quad (36)\]

\[NW_{H,t}^F = \gamma^F V_{H,t}^F + W_{H,t}^F \quad (37)\]

Where \(V_{H,t}^E\) and \(V_{H,t}^F\) represent the equity of entrepreneurs and financial intermediaries respectively:

\[V_{H,t}^E = \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] (1 - \tau^E_H ) R^{E}_{HH,t+1} Q_{H,t} K_{HH,t-1} + \]

\[+ \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] \tau^E_H R^{E}_{HH,t+1} Q_{H,t} K_{HH,t-1} \quad (38)\]

\[V_{H,t}^F = \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] (1 - \tau^F_H ) R^{F}_{H,t} \left\{ \left[ 1 - \tau^E_H \right] (Q_{H,t-1} K_{HH,t-1} - NW_{H,t-1}^E) \right\} + \]

\[+ \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] \tau^F_H R^{F}_{H,t} \left\{ \left[ 1 - \tau^E_H \right] (Q_{H,t-1} K_{HH,t-1} - NW_{H,t-1}^E) \right\} \quad (39)\]

\[+ \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] \tau^E_H R^{E}_{H,t} \left\{ \left[ 1 - \tau^E_H \right] (Q_{H,t-1} K_{HH,t-1} - NW_{H,t-1}^E) \right\} + \]

\[+ \left[ 1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E) \right] \tau^E_H R^{E}_{H,t} \left\{ \left[ 1 - \tau^E_H \right] (Q_{H,t-1} K_{HH,t-1} - NW_{H,t-1}^E) \right\} \quad (40)\]

\(^{20}\)The presence of wages guarantees that net worth is non-zero in steady state, but does not have a significant effect on the dynamics of net worth given the small share of entrepreneurs’ and bankers’ labor in the production function.
Hence, $V^E_{H,t}$ represents the retained earnings of the domestic entrepreneurial sector, derived from their capital investment using funds from domestic and foreign financial institutions. $V^F_{H,t}$ represents retained earnings of financial institutions on their portfolio of loans to domestic and foreign entrepreneurs. Agents leaving the market at any time period consume the entire value of their assets, hence consumption of entrepreneurs and financial intermediaries is given by:

\begin{equation}
C^E_{H,t} = (1 - \gamma^E) V^E_{H,t}
\end{equation}

\begin{equation}
C^F_{H,t} = (1 - \gamma^F) V^F_{H,t}
\end{equation}

And it has the same composition as households’ consumption.

### 3.6 Exchange rate and terms of trade

Prices in the tradable sector are set in the producers’ currency. Furthermore, it is assumed that the international law of one price holds, implying that the price of the same good sold in the two countries is equalized using the nominal exchange rate $S_t$ (defined as the price of foreign currency in terms of domestic currency). This implies that:

\begin{equation}
P^*_H \frac{1}{S_t} = P^*_F \frac{1}{S_t}
\end{equation}

I define the real exchange rate:

\begin{equation}
\varepsilon_t = \frac{S_t P^*_F}{P^*_t}
\end{equation}

I define the terms of trade as the ratio between import and export prices in domestic currency, which, given the law of one price, can be expressed as:

\begin{equation}
TOT_t = \frac{S_t P^*_F}{P^*_t}
\end{equation}

### 3.6.1 Monetary and fiscal policy

Monetary policy sets the short-term interest rate in both economies, according to endogenous economic developments. The central bank in the Home country sets the short-term nominal interest rate according to a rule of the following general form:

\begin{equation}
\frac{R^*_n}{R^*} = \left( \frac{R^*_n}{R^*} \right)^{\rho^e} \left[ \frac{\pi^e_{H,t}}{\pi^e_H} \right]^{\rho^e} \left( \frac{S_t}{S} \right)^{1-\rho^e} \exp(\xi_{R,t})
\end{equation}

Where variables without time subscript refer to steady state values. In particular, for $\rho^e_S = 0$ the Home country implements a floating exchange rate regime with inflation targeting. On the other hand, it follows a fixed exchange rate by setting $\rho^e_S$ so large that $R^*_n = R^*_t$. $\xi_{R,t}$ represents an exogenous monetary policy shock.
The Foreign country’s monetary policy sets the policy rate according to the following feedback rule:

\[
\frac{R_{t}^{n*}}{R_{t-1}^{n*}} = \left( \frac{R_{t-1}^{n*}}{R_{t-1}^{n*}} \right)^{\rho_{\pi}^{*}} \left( \frac{\pi_{F,t}}{\pi_{F}} \right)^{\rho_{\pi}^{*}} \left( \frac{Y_{F,t}}{Y_{F}} \right)^{\rho_{y}^{*}} \exp(\xi_{R^{n*},t})
\] (47)

The fiscal authority aims at attaining a balanced budget in every period:

\[
G_{t} = T_{t}
\] (48)

### 3.7 Market clearing and equilibrium

Market clearing in each country requires that total production equals total absorption. In particular, domestic output is used for home consumption, home investment, government expenditure and exports. Furthermore, a small fraction of output is lost in each period due to monitoring costs incurred by lenders and financial institutions. The total amount of monitoring costs is given by the following equation:

\[
M_{t} = \mu^{E}G^{E} \left( \omega_{HH,t}^{E} \right) R_{HH,t}^{E} (1 - \tau_{H}^{E}) Q_{H,t-1}K_{H,H,t-1} + \mu^{E}G^{E} \left( \omega_{FH,t}^{E} \right) R_{FH,t}^{E} \varepsilon_{t-1}^{E} Q_{F,t-1}K_{F,H,t-1} + \\
+ \mu^{F}G^{F} \left( \omega_{HH,t}^{F} \right) R_{H,t}^{F} (1 - \tau_{H}^{F}) \left[ (1 - \tau_{H}^{F}) \left( Q_{H,t-1}K_{H,H,t-1} - NW_{H,H,t-1}^{E} \right) + \right] + \\
+ \mu^{F}G^{F} \left( \omega_{FH,t}^{F} \right) \tau_{F,t-1}^{E} R_{F,H,t}^{E} \left[ \frac{1}{\varepsilon_{t-1}^{E}} (1 - \tau_{E}^{H}) \left( Q_{H,t-1}K_{H,H,t-1} - NW_{H,F,t-1}^{E} \right) + \\
(1 - \tau_{E}^{F}) \left( Q_{F,t-1}K_{F,H,t-1} - NW_{F,F,t-1}^{E} \right) \right]
\]

Where the first two terms on the right-hand side represent monitoring costs incurred by Home financial intermediaries involved in credit contracts with Home and Foreign entrepreneurs, and the last two terms represent the cost incurred by domestic lenders in the contracts with Home and foreign financial intermediaries.

The Home country’s resource constraint can then be written as:

\[
Y_{H,t} = \Delta_{H,t} \left( C_{H,t} + C_{H,t}^{E} + C_{H,t}^{F} + I_{H,t} + G_{t} + M_{t} \right) + \Delta_{H,t}^{*} \frac{(1 - n)}{n} \left( C_{H,t}^{E} + C_{H,t}^{E} + C_{H,t}^{F} + I_{H,t}^{*} \right)
\] (49)

Where \( \Delta_{H,t} \) and \( \Delta_{H,t}^{*} \) are indexes of price dispersion implied by the staggered price setting defined as:

\[
\Delta_{H,t} = \int_{0}^{1} \left( \frac{P_{i}(jH)}{P_{t}} \right)^{-\phi_{H}} dj_{H}
\]

\[
\Delta_{H,t}^{*} = \int_{0}^{1} \left( \frac{P_{H,t}(jH)}{P_{t}^{*}} \right)^{-\phi_{H}} dj_{H}
\]

and whose laws of motion are given by:
\[
\Delta_{H,t} = (1 - \theta_H) \left[ \frac{1 - \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{1-\varepsilon_H}}{1 - \theta_H} \right]^{\varepsilon_H} + \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{-\varepsilon_H} \Delta_{H,t-1} \tag{50}
\]

\[
\Delta_{H,t}^* = (1 - \theta_H) \left[ \frac{1 - \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{1-\varepsilon_H}}{1 - \theta_H} \right]^{\varepsilon_H} + \theta_H \left( \frac{1}{\pi_{H,t}^*} \right)^{-\varepsilon_H} \Delta_{H,t-1}^* \tag{51}
\]

Market clearing in the labor market requires that total labor supply equals demand, a condition represented by equation (6). Furthermore, for the capital market to be in equilibrium, the total capital investment by entrepreneurs (borrowing domestically and abroad) has to equal the aggregate capital production, i.e.:

\[
K_t = (1 - \tau_{E}^H) K_{HH,t} + \tau_{E}^E K_{HF,t} \tag{52}
\]

### 3.8 Steady state and calibration

Before simulating the model, I calculate the deterministic steady state by solving the linear system of static equations implied by the model (cfr. Appendix C).

<table>
<thead>
<tr>
<th>Table 1: Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>( \varphi )</td>
</tr>
<tr>
<td>( \sigma )</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \lambda )</td>
</tr>
<tr>
<td>( \omega_E )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \theta_H )</td>
</tr>
<tr>
<td>( \epsilon )</td>
</tr>
<tr>
<td>( \mu_{E,H} )</td>
</tr>
<tr>
<td>( \mu_{F,H} )</td>
</tr>
<tr>
<td>( \gamma_p )</td>
</tr>
<tr>
<td>( \gamma_F )</td>
</tr>
</tbody>
</table>

The model parameters are calibrated following the literature. Concerning households’ preferences, I set the intertemporal discount factor \( (\beta) \) to 0.99, which corresponds to a yearly risk-free interest rate of 4\%. The intertemporal elasticity of substitution \( (\sigma) \) is set to 1, so as the elasticity of labor supply \( (\varphi) \) following Christiano, Eichenbaum, and Evans (1997). In order to obtain a steady state labor supply of 0.33 the coefficient on labor in the utility function \( (\chi_H) \) is calibrated to 9.02. Regarding the composition of consumption, I
set the share of imported goods in the consumption basket at 0.4, which implies a degree of home bias and the consumption habit parameter at 0.5. Finally, note that the Cobb-Douglas specification of the consumption aggregator implies unit elasticity of substitution between domestic and foreign goods.21

On the production side, the rate of depreciation of capital ($\delta$) is set to 0.025, implying a yearly depreciation rate of 10%. Furthermore, I set the adjustment cost parameter, $\kappa$, to 2.5. The parameters of the production function are chosen such that the share of capital in production is 0.35, while the shares of labor by entrepreneurs and financial intermediaries ($\Omega^E$ and $\Omega^F$ respectively) is 0.01. Following the standard estimate used in the literature (Chari, Kehoe and McGratten (2000)), I assume a Calvo price stickiness parameter $\theta_H$ equal to 0.75, implying that price adjustment happens, on average, every four quarters. Furthermore, I assume that firms that do not optimally chose their price in a given period adjust their price to past inflation with a coefficient of 0.2. Finally, the elasticity of substitution between varieties of domestic goods ($\epsilon$) is set to 6, implying a 20% price markup. The parameters of the wage setting process imply a higher persistence of wages compared to prices. I set the parameter representing the elasticity of substitution between labor types ($\epsilon_w$) at 10, implying an 11% markup and the wage stickiness parameter ($\theta_w$) equal to 0.94 (Schmitt-Grohé and Uribe (2006)).

The parameters pertaining to financial frictions in the banking and entrepreneurial sector are calibrated in order for the steady state values of key financial variables in the model to match time series data relative to Europe for the financial and non-financial sector. In particular, the parameters related to the cost of monitoring banks’ and entrepreneurial output ($\mu^F$ and $\mu^E$ respectively), the volatility parameters of banks’ and entrepreneurs’ idiosyncratic productivity ($\sigma^F$ and $\sigma^E$) and the survival probabilities ($\gamma^E$ and $\gamma^F$) are calibrated in order to match European data on leverage, lending spreads and default probabilities. European data on bank default reveal that the expected short-term (1 year) default probability of banks averaged 0.6% between 2000 and 2007 (Fiordelisi et al. (2010)): hence I set the quarterly default probability of financial intermediaries to 0.0015. Following Faia (2010), I set the annual steady state default of entrepreneurs to 3%, implying a quarterly value of 0.0075. Steady state equity-to-assets ratios for entrepreneurs and banks are calibrated according to the micro-level data reported in Kalemli-Ozcan et al. (2011). In particular, I set the inverse leverage ratios to 0.4904 and 0.1040 for entrepreneurs and banks respectively, implying leverage ratios of 2.0939 and 9.6129. Finally, I calibrate the entrepreneurial and financial intermediaries’ lending spreads according to Eurostat data on European interest rates between 2000 and 2007. Hence I target the steady state level of the spread between the lending rates of entrepreneurs and banks at 0.0052 quarterly, and the spread between financial intermediaries’ lending rate and the risk free rate at 0.0003 quarterly. The calibrated values of the model parameters are reported in Table 1, while Table 2 reports the key steady state values resulting from the calibration.

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21 For a similar specification see for example Kolasa and Lombardo (2011).
Table 2: Steady State

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>1.0101</td>
</tr>
<tr>
<td>( C/Y )</td>
<td>0.5382</td>
</tr>
<tr>
<td>( I/Y )</td>
<td>0.1821</td>
</tr>
<tr>
<td>( G/Y )</td>
<td>0.2</td>
</tr>
<tr>
<td>( H )</td>
<td>0.33</td>
</tr>
<tr>
<td>( QK/NW_e )</td>
<td>0.4904</td>
</tr>
<tr>
<td>( QK/NW_f )</td>
<td>0.1040</td>
</tr>
<tr>
<td>( R^k/R )</td>
<td>1.0049</td>
</tr>
<tr>
<td>( Z^E - R )</td>
<td>0.0056</td>
</tr>
<tr>
<td>( Z^E - Z^F )</td>
<td>0.0052</td>
</tr>
<tr>
<td>( Z^F - R )</td>
<td>0.0004</td>
</tr>
<tr>
<td>( F(\bar{\omega}_E) )</td>
<td>0.0075</td>
</tr>
<tr>
<td>( F(\bar{\omega}_F) )</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

The form of the interest rule allows for a variety of different types of monetary policy stances of the small open economy. Table 3 reports the two different rules I am considering in the analysis, together with the parameters of the Taylor rule of the foreign economy, and the values of the financial integration parameters used in the model simulations.

Table 3: Overview of alternative models

<table>
<thead>
<tr>
<th>Financial Integration</th>
<th>( \tau_E )</th>
<th>( \tau_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No financial integration</td>
<td>0; 0</td>
<td>0; 0</td>
</tr>
<tr>
<td>Partial financial integration</td>
<td>0; 0.2</td>
<td>0; 0.2</td>
</tr>
<tr>
<td>Full financial integration</td>
<td>0.2; 0.2</td>
<td>0.2; 0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy Type</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary policy F</td>
<td>( \rho_r = 0.8; \rho_\pi = 1.5; \rho_y = 0 )</td>
</tr>
<tr>
<td>Monetary policy H</td>
<td>( \rho_r = 0.8; \rho_\pi = 1.5; \rho_S \rightarrow \infty )</td>
</tr>
<tr>
<td>Fixed exchange rate</td>
<td>( \rho_r = 0.8; \rho_\pi = 1.5; \rho_S = 0 )</td>
</tr>
<tr>
<td>Flexible exchange rate</td>
<td>( \rho_r = 0.8; \rho_\pi = 1.5; \rho_S = 0 )</td>
</tr>
</tbody>
</table>

4 Impulse response analysis

In this section I present the simulated path of the main real and financial variables of the small open economy in response to nominal, real and financial shocks. The small open economy emerges as a limit, when the relative size of the Home country, \( n \), tends to zero\(^{22}\). Hence, the Foreign country becomes relatively closed, but as long as there is a positive degree of home bias, the Home economy continues to consume foreign produced goods. This allows to model the small open economy in a consistent way, retaining its trade and financial linkages with the rest of the world. In fact, as shown by Batini et al. (2007), although the

\(^{22}\)For a similar approach, see Batini et al. (2007), De Paoli (2009) and Corsetti and Müller (2011).
Foreign economy becomes relatively closed, its demand for exports (i.e. goods produces in the Home country) is not zero. Hence, while the small economy becomes invisible to the rest of the world, the reverse is not true.

In figures 1 to 4, the responses for the Home country are presented for different exchange rate regimes and degrees of financial integration. In particular, in the case of financial autarky the coefficients $\tau_{HH}^F, \tau_{HH}^E, \tau_{HF}^F, \tau_{HF}^E, \tau_{HH}^F, \tau_{HH}^E$ are set to zero; in the case of banking integration, i.e. a situation where banks can borrow from domestic and foreign lenders but entrepreneurs are constrained to borrow from banks in their own country, I set $\tau_{HH}^F = \tau_{HF}^F = \tau_{FF}^F = \tau_{FH}^F = 0$ and $\tau_{HH}^E = \tau_{HF}^E = \tau_{FF}^E = \tau_{FH}^E = 0$; finally, full financial integration implies fully blown financial linkages between banks, entrepreneurs and lenders in both countries (I set $\tau_{HH}^E = \tau_{HF}^E = \tau_{FF}^E = \tau_{FH}^E = 0$).

4.1 Foreign interest rate shock

Figures 1a and 1b depict the impulse responses corresponding to a monetary policy shock in the Foreign economy, for different exchange rate regimes (flexible and fixed, respectively) in the Home country and different degrees of financial integration.

If the Home currency is allowed to float, the one standard deviation shock to the Foreign interest rate causes the nominal exchange rate to increase on impact, driving also the real exchange rate upwards, implying a real depreciation of the domestic currency. By making foreign goods relatively more expensive, the real depreciation has two effects. First, it drives an expenditure-switching effect towards domestic goods. Secondly, it contributes to the rise in inflation in the Home economy.

The increase in inflation, independently of the degree of financial linkages, leads the Home central bank to increase the nominal interest rate, which results in a real interest rate hike that depresses consumption, investment and output.

However, when financial linkages are turned on, the negative effect on investment is more pronounced. While in the case of financial autarky the financial sector is influenced by the foreign shock only to the extent that this implies a contractionary monetary policy response of the Home central bank, in the presence of financial linkages, the Domestic and Foreign interest rate hikes reinforce each other. In fact, as the risk free return of domestic and foreign lenders increase, the cost of external finance for domestic and foreign banks increases more markedly and it is reflected in a sharper increase in loan rates for financial intermediaries. As these are passed on to final borrowers, entrepreneurs’ loan rates and borrowing spreads increase in both countries. As borrowing costs increase, entrepreneurs undertake fewer projects, thereby reducing investment, the demand for capital and hence asset prices by a larger amount. The combination of declining asset prices and higher borrowing costs decrease the net worth of entrepreneurs and banks worldwide. Figure 1a clearly shows that the decrease in entrepreneurial and banks’ net worth, the increase in lending spreads and the consequent decrease in investment and asset prices are more pronounced the higher the degree of financial linkages. The reason for this is twofold. First, when cross-border borrowing by entrepreneurs is shut off (i.e. the case of bank integration) the foreign interest

---

23 For space reasons, the responses of the corresponding foreign variables are omitted, but are available upon request.
rate shock and the consequent domestic interest rate increase lead to an rise in the loan rate charged by Home banks on domestic entrepreneurs which is higher than in the case of financial autarky. Indeed, in the latter case the foreign monetary policy shock impacts the domestic financial sector only through the domestic monetary policy response. In the case of full financial integration, not only the foreign policy rate matters for the cost of funds of domestic banks, but the deterioration in entrepreneurial net worth in the Foreign country impacts domestic banks negatively affecting their net worth and leverage. Second, the real exchange rate depreciation increases the value of foreign loans held by banks in the Home country. The combination of these effects amplifies the effect of the foreign shock on the domestic economy.

In case the Home country follows a fixed exchange rate, the decline of both real and financial variables is more pronounced. This is due to the fact that, following the initial shock, the central bank in the home country increases its policy rate on impact to defend the currency. In fact, the nominal interest rate increase necessary to keep the exchange rate stable is of the same magnitude of the initial foreign shock, implying that the Home country is fully importing the foreign shock. As the nominal exchange rate is fixed and Home and Foreign inflation decline by the same amount, the real exchange rate is roughly constant. This eliminates, on the real side, expenditure switching effects and, on the financial side, balance sheet effects due to currency denomination of loans. The steeper increase in the real interest rate under a fixed exchange rate regime depresses consumption by a larger amount. Furthermore, the constancy of the real exchange rate implies that consumers demand less of both domestic and imported goods. The sharper decrease in consumption leads to a more pronounced decrease in demand for domestic goods, leading to a larger output drop and a lower demand for capital investment. Hence, output, investment and asset prices are lower than in the flexible exchange rate regime case. On the financial side, it is interesting to notice that the transmission of the shock is identical across degrees of financial integration. On one hand, in response to the foreign monetary policy shock, the domestic central bank increases the nominal interest rate to defend the parity roughly by the same magnitude, thereby "importing" the foreign shock. On the other hand, in this model financial exposures across countries are symmetric. The joint effect of symmetric cross-country financial exposures, a de-facto symmetric shock and a fixed exchange rate lead the response of domestic and foreign variables to be parallel.
Table 4 reports standard deviations for the main real and financial variables under fixed and flexible exchange rate regimes and different degrees of financial linkages. Comparing exchange rate regimes, it is evident that volatility in the domestic economy is much higher when the central bank follows a strict peg: in particular, output is more than three times as volatile compared to the case in which the currency is allowed to float. Also investment and consumption exhibit higher standard deviations. This is the result of the much stronger monetary policy response to the shock necessary to stabilize the exchange rate, which is reflected in the high standard deviation of the nominal interest rate. Furthermore, volatility increases with the degree of financial integration. Under a floating exchange rate regime, financial variables are particularly more volatile under full financial integration, thanks to the immediate transmission of the foreign shock to the domestic economy via the financial sector and the direct spillover from balance sheets of foreign entrepreneurs. However, the degree of financial integration is virtually irrelevant when the small country follows a strict peg, as the impulse-response functions previously revealed.

### 4.2 Foreign productivity shock

Figures 2a and 2b, illustrate the reaction of the main Home country variables in response to a foreign productivity shock. In the foreign country, the negative productivity shock raises marginal costs, which in turn decrease production. This has two consequences: on one side, inflation increases; on the other side, demand for capital decreases, pushing its price downwards. The increase in inflation in the foreign country leads the foreign central bank to increase the nominal interest rate. This, jointly with the decrease in asset prices, deteriorates balance sheet conditions of borrowers in the foreign country: both entrepreneurial and banks’ net worth decrease in the foreign country, leading to an increase in borrowing spreads for both agents. Furthermore, the contractionary Foreign monetary policy depreciates the exchange rate, albeit by a small amount. The increase in Foreign inflation appreciates the foreign currency in real terms, and improves the terms of trade for the Home country. However, as the Home country is relatively small and open, the real depreciation pushes up domestic CPI inflation, which leads the domestic central bank to raise the nominal interest rate. In case the Home country follows a pegged exchange rate policy, the domestic central bank
increases the nominal interest rate in order to counteract the nominal depreciation. In this case, the increase in domestic CPI inflation is more muted, so as the extent of real exchange rate depreciation and the required monetary policy contraction is larger than in the case of flexible exchange rate. While in the first periods the increase in inflation dominates in driving the real interest rate down, after few periods the real interest rate starts increasing, thereby determining a decrease in domestic consumption and a tightening of borrowing conditions for domestic banks. In particular, the real interest rate increases more sharply when the Home country’s monetary policy pegs the exchange rate, leading to a sharper decrease in consumption, investment and GDP.

While the negative effect of the foreign productivity shock on the domestic economy is stronger when the Home country’s central bank adopts a fixed exchange rate regime, the difference in responses between different degrees of financial linkages is more muted. Under financial autarky, the financial sector is affected by the foreign productivity shock through two channels. First, the decrease in demand for capital by domestic firms that reduces the demand for investment and loans, thereby reducing leverage in the domestic financial sector and dampening the financial accelerator effect. Second, the increase in the domestic interest rate that increases the cost of funds for domestic financial intermediaries, accentuating the financial accelerator effect. Hence, the foreign shock is transmitted first to the real economy through a change in international relative prices and its effects on domestic CPI inflation, and then to the credit market through leverage and cost of funds effects. In case of banking integration, the financial accelerator effect is reinforced by the foreign monetary policy contraction that follows the adverse productivity shock. As the foreign interest rate hike is stronger than the domestic one, the cost of foreign borrowing increases by more than in the case of financial autarky. Furthermore, the real exchange rate depreciation acts in the same direction, feeding the financial accelerator effect and resulting in a sharper decrease in net worth and increase in borrowing spreads. In case of full financial integration, the increase in financial acceleration caused by the foreign monetary policy contraction and real exchange rate depreciation is somewhat counteracted by the decrease in leverage of foreign entrepreneurs\footnote{Following the adverse productivity shock, foreign entrepreneurs demand less loans given the decrease in overall demand for capital, thereby decreasing leverage.}. Hence, the leverage of domestic financial intermediaries lowers by a smaller amount, so as the lending spread of entrepreneurs, whereas their net worth decreases by less than in case of banking integration. As a result, entrepreneurial net worth decreases more in case of banking and full financial integration, leading to a more muted response of investment in case of financial autarky. However, this effect is quantitatively quite small.
<table>
<thead>
<tr>
<th></th>
<th>Financial Autarky</th>
<th>Bank Integration</th>
<th>Full Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Float</td>
<td>Fix</td>
<td>Float</td>
</tr>
<tr>
<td>Output</td>
<td>0.1409</td>
<td>0.1654</td>
<td>0.1435</td>
</tr>
<tr>
<td>Investment</td>
<td>0.1540</td>
<td>0.1683</td>
<td>0.1647</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0437</td>
<td>0.0482</td>
<td>0.0450</td>
</tr>
<tr>
<td>Int. Rate</td>
<td>0.0244</td>
<td>0.0241</td>
<td>0.0246</td>
</tr>
<tr>
<td>Asset Price</td>
<td>0.2970</td>
<td>0.3309</td>
<td>0.3175</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0365</td>
<td>0.0203</td>
<td>0.0368</td>
</tr>
<tr>
<td>Leverage E</td>
<td>0.0650</td>
<td>0.0679</td>
<td>0.0698</td>
</tr>
<tr>
<td>Leverage B</td>
<td>0.0787</td>
<td>0.0893</td>
<td>0.0873</td>
</tr>
<tr>
<td>Spread E</td>
<td>0.0280</td>
<td>0.0312</td>
<td>0.0335</td>
</tr>
</tbody>
</table>

The qualitative results are confirmed by the analysis of volatilities reported in Table 5. For all degrees of financial integration, the main real and financial variables are more volatile under fixed exchange rate regime, with the exception of inflation. This results from the fact that, if the exchange rate is allowed to float, the foreign productivity shock affects domestic inflation through the real exchange rate depreciation, effect that is less pronounced when the exchange rate is fixed. Furthermore, I find that the cost of fixing the exchange rate (i.e. the difference in volatility between fixed and flexible exchange rate regime) is more pronounced under financial autarky: in this case, output, investment, consumption and asset prices are relatively more volatile under a strict peg. Hence, the more financially integrated the small economy is with the rest of the world, the less is the cost of pursuing a fixed exchange rate policy. In fact, the presence of financial linkages increases the business cycle correlation between the two countries, and triggers similar monetary policy responses (i.e. a monetary policy tightening in both countries), under a flexible exchange rate regime. If the small country follows a strict peg, it is forced to import the foreign monetary policy stance to satisfy the interest parity condition and to raise the interest rate by the same amount as the foreign central bank. This implies that, under both exchange rate regimes, the central banks in the two countries are somewhat correlated, and this reduces the cost of pegging. On the other hand, a floating exchange rate delivers higher volatility of financial variables due to its effect on balance sheets of financial intermediaries in the small economy, thereby reducing the cost of pegging the currency.

### 4.3 Foreign financial shocks

Figures 3a, 3b and 4a and 4b illustrate the impulse-responses of the variables of interest to a shock to the idiosyncratic productivity of foreign banks and entrepreneurs respectively. The shock increases the volatility of the idiosyncratic productivity of banks and entrepreneurs, and can be interpreted as an increase in the riskiness of borrowers. In fact, as the distribution of the stochastic idiosyncratic productivity becomes more dispersed, uncertainty about the realization of productivity increases, increasing borrowers’ default probability and forcing lenders to charge a higher loan premium.
Table 6: Standard Deviations - Foreign $\sigma_B$ shock

<table>
<thead>
<tr>
<th></th>
<th>Financial Autarky</th>
<th>Bank Integration</th>
<th>Full Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Float</td>
<td>Fix</td>
<td>Float</td>
</tr>
<tr>
<td>Output</td>
<td>0.0312</td>
<td>0.0335</td>
<td>0.0399</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0387</td>
<td>0.0349</td>
<td>0.0060</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0313</td>
<td>0.0307</td>
<td>0.0268</td>
</tr>
<tr>
<td>Int. Rate</td>
<td>0.0182</td>
<td>0.0233</td>
<td>0.0160</td>
</tr>
<tr>
<td>Asset Price</td>
<td>0.0428</td>
<td>0.0382</td>
<td>0.0539</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0161</td>
<td>0.0203</td>
<td>0.0140</td>
</tr>
<tr>
<td>Leverage E</td>
<td>0.0123</td>
<td>0.0119</td>
<td>0.0062</td>
</tr>
<tr>
<td>Leverage B</td>
<td>0.0027</td>
<td>0.0026</td>
<td>0.0307</td>
</tr>
<tr>
<td>Spread E</td>
<td>0.0018</td>
<td>0.0017</td>
<td>0.0235</td>
</tr>
</tbody>
</table>

The shock to the foreign bank’s idiosyncratic productivity leads to an increase in banks’ borrowing spreads, which are passed on to entrepreneurs, rising their loan spreads. As borrowing costs rise, entrepreneurs in the foreign economy invest less, causing a decrease in the capital stock and in the price of capital. This in turn implies a decrease in foreign banks’ and entrepreneurs’ net worth, which reinforces the initial shock. The simultaneous decrease in output and inflation reveals that the riskiness shock has the effect of a demand shock on the foreign economy, as prices and quantities move in the same direction. The foreign central bank reacts to the fall in inflation by lowering the policy rate; however, in the first period, this is not sufficient for lowering the real interest rate, which starts decreasing only in the second period after the shock, falling below steady state values right after. The decrease in the real interest rate, by lowering the external finance premium for foreign banks, mitigates the negative financial acceleration effect triggered by the original shock. Furthermore, it increase foreign consumption, partially counteracting the initial negative effect on aggregate demand.

The transmission of the foreign shock differs across degrees of financial linkages. In case of financial autarky, the shock is mainly transmitted through trade channels. The decrease in foreign output, investment and GDP implies a decline in Foreign demand for Home produced goods, which depresses production in the Home country. As a result, the shock is imported as a demand shock, and domestic inflation declines, leading the Home central bank to enact a monetary expansion. As the domestic nominal interest rate decreases only slightly on impact, the uncovered interest parity implies that the nominal exchange rate slightly depreciates. As a consequence of the domestic monetary policy easing, the real interest rate starts decreasing, after a brief upward jump on impact, positively impacting consumption. However, under financial autarky, the foreign banking sector shock has virtually no effect on domestic investment and on the domestic financial sector, as lending spreads, leverage ratios and net worths are unaffected. Under banking and full financial integration the foreign shock spills over to the domestic economy directly thorough the international credit market: in particular, the tighter the financial interlinkages, the stronger the transmission of the shock. Under banking integration, foreign banks stipulate credit contract with domestic lenders; hence, the adverse shock to foreign banks’ productivity increases the lending rate charged to foreign banks borrowing domestically. Although this has no direct effect on the domestic
financial sector, it worsens the recession in the foreign economy, implying lower investment and output, which leads to a decreased demand for Home exports, which depresses Home output. Hence, in case of banking integration, banks’ and entrepreneurial lending spreads increase more markedly in the Home country, while net worths decrease more. In the case of full financial integration, the negative foreign shock is more heavily transmitted to the Home country by the credit relationship between Home entrepreneurs and Foreign banks. Here, the shock in the foreign banking sector translates in higher lending rates for domestic entrepreneurs borrowing from abroad, thereby decreasing their ability to loan funds. Hence, investment and asset prices decrease even further.

The difference in responses between fixed and flexible exchange rate regimes are only marginal, as we can see from Table 6. This is due to the fact that the shock, even in the case of floating currency, does not lead to sensible differences in real exchange rate fluctuations.

Furthermore, Table 6 confirms the highest volatility of the main real and financial variables in the case of full financial integration. This effect arises through two channels. First, the increase in borrowing spreads in the foreign country translates into an increase in the cost of external finance for banks and entrepreneurs in the Home country borrowing abroad, which decreases their willingness to borrow. Secondly, the decrease in foreign banks’ and entrepreneurial net worth deteriorates balance sheet conditions in the Home country, and worsens borrowing conditions even further.

Figures 4a and 4b report the impulse responses relative to a foreign entrepreneurial idiosyncratic volatility shock. Albeit in the foreign economy the shock has similar consequences as the idiosyncratic productivity shock affecting banks, its transmission to the Home country differs according to the degree of financial integration. In the foreign country, the shock results in an increase in the external finance premium of foreign entrepreneurs, a worsening of their borrowing conditions and a decrease in investment and asset prices. As before, the shock acts on the supply of credit, by reducing loans and increasing their price. Foreign output decrease because of the decrease in demand for investment goods and the reduction in capital stock. As before, foreign inflation falls, inducing the foreign central bank to ease monetary policy, which rises the real interest rate after few periods. The monetary policy action causes foreign consumption to rise, the foreign currency to appreciate through the uncovered interest parity. Furthermore, the monetary policy easing contributes to counteract the negative spiral of the financial accelerator, by easing credit conditions. Once again, the decline in Foreign demand for Home goods leads domestic output to decrease and inflation to fall, leading the Home central bank to lower its policy rate.

However, under financial autarky, the foreign banking sector shock has virtually no effect on domestic investment and on the domestic financial sector, as lending spreads, leverage ratios and net worths are unaffected. The case of banking integration is largely similar, because the domestic credit market is still insulated from the shock to entrepreneurial riskiness, as Home banks only lend to Home entrepreneurs: therefore, the shock is still transmitted through real channels. The most interesting case is that of full financial integration. Now the shock to foreign entrepreneurs directly affects banks in the Home country through their international loan portfolio. As a consequence of the increase in Home banks’ leverage, borrowing spreads for Home banks rise, and are passed on to entrepreneurs, even to those borrowing domestically. Then, the shock is transmitted similarly in the two countries, passing through increases in loan rates and decrease in loan demand. However, in case of full
financial integration, the joint effect of the monetary policy easings in the two countries acts as to counteract the negative effect of the foreign shock on the Home country. In fact, in both countries the decrease in interest rates leads to an easing of borrowing conditions. While in the Foreign country, where the shock originates and its effects are stronger, this is not sufficient to completely offset the shock, the Home country, where financial variables react less strongly than in the Foreign country because of asymmetric cross border exposures, benefits from the two monetary policy actions. This is evident by looking at the behavior of entrepreneurial spreads and net worth in the Home country. After a deterioration in the first periods after the shock, spreads rapidly decrease and fall below steady state values; net worth, after an initial fall, rises above the zero line.

Table 7: Standard Deviations - Foreign $\sigma_E$ shock

<table>
<thead>
<tr>
<th></th>
<th>Financial Autarky</th>
<th>Bank Integration</th>
<th>Full Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Float</td>
<td>Fix</td>
<td>Float</td>
</tr>
<tr>
<td>Output</td>
<td>0.0077</td>
<td>0.0079</td>
<td>0.0077</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0143</td>
<td>0.0130</td>
<td>0.0143</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0108</td>
<td>0.0106</td>
<td>0.0108</td>
</tr>
<tr>
<td>Int.Rate</td>
<td>0.0056</td>
<td>0.0073</td>
<td>0.0056</td>
</tr>
<tr>
<td>Asset Price</td>
<td>0.0151</td>
<td>0.0136</td>
<td>0.0153</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0050</td>
<td>0.0064</td>
<td>0.0050</td>
</tr>
<tr>
<td>Leverage E</td>
<td>0.0038</td>
<td>0.0036</td>
<td>0.0038</td>
</tr>
<tr>
<td>Leverage B</td>
<td>0.0011</td>
<td>0.0009</td>
<td>0.0012</td>
</tr>
<tr>
<td>Spread E</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Comparing the impulse-responses under different exchange rate regimes (cfr Table 7) reveals that, although when the Home central bank pegs the currency output decreases more and investment increases less, the difference is not pronounced. Once again, this is due to the fact that the initial shock does not trigger sensible real exchange rate movements that induce expenditure switching and balance sheet effects.

4.4 Monetary policy trade-offs

The central bank of a small open economy faced with real and financial frictions is faced with two fundamental trade-offs. First, the trade-off between inflation and output volatility, which exists when greater inflation stability can be obtained only at the expense of increased output volatility. In closed economies, such trade-off is present when the economy is hit by cost-push shocks: the central bank can counteract the increase in inflation only by incurring a greater output contraction. On the contrary, following a demand shock that leads output and inflation to move in the same direction, the optimal response of the central bank is to fully neutralize it, stabilizing at the same time both output and inflation\(^{25}\). Furthermore, the intensity of the output-inflation trade-off increases with the degree of rigidities in the model:

Blanchard and Gali’ (2007) show that the presence of wage rigidities in addition to price rigidities aggravates the trade-off faced by the central bank. Paustian (2004), shows that sticky wages lead to the existence of an output-inflation trade-off even when the economy is subject to cost-push shocks. In this case, if the central bank targets inflation and reacts to the shock by rising the interest rate, it incurs in a larger loss in output because the monetary policy reaction, by dampening the increase in inflation, prevents a fall in the real wage that would mitigate the output fall.

In the open economy context, the presence of the real exchange rate affects the trade-off faced by monetary policy: in particular, in this case the output-inflation trade-off is present even for demand shocks. The reason is that real exchange rate movements have an effect on aggregate demand, inflation and the monetary policy instrument. If the real exchange rate had effects only on aggregate demand, stabilizing exchange rate disturbances would be optimal, and wouldn’t lead to any trade-off. However, real exchange rate movements exert an effect on inflation, through the change in the price of imported goods. Furthermore, the nominal interest rate has an effect on the exchange rate, through the uncovered interest parity. Hence, if the central bank tries to stabilize output following a demand shock, the monetary policy action will affect the real exchange rate, and through it inflation, thereby introducing a trade-off (Walsh (1999)).

Concerning the relationship between the output-inflation trade-off and monetary policy in open economies, previous studies conclude that such trade-off is accentuated when the central bank implements a fixed exchange rate policy. This results from the fact that a central bank that aims at maintaining the exchange rate stable has to enact a stronger monetary policy reaction. For example, facing a shock with devaluation pressures, the central bank is forced to increase the interest rate to defend the parity. Albeit this reduces inflation, it also dampens the positive effect on output stemming from expenditure switching effects. Furthermore, in a model with the financial accelerator, the contractionary monetary policy action, by increasing the cost of external finance, triggers a negative spiral of disinvestment, exacerbating the negative effect on output and thereby worsening the output-inflation trade-off. However, as shown by Devereux, Lane and Xu (2006), the intensity of such trade-off depends on the degree of exchange rate pass-through in the economy. With high pass-through, exchange rate fluctuations are swiftly transmitted to consumer price inflation. Hence, a fixed exchange rate stabilizes inflation and the exchange rate, but implies higher output volatility. With a low degree of pass-through, the expenditure switching effect is more muted and exchange rate movements do not immediately destabilize the price level. Hence, monetary policy can stabilize inflation and still afford exchange rate flexibility to stabilize the economy facing external shocks.

The second trade-off a central bank faces is between real and financial stability and it relates to the key importance of interest rate movements in affecting both the real and financial sector of the economy. On the real side, changes in the interest rate influence the intertemporal consumption and saving decisions of households. On the financial side, the risk-free interest rate affects the price of credit, by altering the opportunity cost of

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27 See also Devereux (2001).
lenders of engaging in a financial contract and, ultimately, asset prices and investment. These effects work in the same direction and, say, an expansionary monetary policy boosts economic activity through its joint effect on consumption and investment. However, two issues arise. First, an attempt of the central bank to stabilize inflation might result in increased financial vulnerability. For example, by lowering the interest rate to counteract a shock leading to deflationary pressures, the central bank induces a decrease in borrowing rates which encourages borrowing, thereby increasing leverage in the economy. Second, when the economy is hit by a disturbance that leads to inflationary pressures and a decrease in output (say, a negative foreign productivity shock), the central bank reacts by increasing the policy rate. In doing so, it stabilize inflation, but it worsens credit conditions, thereby discouraging investment and prolonging the negative output effect of the shock.

Table 8: Sacrifice Ratios

<table>
<thead>
<tr>
<th></th>
<th>Financial Autarky</th>
<th>Bank Integration</th>
<th>Full Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Float</td>
<td>Fix</td>
<td>Float</td>
</tr>
<tr>
<td>Foreign interest rate</td>
<td>0.9617</td>
<td>3.3663</td>
<td>0.9990</td>
</tr>
<tr>
<td>Foreign productivity</td>
<td>3.8576</td>
<td>8.1643</td>
<td>3.9010</td>
</tr>
<tr>
<td>Foreign sigmaB</td>
<td>1.9434</td>
<td>1.6506</td>
<td>2.8437</td>
</tr>
<tr>
<td>Foreign sigmaE</td>
<td>1.5538</td>
<td>1.2434</td>
<td>1.5489</td>
</tr>
</tbody>
</table>

In Table 8, I compute the sacrifice ratios, i.e. the ratio between output volatility and inflation volatility, for different degrees of financial integration, exchange rate regimes and shocks. Intuitively, the higher the ratio, the greater the trade-off the central bank has to face, since in order to stabilize inflation it has to accept increased output volatility. The table shows that, for all degrees of financial integration, in case of foreign monetary policy and productivity shocks, the output-inflation trade-off is greater when the Home country’s central bank pegs the exchange rate. In both cases, this results from the stronger monetary policy response the domestic central bank has to enact to preserve the parity in case of fixed exchange rate. The foreign monetary policy shock has the effect of depreciating the domestic currency and increasing inflation. Under flexible exchange rate the central bank reacts to the inflation increase by increasing the interest rate, but still allowing the currency to depreciate and, through expenditure switching effects, mitigate the negative effect of the shock on output. Under fixed exchange rate, the domestic central bank increases the interest rate more markedly in order to keep the nominal value of the currency stable. Hence, not

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28 A growing literature has explored the issue as to whether central banks should respond directly to credit conditions, by including financial variables in the policy rule. A non-exhaustive list of such studies include Goodfriend and McCallum (2007), De Fiore and Tristani (2009), Faia and Monacelli (2007), Merola (2010), Curdia and Woodford (2010), Woodford (2010), Davis and Huang (2012). However, a consensus on the matter has not been reached. While Goodfriend and McCallum (2007), De Fiore and Tristani (2009), Curdia and Woodford (2010), Woodford (2010) conclude that in the presence of credit frictions, it is optimal to include a spread adjustment in the Taylor Rule, others claim that doing so has a negligible impact on welfare, and that targeting inflation is always optimal (Faia and Monacelli (2007), Merola (2010)). Finally, Davis and Huang (2012) argue that modifying the Taylor rule to include financial variables optimal only fluctuations in credit spreads are caused by exogenous disturbances to the credit sector.
only the expenditure switching effect is absent, but real wages increase instead of falling, thereby amplifying the volatility of output. The case of a foreign productivity shock is broadly similar, however here the central bank’s response under fixed exchange rate is not as strong as to imply a reversal in the behavior of inflation. In any case, the reduced expenditure switching effect and the smaller decrease in real wages contribute to accentuate output volatility under fixed exchange rate.

Foreign banks’ and entrepreneurs’ riskiness shocks imply a lower trade-off than the productivity shock. This is because they act as demand shocks, implying a simultaneous decrease in inflation and economic activity. Furthermore, the table shows that in case of financial shocks, the trade-off decreases when the central bank opts for a pegged exchange rate regime: however, the difference is very small.

It is interesting to notice that, for all shocks, the monetary policy trade-off increases with the degree of financial integration, meaning that, the more an economy is financially linked to another, the central bank has to accept higher output volatility in order to stabilize inflation.

5 Sensitivity analysis

In what follows, I report synthetic results obtained by perturbing the model on several grounds, in order to test the robustness of my findings to alternative model specifications and parametrizations.

First, I test the sensitivity of my results to a specification of the model where credit contracts are stipulated in nominal terms. As noted by Dib, Mendicino and Zhang (2008), nominal assets introduce private risk given by uncertain returns. In fact, the external finance premium in non-indexed contracts is directly linked to expected inflation and, if foreign borrowing is allowed, to expected movements in the nominal exchange rate. Furthermore, inflation increases the value of nominal net worth and decreases the real value of debt. Hence, nominal contracts introduce a redistribution of wealth between borrowers and lenders due to unexpected fluctuations in debt services. Moreover, with nominal contracts, disinflation reinforces the financial accelerator mechanism, thereby dampening the expansionary effects of a decrease in prices. Therefore, from a monetary policy point of view, nominal contracts give a central bank an incentive not to respond aggressively to a shock with inflationary pressures.

Figures 5a to 5d report the results of model simulations with nominal debt contracts. Comparing these results with the impulse-responses of the baseline model, it is clear that lack of debt indexation does not change my conclusions regarding the performance of fixed and flexible exchange rate regimes and the role of financial interlinkages, it only results in a slight amplification in the effect of all shocks.

Secondly, I change the relative size of the small country. In the baseline simulations I reduced the size of the small economy to zero, implying that its developments had no repercussions on the foreign country. Now I set the relative size of the small country to 0.25: while the Home country is still relatively smaller, it is not completely invisible in the eyes
of the foreign country. This allows for stronger feedback effects between countries, which operate mainly through trade channels. Allowing for greater intercountry feedbacks implies that the Foreign country is going to be influenced by changes in international relative prices, because now the share of imports from the Home country is not negligible in the Foreign consumption basket, and this is going to have repercussions on the small country. Therefore, the real exchange rate, which can be shown to be dependent on the openness parameters of the two countries, in turn related to the relative country size in this model\textsuperscript{29}, is going to be influenced by domestic developments. The impulse-response functions (not shown for space reasons but available upon request) show that the real exchange rate in this version of the model is less volatile than in the baseline. However, this does not alter the conclusions regarding the better performance of flexible exchange rates in isolating the economy from external shocks, nor the interaction between exchange rate regimes and degrees of financial integration.

\textsuperscript{29} see for example Kolasa and Lombardo (2011) for a proof.
Table 9: Volatilities Nominal Contract Model

<table>
<thead>
<tr>
<th></th>
<th>Financial</th>
<th>Autarky</th>
<th>Bank Integration</th>
<th>Full Integration</th>
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<td>Float</td>
<td>Fix</td>
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<td>0.9700</td>
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<td>0.0901</td>
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<td>0.5829</td>
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<td>1.1065</td>
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<td>0.0474</td>
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<td></td>
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<tr>
<td>Output</td>
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<td>0.0228</td>
<td>0.0274</td>
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<td>0.0056</td>
<td>0.0021</td>
<td>0.0037</td>
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</table>

As a third robustness check, change the parametrization of the financial accelerator mechanism, since my calibration based on European data yields different values for the parameters representing monitoring costs and idiosyncratic volatilities in the banking and entrepreneurial sectors than in the paper by Ueda (2012), as shown in Table 10:
Table 10: Alternative Calibration Financial Accelerator

<table>
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<th>Baseline calibration</th>
<th>Ueda (2012) calibration</th>
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<tr>
<td>$[0.5\text{ex}] \mu_{E,H}$</td>
<td>0.033</td>
<td>0.013</td>
</tr>
<tr>
<td>$\mu_{E,F}$</td>
<td>0.033</td>
<td>0.013</td>
</tr>
<tr>
<td>$\mu_{F,H}$</td>
<td>0.243</td>
<td>0.033</td>
</tr>
<tr>
<td>$\mu_{F,F}$</td>
<td>0.243</td>
<td>0.033</td>
</tr>
<tr>
<td>$\gamma_E$</td>
<td>0.985</td>
<td>0.984</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>0.969</td>
<td>0.963</td>
</tr>
<tr>
<td>$QK/NW_e$</td>
<td>0.490</td>
<td>0.5</td>
</tr>
<tr>
<td>$QK/NW_f$</td>
<td>0.104</td>
<td>0.1</td>
</tr>
<tr>
<td>$R^k/R$</td>
<td>1.0049</td>
<td>1.005</td>
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<tr>
<td>$Z^E - Z^F$</td>
<td>0.0052</td>
<td>0.023\text{*}^{0.25}</td>
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<tr>
<td>$Z^F - R$</td>
<td>0.0004</td>
<td>0.006\text{*}^{0.25}</td>
</tr>
<tr>
<td>$F(\tilde{\omega}_E)$</td>
<td>0.0075</td>
<td>0.02</td>
</tr>
<tr>
<td>$F(\tilde{\omega}_F)$</td>
<td>0.0015</td>
<td>0.02</td>
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</table>

As the alternative calibration does not alter the results, I omit the impulse-response functions for space reasons.

6 Conclusion

This paper examines the interaction between financial interlinkages and exchange rate regimes in a small open economy, obtained as the limit of a two-country DSGE model with real and financial frictions. In particular, it compares the performance of fixed versus flexible exchange rate regimes in stabilizing a small open economy facing foreign nominal, real and financial shocks.

The simulation results lead to a clear understanding of the mechanism through which foreign shocks spill over to the small economy. While under financial autarky, the domestic economy is affected by foreign disturbances to the extent that they influence international relative prices and demand, financial linkages provide an additional transmission channel. In particular, the domestic and foreign monetary policy responses to a shock are of key importance in determining the role of financial linkages under the two exchange rate regimes. While the role of financial linkages in magnifying the spillover of foreign shocks has been studied in the recent literature, the role of exchange rate regimes is an issue not previously explored. I find that, overall, themundellian argument in favor of flexible exchange rates holds irrespectively of the degree of financial integration. In fact, for all shocks, a fixed exchange rate policy delivers larger output losses and higher volatility of real and financial variables. Furthermore, my results reveal that the cost of pegging the exchange rate decreases when the degree of financial integration increases. Finally, I find that the presence of financial linkages increases the trade-off between inflation and output volatility faced by the central bank of a small open economy.

The present study constitutes a first attempt to study monetary policy and financial integration in a New Keynesian model with cross-border lending. The analysis can be extended
in several interesting directions, which will be the subject of future research.

First, it would be worth focusing on the characterization of the optimal monetary policy for the small open economy, and assess whether the inclusion of domestic and/or foreign financial variables in the taylor rule would lead to better stabilization performances and welfare gains. Second, an interesting question to explore in this setting is whether financial linkages increase the incentive of a small country to join a monetary union. Third, testing the ability of the model to account for the transmission of the financial crisis through Bayesian estimation would lead to interesting insights on the appropriateness of the chosen modeling strategy in accounting for actual economic dynamics.

References


7 Appendix

7.1 Model equations

In what follows, the complete set of stationary equilibrium relationships are presented. In equilibrium, only relative prices are defined. In every country, nominal prices are deflated by the composite price index, resulting in the following definitions:
\[ p_{HH,t} = \frac{P_{H,t}}{P_t}; \quad p_{HF,t} = \frac{P_{F,t}}{P_t}; \]
\[ p_{FH,t} = \frac{P_{H,t}^*}{P_t}; \quad p_{FF,t} = \frac{P_{F,t}^*}{P_t^*}; \]

Hence, the following relationships linking relative prices and inflation are derived:

\[ \pi_{HH,t} = \frac{p_{HH,t}}{P_{HH,t-1}} \pi_{H,t} \]
\[ \pi_{HF,t} = \frac{p_{HF,t}}{P_{HF,t-1}} \pi_{H,t} \]
\[ \pi_{FF,t} = \frac{p_{FF,t}}{P_{FF,t-1}} \pi_{F,t} \]
\[ \pi_{FH,t} = \frac{p_{FH,t}}{P_{FH,t-1}} \pi_{F,t} \]

Furthermore, the equations of the domestic and foreign CPI level are rewritten as:

\[ 1 = p_{HH,t}^{(1-\gamma)} \]
\[ 1 = p_{FF,t}^{(1-\gamma)} \]

In addition, the relationships between relative prices, the real exchange rate and the terms of trade can be formulated as:

\[ p_{FH,t} = \frac{1}{\varepsilon_t} p_{HH,t} \]
\[ p_{HF,t} = \varepsilon_t p_{FF,t} \]
\[ \varepsilon_t = \frac{S_t}{S_{t-1}} \frac{\pi_{F,t}}{\pi_{H,t}} \]
\[ TOT_t = \varepsilon_t \frac{p_{HF,t}}{p_{HH,t}} \]

The complete equilibrium conditions for the Home country are presented. An analogous set of equations holds for the Foreign economy.

\[ \mu_t = (C_t - hC_{t-1})^{-\sigma} \]
\[ \mu_t = \beta E_t \left\{ \mu_{t+1} \frac{R_t}{\pi_{t+1}} \right\} \]
\[ w_t = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi_H H_t^2}{\mu_t} \]
\[
C_{H,t} = \gamma \left( \frac{P_{H,t}(j_H)}{P_{H,t}} \right)^{-\varepsilon} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t
\]
\[
C_{F,t} = (1 - \gamma) \left( \frac{P_{F,t}(j_F)}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t
\]
\[
K_{t}^w = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \bar{W}_t \mu_t H_t \left( \frac{W_t}{\bar{W}_t} \right)^{\varepsilon} + \beta \theta_w \left( \pi_{H,t+1} \bar{W}_{t+1} \right)^{\varepsilon - 1} K_{t+1}^w
\]
\[
F_t^w = \chi H \left( H_t \right)^{\varphi} \left( \frac{W_t}{\bar{W}_t} \right)^{\varepsilon} H_t + \beta \theta_w \left( \pi_{H,t+1} \bar{W}_{t+1} \right)^{\varepsilon - 1} F_{t+1}^w
\]
\[
K_t^w = F_t^w
\]
\[
W_t = \left[ (1 - \theta_w) \bar{W}_t^{1-\varepsilon} + \theta_w (W_{t-1}(i) \pi_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}
\]
\[
\varepsilon_t = \frac{\mu_t - \sigma}{\mu_t + \sigma}
\]
\[
Y_{H,t} = \Lambda_t K_{H,t} \gamma \alpha
\]
\[
W_t = MC_t (1 - \alpha) \frac{Y_t}{H_t}
\]
\[
r_t^K = MC_t \gamma \alpha \frac{Y_t}{K_t}
\]
\[
P_{H,t} = \left[ \theta_H \left( P_{H,t-1} \pi_{t-1} \gamma_H \left( \pi_{t} \right) \right)^{1-\varepsilon_H} + (1 - \theta_H) \pi_{t+1}^{1-\varepsilon_H} \right]^{\frac{1}{1-\varepsilon_H}}
\]
\[
\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon_H}{\varepsilon_H - 1} \frac{F_{H,t}}{D_{H,t}}
\]
\[
F_{H,t} = \frac{\Lambda_t}{P_t} P_{H,t} \lambda c_{1,t} Y_t^H + \beta \theta_H E_t \left\{ (\pi_{t+1}^H \varepsilon_H) F_{H,t+1} \right\}
\]
\[
D_{H,t} = \frac{\Lambda_t}{P_t} P_{H,t} Y_t^H + \beta \theta_H E_t \left\{ (\pi_{t+1}^H \varepsilon_H - 1) D_{H,t+1} \right\}
\]
\[
1 = \frac{1}{q_{H,t}} \left[ 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_{t+1}}{I_{t-1}} - 1 \right) \right] + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} q_{H,t+1} \left[ \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \right\}
\]

44
\[ K_{t+1} = \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]^2 I_t + (1 - \delta) K_t \]

\[ I_{H,t} = \gamma \left( \frac{P_{H,t}(jH)}{P_{H,t}} \right)^{-\varepsilon} \left( \frac{P_{H,t}}{P_t} \right)^{-1} I_t \]

\[ I_{F,t} = (1 - \gamma) \left( \frac{P_{F,t}(jF)}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_t} \right)^{-1} I_t \]

\[ R_{H,t}^E = \frac{\tau_t^K + (1 - \delta) Q_{H,t}}{Q_{H,t-1}} \]

\[ R_{F,t}^E = \frac{\tau_t^K + (1 - \delta) Q_{F,t}}{Q_{F,t-1}} \]

\[ \omega_{H,t}^{E,F} R_{F,H,t}^E Q_{H,t} K_{H,t} = Z_{H,t}^E (Q_{H,t} K_{H,t,t} - NW_{H,t}) \]

\[ \omega_{F,t}^{E,F} R_{F,F,t}^E Q_{F,t} K_{F,t} = Z_{F,t}^E (Q_{F,t} K_{F,t,t} - NW_{F,t}) \]

\[ 0 = \left\{ \begin{array}{l} R_{H,t}^{F,F} \left[ \left( 1 - \tau_t^E \right) (Q_{H,t} K_{H,t,t} - NW_{H,t}) + \tau_t^E \varepsilon_t (Q_{F,t} K_{F,t,t} - NW_{F,t}) \right] = Z_{H,t}^{F,F} I_t \\ + \omega_{H,t}^{E,F} R_{F,H,t}^E Q_{H,t} K_{H,t} = Z_{H,t}^E (Q_{H,t} K_{H,t,t} - NW_{H,t}) \end{array} \right. \]

\[ 0 = \left\{ \begin{array}{l} R_{F,t}^{F,F} \left[ \left( 1 - \tau_t^E \right) (Q_{H,t} K_{H,t,t} - NW_{H,t}) + \tau_t^E \varepsilon_t (Q_{F,t} K_{F,t,t} - NW_{F,t}) \right] = Z_{H,t}^{F,F} I_t \\ + \omega_{F,t}^{E,F} R_{F,F,t}^E Q_{F,t} K_{F,t} = Z_{F,t}^E (Q_{F,t} K_{F,t,t} - NW_{F,t}) \end{array} \right. \]
\[
(1 - \tau_H^E) \Phi^F (\omega_{HH,t+1}^F) \cdot \left[ (1 - \tau_H^E) R_{HH,t+1}^E Q_{H,H,H,t}^H K_{HH,t}^H \Phi^E (\omega_{HH,t+1}^E) + \right] = \\
R_t (1 - \tau_H^E) \left[ (1 - \tau_H^E) (Q_{H,H,H,t}^H - NW_{H,t}^E) + \tau_F \varepsilon_t (Q_{F,F,H,t}^F - NW_{F,F,t}^E) - NW_{H,t}^E \right] \\
\tau_H^E \Phi^F (\omega_{HH,t+1}^F) \cdot \left[ (1 - \tau_H^E) R_{HH,t+1}^E Q_{H,H,H,t}^H K_{HH,t}^H \Phi^E (\omega_{HH,t+1}^E) + \right] = \\
R_t^* \varepsilon_t \left[ (1 - \tau_H^E) (Q_{F,F,H,t}^F - NW_{F,F,t}^E) + \tau_F \varepsilon_t (Q_{F,F,H,t}^F - NW_{F,F,t}^E) - NW_{H,t}^E \right] \\
\left[ 1 - \Gamma_{HH}^E (\omega_{HH,t+1}^E) \right] R_{HH,t+1}^E Q_{H,H,H,t}^H K_{HH,t}^H = R_{HH,t+1}^E NW_{H,t}^E \\
\left[ 1 - \Gamma_{FH}^E (\omega_{HH,t+1}^F) \right] R_{FH,t+1}^E Q_{F,F,H,t}^F K_{FH,t}^H = R_{FH,t+1}^E NW_{F,t}^E \\
NW_{H,t}^F = \gamma^E V_{H,t}^E + W_{H,t}^E \\
NW_{H,t}^F = \gamma^E V_{H,t}^F + W_{H,t}^F \\
V_{H,t}^E = [1 - \Gamma_{HH,t+1}^E (\omega_{HH,t}^E)] (1 - \tau_H^E) R_{HH,t+1}^E Q_{H,H,H,t+1}^H K_{HH,t+1}^H + \\
+ [1 - \Gamma_{FH,t+1}^E (\omega_{HH,t}^F)] \tau_H^E R_{FH,t+1}^E Q_{H,H,H,t+1}^H K_{HH,t+1}^H \\
V_{H,t}^F = [1 - \Gamma_{HH,t+1}^F (\omega_{HH,t}^F)] (1 - \tau_H^F) R_{H,F,t+1}^F \left\{ (1 - \tau_H^F) (Q_{H,H,H,t+1}^H - NW_{H,t+1}^E) + \tau_F \varepsilon_t (Q_{F,F,H,t+1}^F - NW_{F,F,t+1}^E) \right\} + \\
+ [1 - \Gamma_{FH,t+1}^F (\omega_{HH,t}^F)] \tau_H^F R_{H,F,t+1}^F \left\{ (1 - \tau_H^F) (Q_{H,H,H,t+1}^H - NW_{H,t+1}^E) + \tau_F \varepsilon_t (Q_{F,F,H,t+1}^F - NW_{F,F,t+1}^E) \right\} \\
C_{H,t}^E = (1 - \gamma) V_{H,t}^E \\
C_{H,t}^F = (1 - \gamma) V_{H,t}^F \\
C_{H,t}^E = \gamma \left( \frac{P_{H,t}(j_H)}{P_{H,t}} \right)^{-\varepsilon} \left( \frac{P_{H,t}}{P_{t}} \right)^{-1} C_{t}^E \\
C_{F,t}^E = (1 - \gamma) \left( \frac{P_{F,t}(j_F)}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_{t}} \right)^{-1} C_{t}^E \\
C_{H,t}^F = \gamma \left( \frac{P_{H,t}(j_H)}{P_{H,t}} \right)^{-\varepsilon} \left( \frac{P_{H,t}}{P_{t}} \right)^{-1} C_{t}^F \\
C_{F,t}^F = (1 - \gamma) \left( \frac{P_{F,t}(j_F)}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_{t}} \right)^{-1} C_{t}^F
\[ K_t = (1 - \tau^E) K_{HH,t} + \tau^E K_{HF,t} \]

\[ Y_{H,t} = \Delta_{H,t} \left( C_{H,t} + C^E_{H,t} + C^F_{H,t} + I_{H,t} + G_t + M_t \right) + \Delta^*_H \frac{(1-n)}{n} \left( C^*_H + C^F_{H,t} + C^F_{H,t} + I^*_H \right) \]

\[ M_t = \mu^E G^E (\bar{\omega}^E_{H,t}) R^E_{H,t} (1 - \tau^E) Q_{H,t-1} K_{HH,t-1} + \mu^E G^E (\bar{\omega}^E_{HF,t}) R^E_{HF,t} \varepsilon_{t-1} (1 - \tau^E) + \mu^F G^F (\bar{\omega}^F_{HF,t}) (1 - \tau^E) \varepsilon_{t-1} \varepsilon_{t-1} (1 - \tau^E) + \mu^F G^F (\bar{\omega}^F_{HF,t}) \tau^F_{HF,t} + \mu^F G^F (\bar{\omega}^F_{HF,t}) \tau^F_{HF,t} \]

\[ \Delta_{H,t} = (1 - \theta_H) \left[ \frac{1 - \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{1-\varepsilon_H}}{1 - \theta_H} \right] + \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{-\varepsilon_H} \Delta_{H,t-1} \]

\[ \Delta^*_{H,t} = (1 - \theta_H) \left[ \frac{1 - \theta_H \left( \frac{1}{\pi_{H,t}} \right)^{1-\varepsilon_H}}{1 - \theta_H} \right] + \theta_H \left( \frac{1}{\pi_{H,t}^*} \right)^{-\varepsilon_H} \Delta^*_{H,t-1} \]

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_r} \left[ \left( \frac{\pi_{H,t}}{\pi_H} \right)^{\rho_s} \left( \frac{S_t}{S} \right) \frac{1}{1-\rho_s} \right]^{(1-\rho_r)} \exp(\xi_{R,t}) \]

### 7.2 Expressions related to optimal credit contract

As in BGG(1999), the idiosyncratic shocks of domestic and foreign entrepreneurs and financial intermediaries \((\omega^E_{HH,t}, \omega^E_{HF,t}, \omega^E_{HF,t}, \omega^F_{HF,t}, \omega^F_{HF,t}, \omega^F_{HF,t}, \omega^F_{HF,t}, \omega^F_{HF,t})\) are log-normally distributed with \(E(\omega_{ij,t}) = 1\). I denote \(f(\omega_{ij})\) the probability distribution function and \(F(\omega_{ij})\) the cumulative distribution function of \(\omega_{ij}\). Hence:

\[ f \left( \bar{\omega}_{ij} - \frac{\sigma^2}{2}, \sigma \right) = \frac{1}{\bar{\omega}_{ij} \sigma \sqrt{2\pi}} e^{-\frac{(\log \bar{\omega}_{ij} + \frac{\sigma^2}{2})^2}{2\sigma^2}} = \frac{1}{\bar{\omega}_{ij} \sigma} \text{Npdf} \left( \frac{\log \bar{\omega}_{ij} + 0.5\sigma^2}{\sigma} \right) \]

\[ F \left( \bar{\omega}_{ij} - \frac{\sigma^2}{2}, \sigma \right) = \frac{1}{\sqrt{2\pi}} \int_0^{\bar{\omega}_{ij}} e^{-t^2} dt = \text{Ncdf} \left( \frac{\log \bar{\omega}_{ij} + 0.5\sigma^2}{\sigma} \right) \]
\[ G(\omega_{ij}; \sigma) = \int_{0}^{\bar{\omega}_{ij}} \omega_{ij} f(\omega_{ij}) d\omega_{ij} = Ncdf \left( \frac{\log \bar{\omega}_{ij} - 0.5\sigma^2}{\sigma} \right) \]

\[ \Gamma(\omega_{ij}; \sigma) = \bar{\omega}_{ij} [1 - F(\bar{\omega}_{ij})] + G(\omega_{ij}; \sigma) \]

\[ \Gamma'(\omega_{ij}; \sigma) = [1 - F(\bar{\omega}_{ij})] \]

\[ G'(\omega_{ij}; \sigma) = \bar{\omega}_{ij} f(\bar{\omega}_{ij}) \]

\[ \Phi(\omega_{ij}; \sigma) = \Gamma(\omega_{ij}; \sigma) - \mu_{ij} G(\omega_{ij}; \sigma) \]

\[ \Phi'(\omega_{ij}; \sigma) = \Gamma'(\omega_{ij}; \sigma) - \mu_{ij} G'(\omega_{ij}; \sigma) \]

### 7.3 Steady state

As the model is not solvable in closed form, I perform the analysis by linearizing the model equations around the non-stochastic steady state with zero inflation and no exchange rate depreciation. In the deterministic steady state, all shocks are equal to their mean values, and all (gross) inflation rates are equal to 1. Furthermore, marginal costs and markups are the same for all firms in the economy, hence all relative prices are equal to 1 and price dispersion is equal to 1. Finally, in the steady state consumption is equalized across countries and the net foreign asset position is zero.

Assume \( A_{NT} = A_H = A^*_F = A_F = 1 \).

\[ \pi_t = \pi^*_t = \pi^H_t = \pi^F_t = \pi^N_{NT} = 1 \]

Normalize nominal exchange rate:

\[ S_t = 1 \]

Use Consumption Euler:

\[ R_t = \frac{1}{\beta} = R^*_t \]

From the Tobin’s Q equation I obtain:

\[ Q_t = Q^*_t = 1 \]

### 7.3.1 Credit markets and financial frictions

The steady state of the credit market is computed assuming target values for six quantities:

1. The risk premium for entrepreneurs \((R^E - R)\),
2. The leverage ratio for financial intermediaries \(\frac{NW^E}{QK}\),
3. The leverage ratio of entrepreneurs \(\frac{NW^E}{QK}\),
4. The annualized default probability of financial intermediaries \(F(\bar{\omega}^E_{ij})\),
5. The annualized default rate of entrepreneurs \(F(\bar{\omega}^E_{ij})\),
6. The spread between the FI’s loan rate and the FI’s borrowing rate \((Z^E - Z^F)\)

and, finally, (7) the spread between the FI’s borrowing rate and the risk free rate \((Z^F - R)\). I choose the value of parameters related to monitoring costs in the contract between banks and entrepreneurs \((\mu^E)\) and between banks and lenders \((\mu^F)\), volatility of the idiosyncratic...
shocks \((\sigma^E, \sigma^F)\), steady state threshold productivity levels \((\bar{\omega}^E, \bar{\omega}^F)\) and survival rate of entrepreneurs and financial intermediaries \((\gamma^E, \gamma^F)\) to match the aforementioned steady state quantities.

In the calibration I assume that countries are fully symmetric, including in the degree of financial frictions, both within and between countries. This implies that domestic (foreign) financial intermediaries incur the same cost of monitoring domestic and foreign lenders, and domestic (foreign) lenders are subject to the same cost of monitoring domestic and foreign financial intermediaries. Furthermore, these costs are equalized across countries.

Given values of \(\sigma^E, \sigma^H \) and \(\sigma^F, \sigma^F \), and a target value for the default probabilities in each sector \(F(\bar{\omega}^E; \sigma^E), F(\bar{\omega}^E; \sigma^E)\) and \(F(\bar{\omega}^H; \sigma^H), F(\bar{\omega}^F; \sigma^F)\), I can calculate the threshold productivity levels:

\[
\bar{\omega}_{ij} = Ncdf^{-1} \left( \frac{\log \bar{\omega}_{ij} + 0.5\sigma^2}{\sigma} \right)
\]

Now I can calculate the following quantities:

\[
F'_\omega(\bar{\omega}_{ij}; \sigma^j) = f(\bar{\omega}_{ij}; \sigma^j) = \frac{1}{\bar{\omega}_{ij}\sigma^j} Npdf \left( \frac{\log \bar{\omega}_{ij} + 0.5\sigma^2}{\sigma} \right)
\]

\[
G(\bar{\omega}_{ij}; \sigma^j) = \int_0^{\bar{\omega}_{ij}} \omega_{ij} f(\omega_{ij}) d\omega = Ncdf \left( \frac{\log \bar{\omega}_{ij} - 0.5\sigma^2}{\sigma} \right)
\]

\[
\Gamma(\bar{\omega}_{ij}; \sigma) = \bar{\omega}_{ij} (1 - F(\bar{\omega}_{ij}; \sigma)) + G(\bar{\omega}_{ij}; \sigma)
\]

\[
\Gamma'_\omega(\bar{\omega}_{ij}; \sigma) = 1 - F(\bar{\omega}_{ij})
\]

\[
G'_\omega(\bar{\omega}_{ij}; \sigma) = \bar{\omega}_{ij} F'_\omega(\bar{\omega}_{ij}; \sigma)
\]

Given \(\mu^j\), I can calculate:

\[
\Phi(\bar{\omega}_{ij}; \sigma^j) = \Gamma(\bar{\omega}_{ij}; \sigma) - \mu_{ij} G(\bar{\omega}_{ij}; \sigma)
\]
\[
\Phi'_\omega(\bar{\omega}_{ij}; \sigma) = \Gamma'_\omega(\bar{\omega}_{ij}; \sigma) - \mu_{ij} G'_\omega(\bar{\omega}_{ij}; \sigma)
\]

Assume countries are symmetric, i.e. they have the same degree of financial frictions \((\sigma^E = \sigma^E, \sigma^F = \sigma^F, \mu^E = \mu^E, \mu^F = \mu^F, \bar{\omega}^E = \bar{\omega}^E, \bar{\omega}^F = \bar{\omega}^F)\) and the same degree of openness \((\tau_H^F = \tau_F^F, \tau^E_H = \tau^E_F)\).

Then from the first order conditions of the optimal contract in country H (recall that in steady state, \(R_t = R_t^* = \frac{1}{\beta}\)):
\[
R_H^E \left[ (1 - \Gamma (\bar{\omega}^E)) \Phi' (\bar{\omega}^E) + \Gamma' (\bar{\omega}^E) \Phi (\bar{\omega}^E) \right] \cdot \left[ (1 - \Gamma (\bar{\omega}^F)) (1 - \tau^F) + (1 - \Gamma (\bar{\omega}^F)) \tau^F \right] + \\
(1 - \tau^F) \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \left[ \Gamma' (\bar{\omega}^E) \Phi (\bar{\omega}^E) R_H^E \Phi (\bar{\omega}^E) + \Phi (\bar{\omega}^F) \Phi' (\bar{\omega}^E) (1 - \Gamma (\bar{\omega}^E)) R_H^E \right] + \\
\tau^F \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \left[ \Gamma' (\bar{\omega}^E) \Phi (\bar{\omega}^E) R_H^E \Phi (\bar{\omega}^E) + \Phi (\bar{\omega}^F) \Phi' (\bar{\omega}^E) (1 - \Gamma (\bar{\omega}^E)) R_H^E \right] = \\
= \frac{1}{\beta} (1 - \tau^F) \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \Gamma' (\bar{\omega}^E) + \frac{1}{\beta} \tau^F \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \Gamma' (\bar{\omega}^E)
\]

\[
R_H^E \left\{ \left[ (1 - \Gamma (\bar{\omega}^E)) \Phi' (\bar{\omega}^E) + \Gamma' (\bar{\omega}^E) \Phi (\bar{\omega}^E) \right] \cdot \left[ (1 - \Gamma (\bar{\omega}^F)) (1 - \tau^F) + (1 - \Gamma (\bar{\omega}^F)) \tau^F \right] + \\
\frac{\Gamma' (\bar{\omega}^F)}{\Phi' (\bar{\omega}^F)} \Phi' (\bar{\omega}^E) (1 - \Gamma (\bar{\omega}^E)) + \frac{\Gamma' (\bar{\omega}^F)}{\Phi' (\bar{\omega}^F)} \Gamma' (\bar{\omega}^E) \Phi (\bar{\omega}^F) \Phi (\bar{\omega}^E) \right\} = \\
= R \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \Gamma' (\bar{\omega}^E)
\]

\[
\frac{R_H^E}{R} = \frac{\Gamma' (\bar{\omega}^E)}{\Phi' (\bar{\omega}^E)} \Gamma' (\bar{\omega}^E)
\]

And, similarly:

\[
\frac{R_H^E}{R} = \frac{\Gamma' (\bar{\omega}^F)}{\Phi' (\bar{\omega}^F)} \Gamma' (\bar{\omega}^F)
\]

Given symmetry, it results that:

\[
\frac{R_H^E}{R} = \frac{R_H^E^*}{R^*} = \frac{R_F^E}{R^*} = \frac{R_F^E^*}{R^*}
\]

Part. constraint entrepreneurs borrowing from FI H

\[
K = (1 - \tau^E) K_H + \tau^E K_F
\]

\[
\left[ 1 - \Gamma (\bar{\omega}^E) \right] R_H^E Q K_H = R_H^E N W^E \tag{53}
\]

\[
\frac{N W^E}{Q K_H} = \left[ 1 - \Gamma_H^E (\bar{\omega}^E) \right]
\]

50
\[
[1 - \Gamma (\bar{\omega}^E)] \frac{R_H^{E*} Q^* K_H^*}{NW_{E*}} = R_H^{E*} NW_{E*}
\]
\[
\frac{NW_{E*}}{Q* K_H^*} = [1 - \Gamma (\bar{\omega}^E)]
\]

Part. constraint entrepreneurs borrowing from FI F:

\[
K_{t}^* = \tau^E K_{H,*t}^* + (1 - \tau^E) K_{F,t}^*
\]
\[
[1 - \Gamma (\bar{\omega}^E)] \frac{R_F^{E*} Q K_{F}^*}{NW_{E}} = R_F^{E*} NW_{E}
\]
\[
\frac{NW_{E}}{Q K_{F}^*} = [1 - \Gamma (\bar{\omega}^E)]
\]
\[
[1 - \Gamma (\bar{\omega}^E)] \frac{R_F^{E*} Q^* K_F^*}{NW_{E*}} = R_F^{E*} NW_{E*}
\]
\[
\frac{NW_{E*}}{Q* K_F^*} = [1 - \Gamma (\bar{\omega}^E)]
\]

From previous 4 equations + definition of aggregate capital I get:

\[
\frac{NW_{E}}{Q K} = \left[ \frac{\tau^E}{[1 - \Gamma (\bar{\omega}^E)]} + \frac{(1 - \tau^E)}{[1 - \Gamma (\bar{\omega}^E)]} \right]^{-1} = [1 - \Gamma (\bar{\omega}^E)]
\]
\[
\frac{NW_{E*}}{Q* K^*} = \left[ \frac{\tau^E}{[1 - \Gamma (\bar{\omega}^E)]} + \frac{(1 - \tau^E)}{[1 - \Gamma (\bar{\omega}^E)]} \right]^{-1} = [1 - \Gamma (\bar{\omega}^E)]
\]

Part constraints lenders to FI H:

\[
(1 - \tau^F) R^F \left[ (1 - \tau^E) (Q K_H - NW_{E}) + \tau^E (Q^* K_H^* - NW_{E*}) \right] \cdot \Phi (\bar{\omega}^F)
\]
\[= R(1 - \tau^F) \left[ (1 - \tau^E) (Q K_H - NW_{E}) + \tau^E (Q^* K_H^* - NW_{E*}) - NW_{F} \right]
\]
\[
\tau^F R^F \left[ (1 - \tau^E) (Q K_H - NW_{E}) + \tau^E (Q^* K_H^* - NW_{E*}) \right] \cdot \Phi (\bar{\omega}^F)
\]
\[= R^* \tau^F \left[ (1 - \tau^E) (Q K_H - NW_{E}) + \tau^E (Q^* K_H^* - NW_{E*}) - NW_{F} \right]
\]

The two expressions are equivalent in steady state as \( R = R^* \). Furthermore, if countries are symmetric: \( Q K_H = Q^* K_H^* = Q K \) and \( NW_{E} = NW_{E*} \) so that:

\[
-\frac{R^F}{R} (Q K - NW_{E}) \cdot \Phi (\bar{\omega}^F) + (Q K - NW_{E}) = NW_{F}
\]
\[
(Q K - NW_{E}) \left[ 1 - \frac{R^F}{R} \cdot \Phi (\bar{\omega}^F) \right] = NW_{F}
\]
\[
\left( 1 - \frac{NW_{E}}{Q K} \right) \left[ 1 - \frac{R^F}{R} \cdot \Phi (\bar{\omega}^F) \right] = \frac{NW_{F}}{Q K}
\]
Part constraint FI H:

\[
\left[ (1 - \tau^E) R^E_H Q K^*_H \Phi (\bar{\omega}^E) + \tau^E R^E_H Q^* K^*_H \Phi (\bar{\omega}^E) \right] = \\
R^E \left[ (1 - \tau^E) (Q K^*_H - NW^E) + \tau^E (Q^* K^*_H - NW^{E*}) \right] 
\]

(61)

Again imposing symmetry:

\[
\frac{\Phi (\bar{\omega}^E)}{\left( 1 - \frac{NW^E}{Q K_H} \right)} = \frac{R^E}{R^E}
\]

Similarly for foreign financial intermediaries:

Part constraint lenders to FI F:

\[
\tau^F R^F* \left[ \tau^E (Q K_F - NW^E) + (1 - \tau^E) (Q^* K^*_F - NW^{E*}) \right] \Phi^F (\bar{\omega}^F) = \\
R^F \tau^F \left[ \tau^E (Q K_F - NW^E) + (1 - \tau^E) (Q^* K^*_F - NW^{E*}) - NW^{F*} \right]
\]

(62)

\[
(1 - \tau^F) R^F* \left[ \tau^E (Q K_F - NW^E) + (1 - \tau^E) (Q^* K^*_F - NW^{E*}) \right] \Phi^F (\bar{\omega}^F) = \\
R^F (1 - \tau^F) \left[ \tau^E (Q K_F - NW^E) + (1 - \tau^E_F) (Q^* K^*_F - NW^{E*}) - NW^{F*} \right]
\]

(63)

So that:

\[
\left( 1 - \frac{NW^{E*}}{Q^* K^*} \right) \left[ 1 - \frac{R^F}{R^*} \Phi^F (\bar{\omega}^F) \right] = \frac{NW^{F*}}{Q^* K^*}
\]

Where \( \frac{R^F}{R^*} \) can be derived from the participation constraint of foreign FIs:

\[
\frac{\Phi (\bar{\omega}^E)}{\left( 1 - \frac{NW^E}{Q^* K^*} \right)} = \frac{R^F}{R^E}
\]

Finally, using the default threshold definitions (equations (46) and (47)) I can compute the steady state loan rate to entrepreneurs and to financial intermediaries (interbank rate):

\[
Z^E = Z^E_H = Z^E_F = Z^E_F* = \frac{\bar{\omega}^E R^E}{\left( 1 - \frac{NW^E}{Q K} \right)}
\]

\[
\bar{\omega}^F_H R^F (Q K - NW^E) = Z^F_H \left[ (Q K - NW^E) - NW^F \right]
\]

\[
Z^F = Z^F_H = Z^F_H = Z^F_F = Z^F_F* = \frac{\bar{\omega}^F R^F \left( 1 - \frac{NW^E}{Q K} \right)}{\left[ \left( 1 - \frac{NW^E}{Q K} \right) - \frac{NW^F}{Q K} \right]}
\]

Given the steady state values for the financial side of the model, I can solve for variables pertaining to the real sector.
7.3.2 Real side of the model

Given $R^E$, I can use equation (34) to compute $r^K$:

$$r^K = R^E - (1 - \delta)$$

$$MC_H = \frac{\varepsilon_H - 1}{\varepsilon_H}$$

$$\Delta_H = \Delta^*_H = 1$$

$$\frac{K}{Y} = \frac{\alpha MC}{r^K}$$

$$\frac{K}{H} = \left(\frac{K}{Y}\right)^{1-\alpha}$$

$$W = (1 - \alpha) MC \left(\frac{K}{H}\right)^{\alpha}$$

Fix $H = 0.33$, and solve for $K$:

$$K = \frac{K}{H} H$$

$$Y = \left(\frac{K}{Y}\right)^{-1} K$$

And then calculate:

$$I = \delta K$$

$$NW^E = \left(\frac{NW^E}{QK}\right) K$$

$$NW^F = \left(\frac{NW^F}{QK}\right) K$$

$$V^E = [1 - \Gamma^E (\bar{\omega}^E)] R^E Q K$$

$$V^F = [1 - \Gamma^F (\bar{\omega}^F)] R^F (Q K - NW^E)$$

Then I can back out $\gamma^E$ and $\gamma^F$:

$$\gamma^E = \frac{(NW^E - (1 - \alpha)\Omega_E \cdot MC \cdot Y_H)}{V^E}$$

$$\gamma^F = \frac{(NW^F - (1 - \alpha)\Omega_F \cdot MC \cdot Y_H)}{V^F}$$
\[ C^E = (1 - \gamma^E) \left[ 1 - \Gamma^E (\bar{\omega}^E) \right] R^E Q K \]
\[ C^F = (1 - \gamma^F) \left[ 1 - \Gamma^F (\bar{\omega}^F) \right] R^F (Q K - NW^E) \]

Steady state monitoring costs incurred by entrepeneurs and FIs are given by:
\[ M = \mu^E G (\bar{\omega}^E) R^E Q K + \mu^F G (\bar{\omega}^F) R^F (Q K - NW^E) \]

Now use the goods market clearing conditions for the Home and Foreign countries to solve for the steady state Home and Foreign consumption:
\[
C_F = \frac{1}{\left[ (1 - \gamma^*) - \frac{(1-\gamma)^*}{\gamma} \right]} \left\{ \begin{array}{l} Y_F - (1 - \gamma^*) (I_F + C_F^E + C_F^F) - (1 - \gamma)^* \frac{1}{n} (I_H + C_H^E + C_H^F) - G_F - M_F \nonumber \\ - \frac{n}{(1-n)} \gamma \left[ Y_H - G_H - \gamma (I_H + C_H^E + C_H^F) - M_H - \gamma^* \frac{(1-n)}{n} (I_F + C_F^E + C_F^F) \right] \end{array} \right\}
\]

\[ C_H = \frac{1}{\gamma} \left[ Y_H - \gamma I_H - G_H - \gamma^* \frac{(1-n)}{n} (C_F + I_F + C_F^E + C_F^F) - \gamma C_H^E - M_H - \gamma C_H^F \right] \]

Now I can solve for the remaining variables:
\[ K^p = MC \frac{Y_H}{(1 - \beta \theta_H)} \]
\[ F^p = \frac{Y_H}{(1 - \beta \theta_H)} \]
\[ \tilde{P}_H = \frac{\varepsilon_H}{\varepsilon_H - 1} \frac{K^p}{F^p} \]
\[ \mu = \frac{1}{C_H(1-h)} \]
\[ \chi_H = \frac{\varepsilon_w - 1}{\varepsilon_w} \frac{\mu W}{H^p P} \]
\[ K^w = \frac{\varepsilon_w - 1}{\varepsilon_w} \frac{HW \mu}{(1 - \beta \theta_w)} \]
\[ F^w = \frac{\chi_H H^p}{(1 - \beta \theta_w)} \]
\[ \tilde{W} = W \]
7.4 Figures

Figure 1a: Foreign interest rate shock: Floating Exchange Rate
Figure 1b: Foreign interest rate shock: Fixed Exchange Rate
Figure 2a: Foreign productivity shock: Floating Exchange Rate
Figure 2b: Foreign productivity shock: Fixed Exchange Rate
Figure 3a: Foreign $\sigma_B$ shock: Floating Exchange Rate
Figure 3b: Foreign $\sigma_B$ shock: Fixed Exchange Rate
Figure 4a: Foreign $\sigma_E$ shock: Floating Exchange Rate
Figure 4b: Foreign $\sigma_E$ shock: Fixed Exchange Rate
Figure 5a: Nominal Contract: Foreign interest rate shock
Flexible exchange rate

Fixed exchange rate
Figure 5b: Nominal Contract: ForeignProductivity shock
Flexible exchange rate

Fixed exchange rate
Figure 5c: Nominal Contract: Foreign $\sigma_B$ shock

Flexible exchange rate

Fixed exchange rate
Figure 5d: Nominal Contract: Foreign $\sigma_E$ shock
Flexible exchange rate

Fixed exchange rate